STEADY STREAMING GENERATED BY A MONOCHROMATIC SURFACE WAVE CLOSE TO A ROUGH BOTTOM

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INTRODUCTION

The velocity field within the bottom boundary layer generated by a propagating surface wave is quite important because it allows to determine the bottom shear stress, which is relevant from a practical point of view since, for example, it allows to verify whether the sediments, which make up the bottom, are set into motion and transported by the flow. If the surface wave is of small amplitude and the sea bottom is assumed to be smooth, the flow in the bottom boundary layer was determined by Stokes (1851) and Longuet-Higgins (1953), who showed that weak nonlinear effects generate a steady streaming.

Laboratory experiments by Sleath (1988) showed that a laminar flow over a rough bottom made up of medium sand can be observed up to values of the Reynolds number equal to about 250. Therefore, real situations exist such that the flow regime is laminar and the bottom can be considered hydrodynamically rough. Recently, Blondeaux and Vittori (2023) proposed a simplified model for the oscillating boundary layer close to a rough bottom and derived the solution of the linearized problem. In the present contribution their model is used to solve the problem assuming weak non-linear effects, thus extending the solution by Longuet-Higgins (1953) to the more realistic case of rough bottom.

THE MODEL

Let us consider a two-dimensional surface gravity wave of small but finite amplitude a, angular frequency $\omega = 2\,\pi/T$ (T is the period of the wave), which propagates along the x direction over a constant water depth h_0 (z denotes the upward pointing vertical coordinate). The flow field generated by the propagation of the wave can be split into an irrotational, inviscid part and a viscous part confined in the boundary layers at the free surface and at the bottom. In the present contribution we focus our attention on the bottom boundary layer. Even when the flow can be considered laminar, the flow field close to the bottom is characterized by fluctuations that are due to the small eddies shed by the sand grains of size d, resting on the horizontal bottom. The velocity fluctuations u' and w' are random because of the irregular shape of the sediment grains and of the randomness of the position of the grains. The averaged horizontal velocity component is obtained by using a two-dimensional equation, which is derived from Navier Stokes equation after averaging in the horizontal direction. Beside the plane averaged velocity components $(\hat{u}, \hat{w})$ (hat denotes plane averaged values), the derived momentum equation in the x- direction contains a cross-correlation term $-\rho\widehat{u'w'}$ that is modeled introducing an eddy viscosity that depends on the distance from the wall:

$$-\rho\widehat{u'w'} = \mu_T(z)\frac{\partial \hat{u}}{\partial z}$$

$$\text{with} \quad \mu_T(z) = \mu\, c_1 \left(\frac{z}{d}\right)^n exp\left(-c_2\frac{z}{d}\right) \qquad (1)$$

that describes the random mixing, confined close to the bottom, due to the presence of the sediment grains.

Since the flow around each sediment grain is not resolved, at the bottom a partial-slip boundary condition is forced:

$$\mathbf{v} = s\,\frac{\partial v}{\partial n} \qquad (2)$$

where v denotes the fluid velocity tangent to the boundary, n is a coordinate normal to it, and s is a constant. The constants c_1 and c_2 appearing in (1) are determined by Blondeaux and Vittori (2003) based on the experimental results by Sleath (1970). The complex constant s depends on the roughness of the bottom and is given in Blondeaux and Vittori (2003).

By considering the order of magnitude of the terms of the model equation close to the bottom it turns out that the term describing the horizontal diffusion can be neglected, while the ratio of the order of magnitude of the convective and local acceleration terms is quantified by the parameter

$$\epsilon = \frac{a}{L\ sinh(kh_0)}, \qquad (2)$$

with L and k denoting the wavelength and wavenumber, respectively. The solution is then expanded as power series of the small parameter ϵ and the problems obtained at the leading order and at order ϵ are solved.

At the leading order, the oscillating boundary layer over a rough wall discussed in Blondeaux and Vittori (2023), is recovered.

At order ϵ the solution shows a steady component $\overline{\hat{u}_1}$, that is the subject of the present contribution because of its practical relevance, outlined in the introduction.

RESULTS

The horizontal steady velocity component depends on the roughness parameter $z_r = 2.5\,d$ and on the value of the amplitude of the velocity oscillations outside the bottom boundary layer U_0. Figure 1 shows the vertical profile of $\bar{\hat{u}}/U_0$ for different values of z_r. For small values of the roughness, the horizontal velocity points onshore, it assumes negligible values at the bottom, and it tends to a constant value close to $3/4$ moving away from the bottom. The maximum value of $\bar{\hat{u}}$ is found for z_r equal to about $2.9\,\delta$, where $\delta = \sqrt{2\mu/(\rho\omega)}$ gives the order of magnitude of the thickness of the bottom boundary layer. Indeed, for

$z_r = 0.0125\,\delta$ the velocity profile is practically coincident with that determined by Longuet-Higgins (1953) for a smooth bottom.

