MODELLING NONLINEAR SHORT-WAVE GROUPS AND INFRAGRAVITY WAVES IN PHASE-RESOLVED SIMULATIONS

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INTRODUCTION

Infragravity waves are long waves with typical period greater than 30 seconds. They are induced by the grouping of short waves, and they are bounded to the groups in open sea. After they arrive at the nearshore region, they can be released and become free waves. The generation of the free long waves can be due to abrupt change of bathymetry, wave breaking and so on. Recently, Wang & Liu (2023) proposed a set of improved formulas and a new coupling approach for modelling the wave groups induced infragravity waves in SCHISM-WWMIII, in which the infragravity waves were simulated in phase-resolved long wave model and short-wave groups were considered in the phase-averaged spectral model. The model is shown to be capable of reproducing the release of free long waves due to the abrupt change of bathymetry. However, the shortcoming of this approach is that the short-wave groups are assumed to remain stable. Indeed, rapid energy cascade can occur between the nonlinear interactions of sidebands and carrier waves, leading to the so-called modulational instability. Hence, a phase-resolved numerical simulation tool that is capable of modelling nonlinear short-wave groups with a reasonable computational efficiency is in demand. Nevertheless, to model both long waves with the presence of short-wave groups in the phase-resolved numerical simulation is computationally expensive, as it is often required to resolve the shortest waves with an appropriate discretization scheme.

This study presents a computationally efficient numerical wave model for simulating wave groups and the underlying infragravity waves simultaneously in phase-resolved approach. The new model is based on the potential flow theory and the boundary integral equation (Fructus & Grue, 2007) with a new set of formulations and a successive approximation scheme for estimating the surface vertical velocity considering the water depth variability. For simplicity, the formulas for calculating the vertical velocity and velocity potential at sea bottom at desired order l can be given in the compact form as below $(2 \leq l \leq 7)$:

$$V_l = -\tanh(kh)\frac{k^{l-2}}{(l-1)!}\mathrm{i}\boldsymbol{k}\cdot\mathcal{F}\{\eta^{l-1}\nabla\tilde{\phi}\} + \sum_{j=1}^{l-1}\frac{k^j}{j!}\Big[-\tanh^{\langle j/2\rangle}(kh)\,\mathcal{F}\{\eta^j V_{l-j}\} + \langle\tfrac{j}{2}\rangle\,\mathrm{sech}(kh)\frac{\mathrm{i}\boldsymbol{k}}{k}\cdot\mathcal{F}\{\delta^j\nabla\tilde{\varphi}_{l-j}\}\Big] \quad (1)$$

$$\tilde{\varphi}_l = \sum_{j=1}^{l-1}\frac{k^{j-1}}{j!}\Big[-\langle\tfrac{j}{2}\rangle\, sech(kh)\,\mathcal{F}\{\eta^j V_{l-j}\} - tanh^{\langle j/2\rangle}(kh)\frac{i\boldsymbol{k}}{k}\cdot\mathcal{F}\{\delta^j\nabla\tilde{\varphi}_{l-j}\}\Big] \quad (2)$$

where η is free surface elevation, $\tilde{\phi}$ and $\tilde{\varphi}$ are velocity potential at free surface and sea bottom, respectively, δ is the height of varying bathymetry, and $\langle j/2\rangle$ is the reminder of j divided by 2. Based on the equations above, the boundary value problem can be solved, so that the unknowns η and $\tilde{\phi}$ can be updated in time domain (Wang & Ma, 2015). Owing to the usage of Fast Fourier Transform (FFT), the algorithm is efficient for simulating highly nonlinear and dispersive water waves on arbitrary depth. In contrast to the standard Boussinesq model, the present model is accurate in wave dispersion, hence it is not restrictive for applications in deep water. In comparison with the Higher-Order Spectral (HOS) model (Dommermuth & Yue, 1987), the present model is about 5 times faster (or uses 5 times less FFT operations) than the HOS model for estimating the vertical velocity of a Stoke wave at the limiting wave steepness. After comprehensive validations, the model is used here to simulate the infragravity waves induced by short-wave groups, and the release of free long waves due to the abrupt change of bathymetry.

