WAVE DECAY BY RIGID VEGETATION UNDER ORTHOGONAL WAVE-CURRENT CONDITIONS

Zichen Xu, National University of Singapore, zichen_xu@u.nus.edu
Jiarui Lei, National University of Singapore, jlei@nus.edu.sg

SUMMARY
Coastal vegetation has garnered significant attention as a nature-based solution due to its role in coastal protection. While the wave energy decay by vegetation has been studied under pure-wave and co-directional wave-current conditions, little research has focused on the impact of orthogonal wave-current conditions, which represent propagating waves and longshore currents in natural coastal environments, on wave energy decay. This study introduces the first analytical model that incorporates the current-to-wave velocity ratio (β) to describe wave decay by rigid vegetation under orthogonal wave-current conditions. We conducted experiments measuring wave height reductions in both bare and vegetated basins, encompassing a range of β values from 0.4 to 1.1. Notably, the presence of an orthogonal current exerted a substantial influence on wave decay by vegetation, resulting in an enhanced effect of up to 218% and an inhibited effect of up to 74%. The analysis demonstrates that wave decay generally increases with an increasing β; however, this relationship is not exclusively determined by β. It was also observed that the damping effect is correlated with wave periods. These findings highlight the essential role of orthogonal currents in quantifying wave energy decay by coastal vegetation. Incorporating the effects of orthogonal currents will improve the accuracy of evaluating and designing nature-based solutions for coastal defenses.

ANALYTICAL MODEL
The orthogonal interaction of waves and currents will generate a combined wave-current flow, deviating from the wave propagation direction by an angle of θ. Following Dalrymple's method (Dalrymple et al., 1984), we assume that the wave energy decay is associated with the work done against drag in the direction of wave propagation. To calculate the wave decay, it is essential to project the drag force experienced by rigid structures under the combined wave-current flow onto the wave propagation direction. The current-to-wave velocity ratio, β, is introduced to represent the impact of currents.

$$\beta = \frac{u_c}{\widetilde{u_w}_{mean}} \tag{1}$$

in which u_c is the current velocity and $\widetilde{u_w}_{mean}$ is the mean theoretical magnitude of wave orbital velocity from the bed to the vegetation top.
Assuming C_D is a depth-averaged coefficient, as posited by many researchers such as Losada et al. (2016), we can derive a decay coefficient, $K_{D,wc}$, for wave-current flows. The derived expression is as follows.

$$K_{D,wc} = \frac{8}{3\pi} C_D a_v k \left(\frac{\sinh^3 kl + 3 \sinh kl}{3 \sinh kl\,(\sinh 2kh + 2kh)} \right) f(\beta) \tag{2}$$

$$f(\beta) = \left[-\frac{\beta(\beta^2+1)}{8} F\left(x\middle|-\frac{1}{\beta^2}\right) + \frac{\beta(\beta^2+2)}{8} E\left(x\middle|-\frac{1}{\beta^2}\right) \right] \Big|_{x=0}^{2\pi}, for\ \beta > 0 \tag{3}$$

in which a_v is wave amplitude, k is wave number, h water depth, l is the length of vegetation, $F\left(x\middle|-\frac{1}{\beta^2}\right)$ denotes the elliptic integral of the first kind, and $E\left(x\middle|-\frac{1}{\beta^2}\right)$ denotes the elliptic integral of the second kind.

EXPERIMENTAL SETUP
Laboratory experiments were conducted in a wave-current basin in the Hydraulic Engineering Laboratory at the National University of Singapore (NUS). The wave basin has a unique design that enables the generation of waves with an orthogonal current. A photo of the experimental setup is shown in Figure 1. In total, 76 tests were conducted with 19 different wave-current conditions, covering the current-to-wave velocity ratio from 0.43 to 1.14.