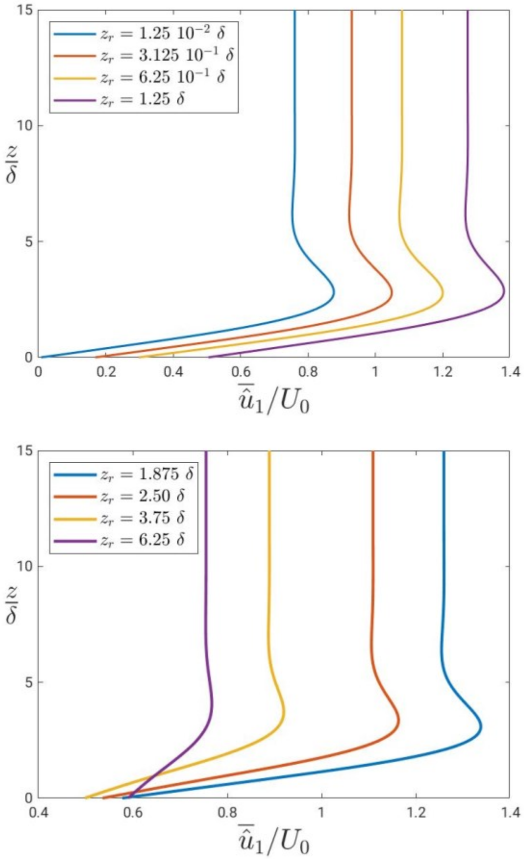

Figure 1 – Steady velocity component generated by a surface gravity wave propagating over a rough bed for different values of the dimensionless roughness.

As the bottom roughness is increased, the steady velocity component intensifies up to values of z_r equal to about $1.25\,\delta$.

Moreover, the slip velocity increases as z_r is increased from $0.0125\,\delta$ to $1.25\,\delta$. This fact appears to be the main cause of the increase of the steady velocity component.

Then, as z_r is further increased, the steady streaming decreases even though the slip velocity at the bottom keeps almost constant. Moreover, the vertical coordinate of the maximum of $\bar{\hat{u}}$ moves away from the bottom as z_r exceeds $1.875\,\delta$. For the roughness sizes presently investigated, which range from that of fine silt to that of very fine gravel, this maximum keeps at a height smaller than 5δ. It is worth pointing out that the steady velocity component ($\bar{\hat{u}}$) always points towards the shore. For larger values of the roughness, as those characteristics of small scale bedforms (ripples), previous investigations suggest that the direction of the steady streaming might reverse (see Vittori (2013) and Sishah (2022)). However, since the oscillatory flow over ripples is characterized by the formation of coherent vortices (e.g., vortex pairs for two-dimensional vortex ripples), it appears questionable to apply the present model assimilating small scale bedforms to large random roughness.

CONCLUSIONS

The steady streaming generated by nonlinear effects in the bottom boundary layer close to a plane rough bottom depends on the size of the sediment at the bottom. Present model shows that the steady velocity component points always in the direction of wave propagation and that for values of the grain size d up to $0.05\,\delta$, with δ being the thickness of the viscous bottom boundary layer, the velocity profile is practically coincident with that predicted by Longuet-Higgins in case of a smooth bottom. If the grain size is increased, the steady velocity component becomes larger and reaches a maximum value that is approximately 70% larger than that predicted by Longuet-Higgins. The maximum of the steady velocity component is attained for $d = 0.6\delta$. A further increase of d leads to a decrease of the steady velocity component that, however, keeps always larger than that predicted for a smooth bottom.

REFERENCES

Blondeaux, Vittori (2023): Wave boundary layer over a rough bottom at moderate Reynolds numbers, Journal of of Hydraulic Engineering, vol. 149.
Longuet-Higgins(1953): Mass transport in water waves, Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences vol. 245.
Sleath (1970): Velocity measurements close to the bed in a wave tank, Journal of Fluid Mechanics vol. 42.
F. Sleath (1988): Transition in oscillatory flow over rough beds, Journal of waterway, port, coastal, and ocean engineering vol. 114,
Stokes (1851): On the effect of the internal friction of fluids on the motion of pendulums, Pitt Press Cambridge, vol. 9.
Vittori, Blondeaux (2013): Steady streaming induced by sea waves over rippled and rough beds, Continental Shelf Research, vol. 65.
Sishah, Vittori (2022): Ranse modeling of the oscillatory flow over two-dimensional rigid ripples, Journal of Geophysical Research: Oceans vol. 127.