RESULTS AND DISCUSSIONS

The short-wave groups to be simulated consist of two components:

$$\eta = a\cos(2\pi f_1 t) + a\cos(2\pi f_2 t) \quad (3)$$

where the wave amplitudes $a = 0.5$ m, frequencies $f_1 = 0.1$ Hz and $f_2 = 0.11$ Hz. Note that the wave groups may subject to modulational instability that is characterized by the Benjamin-Feir Index (BFI):

$$BFI = 2\sqrt{2}\frac{ka}{\Delta k/k} \quad (4)$$

Considering a case in water depth of 50 m, the *BFI* = 0.31 is less than 1, therefore, the wave groups remain stable.

In the simulation, the domain size is set to 512 peak wavelengths (about 80 km) and is discretised into 16384 points. The pneumatic wavemaker is adopted with an localized oscillating pressure distribution being applied on the free surface for generating the short-wave groups. The bound long waves are excited immediately after the short waves propagate away from the generation zone. A snapshot of the wave surface elevation in space after 1.67 hours of propagation is shown in Fig. 1, where a good agreement between the simulated and the theoretically predicted long waves (Longuet-Higgins & Stewart, 1962) can be observed.

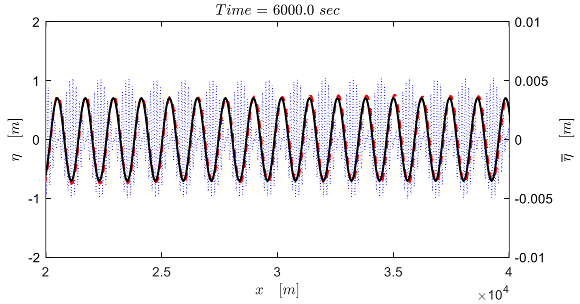

Figure 1 - A snapshot of simulated short-wave groups (left ordinate) and long waves (right ordinate). Blue: short wave groups; Red: simulated bound long waves; Black: bound waves based on Longuet-Higgins's formula of "surf beats".

Next, the release of free long waves over a linearly varying plane slope is simulated. The foot of the slope is located at x_0 = 40 km in the computational domain. The water depth before the slope is $h_0 = 31.9$ m, the length of the slope is $L = 717.8$ m, and the step height is 20 m. In addition, the simulation also considers a canyon-type bathymetry as described by:

$$h(x) = h_0\left\{\frac{3}{2} - \exp\left[-4\ln 2\left(\frac{x - x_0}{L} - \frac{1}{2}\right)^2\right]\right\} \tag{5}$$

where $h_0 = h_1 =$ 31.9 m and $L =$ 2871.3 m. After simulations, the long waves in space at different time instances are extracted and plotted in Fig. 2, in which the envelope based on the theory of Liu (1989) is also presented. The scattered free waves by the varying bathymetry feature a different wavelength and amplitude, and their superimposition with the bound waves create wave groups of different lengths and heights before and after the step. Meanwhile, their envelop agrees very well with the theory of Liu (1989), indicating that the simulations successfully captured the generation of both bound and free long waves under the forcing of short-wave groups and interactions with varying bathymetry. It can be concluded that the present model is a promising simulation tool for modelling short-wave groups and infragravity waves in phase-resolved approach.

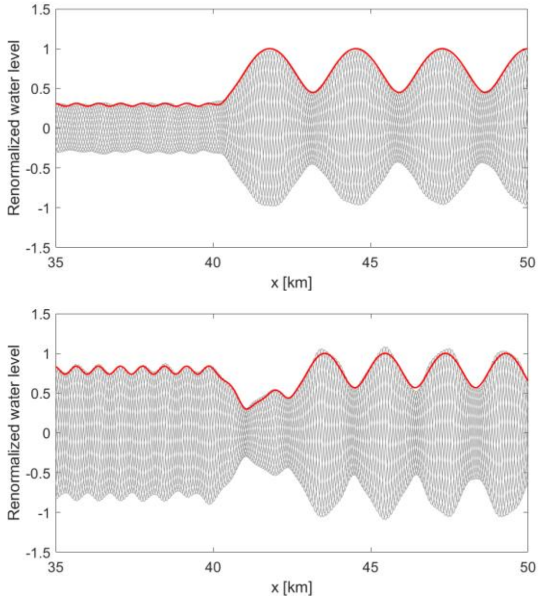

Figure 2 – The simulated surface elevation of long waves (grey) and predicted envelope based on theory of Liu (1989) (red) for the test on plane slope (upper) and canyon (lower).

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