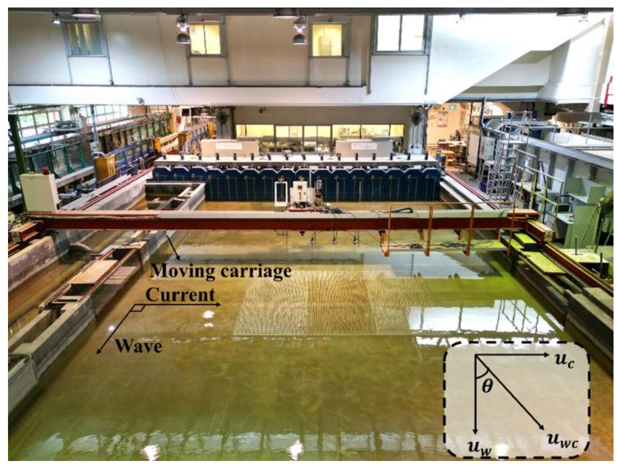

Figure 1 – Photo of the wave-current basin. An inset diagram in the bottom right corner illustrates the direction of the combined wave-current flow and defines the angle θ.

RESULTS
The traditional wave decay model by Dalrymple et al.(1984) can predict wave decay by rigid submerged vegetation well for pure-wave conditions, while its performance for orthogonal wave-current conditions needs to be improved. The performance of the new analytical model (eqn. (2) and (3)) is shown in Figure 2.
The measured $K_{D,wc}$ for each orthogonal wave-current case was normalized by the $K_{D,pw}$ with the same wave condition and compared with β, as illustrated in Figure 2. The existence of the orthogonal current can remarkably influence wave energy decay. $K_{D,wc}/K_{D,pw}$ exhibited a variation from 0.74 to 2.18. Out of the 19 cases, wave decay was enhanced by the orthogonal current in 15. The highest increase occurred in case 1, where $K_{D,wc}$

almost doubled when $\beta = 1.14$ and $T = 0.8$ s, while the decay was suppressed by 26% in case 9 when $\beta = 0.55$ and $T = 1.2$ s.

Generally, our analytical model can predict $K_{D,wc}/K_{D,pw}$ well with a RMSE of 0.32. A higher enhancement of wave decay was observed when β is larger. However, there is remaining scattering when β is relatively small, suggesting that the wave decay under orthogonal wave-current conditions may also be related to other factors, with β not being the sole determinant.

The wave period also plays an important role in impacting wave decay with the existence of an orthogonal current. $K_{D,wc}/K_{D,pw}$ initially decreased with increasing wave period and then exhibited an increasing trend from $T = 1.2$ s. For all the cases of $T = 1.2$ s, wave decay was suppressed by a range from 8% to 26%. However, the reason for this trend is not clear. A parameter that includes both velocity ratio and wave period is possible to be a better factor to explain the variation of $K_{D,wc}/K_{D,pw}$.

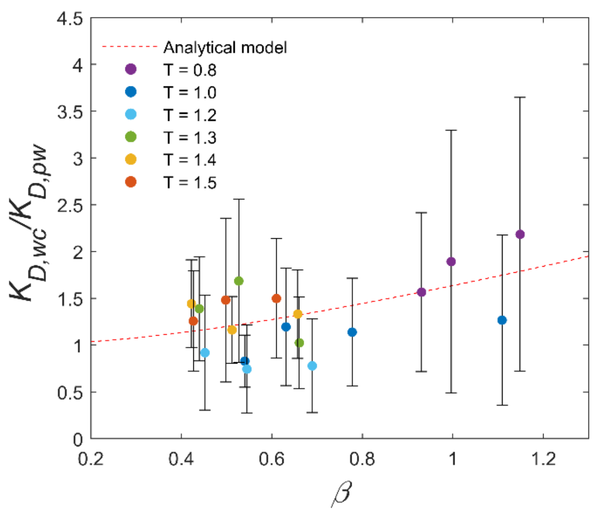

Figure 2 - Graph illustrating the relationship of $K_{D,wc}/K_{D,pw}$ with the velocity ratio, β. The red dashed line represents the analytical model as per eqn. (2) and eqn. (3).

