EVOLUTION OF NONLINEAR WAVES OF MULTIPLE SYSTEMS IN TWO-DIMENSIONAL DIRECTIONAL CROSSING SEAS ON A SLOPE

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INTRODUCTION

As water waves approach coastal areas, various factors and mechanisms, including dispersion, refraction, spatial inhomogeneity, and directional effects, lead to wave deformation. In the study of the rogue/ freak waves, the occurrence of maximum wave height in nonlinear modulated wave trains is related to the four-wave interactions. Lyu et al. (2021, 2023) discussed the roles played by local bathymetry effects and directional effects in the nonlinear evolution process of a random wavefield and quantitatively analyzed their contribution to the occurrence probability of freak waves. However, there is still much to explore in the realm of wave propagation in two dimensions. From observation data and numerical study in general sea states and extreme seas such as tropical cyclones, crossing two or more distinct spectral components will also lead to freak waves (Semedo et al., 2011; Mori, 2012).

In this research, we develop a numerical model to describe the crossing of multiple systems within a random directional wavefield. Building the model upon the extension of Lyu et al. (2023), we simulate the evolution of crossing waves, which are composed of two symmetrical oblique incident wave trains over an uneven bottom.

THEORETICAL MODEL

For a two-dimensional (2D) flow field, we assume the flow is irrotational, inviscid, and incompressible with a free water surface η and finite water depth h. A coordinate system (x, y, z) is defined at still water level, and the evolution of phase-independent wave amplitude A along x coordinate can be given by a modified Nonlinear Schrödinger type equation referring to Lyu et al. (2023):

$$\mathrm{i}\mu A + \mathrm{i}\frac{\partial A}{\partial x} + \lambda\frac{\partial^2 A}{\partial t^2} + \gamma\frac{\partial^2 A}{\partial y^2} = \upsilon|A|^2 A, \qquad (1)$$

where coefficients $\mu, \lambda, \gamma, \upsilon$ are functions of wave number k, angular frequency ω_0 and h. In this model, water depth h is assumed to mildly varies on one dimension, which is corresponding to the principal wave propagation direction. Nevertheless, a small oblique angle between them is allowed in the numerical process if $k_x \approx k$, where k_x is the component of k on x axis.

At the initial condition at $x = x_0$, we give the random Fourier amplitude $\hat{A}$ as the Gaussian distribution:

$$\hat{A}(\omega, x_0, \theta) = \frac{\varepsilon}{2\pi\sigma_\omega\sigma_\theta} \times e^{-\frac{1}{2}\left[\left(\frac{\omega-\omega_0}{\sigma_\omega}\right)^2+\left(\frac{\theta-\theta_m}{\sigma_\theta}\right)^2\right]+\mathrm{i}\psi}, \qquad (2)$$

where ε represents wave steepness, $\sigma_\theta, \sigma_\omega$ are spectral bandwidth of θ, ω, respectively. ψ gives a random phase. θ represents the direction of different wave components. θ_m gives the initial wave principal direction. For a group of random wave trains generated from Eq. (2) with the principal direction θ_m, the surface elevation η_m can be expressed as:

$$\begin{aligned}\eta_m = {} & \varepsilon\mathrm{Re}\left[\frac{1}{2}\bar{A}\exp\big(\mathrm{i}(kx + k_m y - \omega_0 t)\big)\right] \\ & + \varepsilon^2\mathrm{Re}\left[\frac{k\cosh kh}{8\sinh^3 kh}(2\cosh^2 kh \right. \\ & \left. + 1)\bar{A}^2\exp\big(2\mathrm{i}(kx + k_m y - \omega_0 t)\big)\right], \qquad (3)\end{aligned}$$

where $k_m = k\tan\theta_m$. Assuming periodic boundary conditions in time, we integrate Eq. (3) from offshore to onshore and give the evolution of surface elevation in a 2D space-time (2D+T) form.

Based on the above result, we further develop this model to consider the crossing sea case. Since the effect of θ_m does not reflect in the evolution function Eq. (1), we can calculate the η_m from different θ_m as wave components from different sources independently, and the final surface elevation η is their linear superposition:

$$\eta = \sum \eta_m . \qquad (4)$$

NUMERICAL RESULT

First, we simply consider the case that two symmetrical oblique incident wave trains cross, where $m = 1{,}2$, $\theta_1 = -\theta_2$, $\eta = \eta_1 + \eta_2$. In Figure 1, we give the transient surface elevation η, η_1 and η_2 of one sample on a $45\times30L_0$ space (L_0: wavelength) over a flat bottom. The initial condition is given by $\sigma_\theta = 0.4$, $\sigma_\omega = 0.35\omega_0$, wave period $T = 2.5s$, $kh = 3.68$, $\tan\theta_1 = 0.1$.