COMPARISON WITH CO-DIRECTIONAL WAVE-CURRENT STUDIES

The behavior of wave decay by vegetation with the imposed orthogonal currents is different from that of the conditions with co-directional currents. For co-directional studies, the influences from an additional co-directional current to wave decay by vegetation have also been related to the velocity ratio β. It is strengthened when a strong current is imposed, while suppressed when a weak current exists (Schaefer & Nepf, 2022; Hu et al., 2014; Yin et al., 2020; Zhao et al., 2021). However, it is not observed evidently that wave decay decreases with increasing β when orthogonal currents are imposed with a range of β from 0.4 to 1.1. Specifically, a decreasing trend is related to the increasing wave period in this study.

The predicted C_D from a well-accepted empirical model by Hu et al. (2014) has been compared with C_D calibrated from analytical model of this study (eqn. (2) and (3)) in figure 3. By calibrating C_D from traditional K_D model, the impacts from the orthogonal currents, vegetation properties and flow characteristics are entirely attributed to C_D. However, by calibrating C_D from the new analytical model (eqn. (2) and (3)), the requirement to fit a new model for orthogonal wave-current conditions is eliminated. As illustrated in Figure 3, the existing empirical model predicts C_D well for all pure waves and orthogonal wave-current flows with an RMSE of 0.59. Thus, the introduction of β to wave decay by the vegetation model can represent the impact from orthogonal currents to C_D.

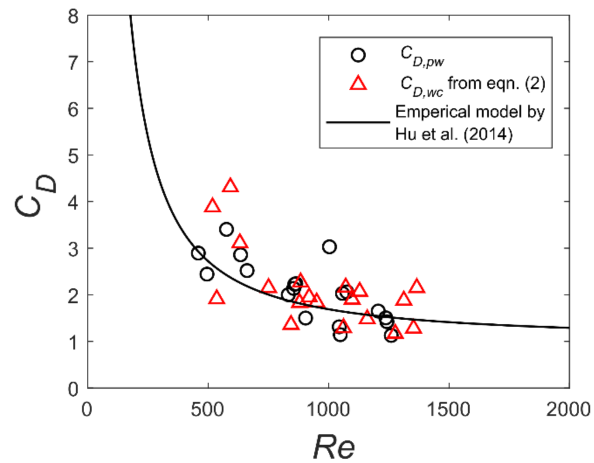

Figure 3 - A comparison between the predicted C_D from the empirical model by Hu et al. (2014) with calibrated C_D values from eqn. (2) and (3).

REFERENCES

Beth Schaefer, R., & Nepf, H. (2022). Wave damping by seagrass meadows in combined wave-current conditions. *Limnology and Oceanography*, *67*(7), 1554-1565. https://doi.org/10.1002/lno.12102

Dalrymple, R. A., Kirby, J. T., & Hwang, P. A. (1984). Wave Diffraction Due to Areas of Energy Dissipation. *Journal of Waterway, Port, Coastal, and Ocean Engineering*, *110*(1), 67-79. https://doi.org/10.1061/(ASCE)0733-950X(1984)110:1(67)

Hu, Z., Suzuki, T., Zitman, T., Uittewaal, W., & Stive, M. (2014). Laboratory study on wave dissipation by vegetation in combined current-wave flow. *Coastal Engineering*, *88*, 131-142. https://doi.org/10.1016/j.coastaleng.2014.02.009

Losada, I. J., Maza, M., & Lara, J. L. (2016). A new formulation for vegetation-induced damping under combined waves and currents. *Coastal Engineering*, *107*, 1-13. https://doi.org/10.1016/j.coastaleng.2015.09.011

Yin, Z., Wang, Y., Liu, Y., & Zou, W. (2020). Wave attenuation by rigid emergent vegetation under combined wave and current flows. *Ocean Engineering*, *213*, 107632. https://doi.org/10.1016/j.oceaneng.2020.107632

Zhao, C., Tang, J., Shen, Y., & Wang, Y. (2021). Study on wave attenuation in following and opposing currents due to rigid vegetation. *Ocean Engineering*, *236*, 109574. https://doi.org/10.1016/j.oceaneng.2021.109574