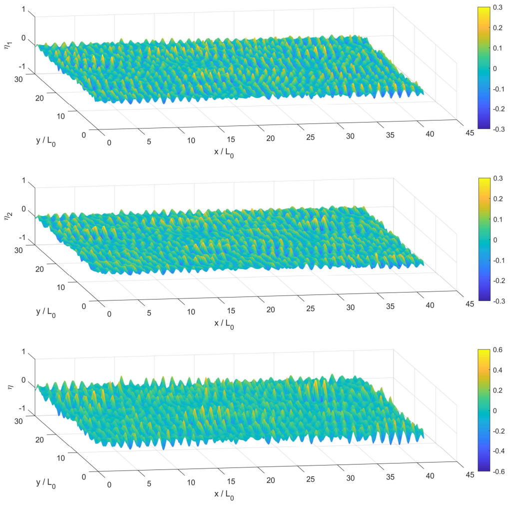

Figure 1 - A sample of 2D surface elevation η and its components η_1, η_2 at $t = 20T$ in constant depth. (top: η_1, middle: η_2, bottom: η)

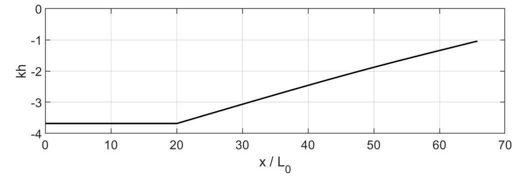

Figure 2 – Mildly changing bottom topography shown as water depth kh with slope $\gamma = 0.01$

For a single irregular wave sample, the dispersion and irregular shape make its characteristics in the evolution process insignificant. Therefore, we conduct a Monte Carlo simulation to investigate the wave height distribution and the evolution of the fourth-order moment kurtosis μ_4 and the third-order moment skewness μ_3, which are considered to be related to the occurrence of freak waves (Mori & Janssen, 2006). Furthermore, we are also interested in the role spatial inhomogeneity plays in the evolution of crossing seas. Therefore, we set the bottom topography mildly decreasing as in Figure 2. The wave trains are incident from the left.

Figures 3 to 5 show the averaged value of μ_4 and μ_3 and expected maximum wave height $H_{\max}$ of time series of η at each grid in space. The result come from the same initial condition as Figure 1 and $\tan\theta_1$ varies from 0 to 0.12. When $\tan\theta_1 = 0$, $\eta_1 = \eta_2$ so η is actually the result from one system that wave is normally incident, which leads to an obvious difference between $\tan\theta_1 = 0$ case and other result. μ_4 from single system are significantly lower than those two systems, while μ_3 and $H_{\max}$ from single system show inverse results. As the $\tan\theta_1$ increase from 0.02, μ_4 correspondingly increases until $\tan\theta_1 = 0.06$ then starts to decrease. In μ_3 and $H_{\max}$, we can see a completely opposite process in that they decrease with the rising $\tan\theta_1$ and turn to increase after $\tan\theta_1 = 0.06$. These results illustrate an interesting process: when the crossing angle is small enough, the wave surface is closer to the results of a single spectral system; as the angle increases, the crossing waves propagating in two symmetric directions eventually reflect a reduction in nonlinear effects in the final result. It should be noted that $H_{\max}$ cannot reflect the wave height distribution and the probability of freak waves; further calculations and discussions are needed for conclusions regarding them. Additionally, the asymmetry in the distribution that μ_3 reflects is shown as positive and negative values in single and double systems, respectively. The cause of this phenomenon is not yet clear.

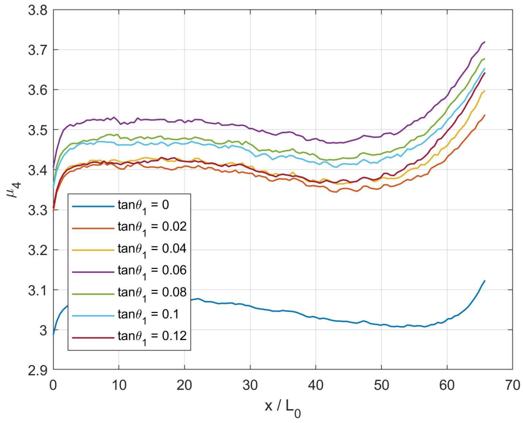

Figure 3 – Averaged kurtosis μ_4 from Monte Carlo simulation from different θ_1

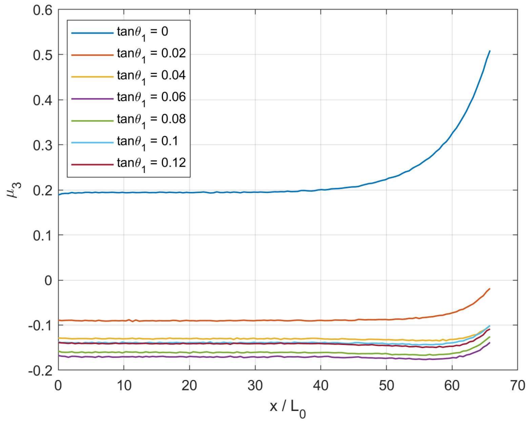

Figure 4 - Averaged skewness μ_3 from Monte Carlo simulation from different θ_1

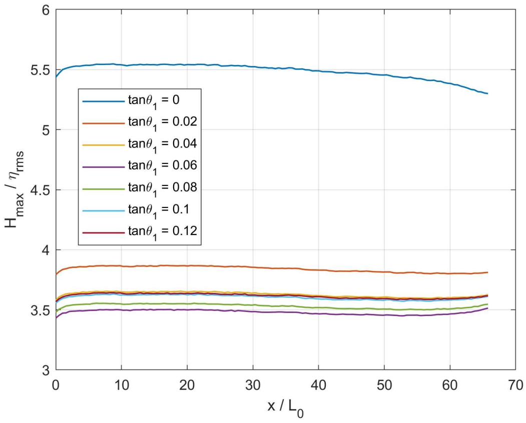

Figure 5 – Averaged maximum wave height from Monte Carlo simulation from different θ_1

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