NUMERICAL MODELLING OF EXTREME WAVES ON VARIOUS BED SLOPES

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INTRODUCTION

The understanding of coastal waves is vital in the design of coastal infrastructure and marine renewables. However, the complexity of interacting physical effects including nonlinear wave amplification, wave breaking, and the influence of bathymetry presents significant challenges in modelling waves in the coastal zone. This has catalysed the development of several numerical models that seek to replicate wave propagation observed in real seas by encapsulating these effects. The present study investigates the performance of Simulating WAves till SHore (SWASH), a non-hydrostatic model based on the solution of the nonlinear shallow water equations (Zijlema et al., 2011), at modelling random waves over a range of bathymetries. The accuracy of the numerical model at capturing the evolution of wave heights is evaluated by comparing numerical predictions to equivalent sea-states generated experimentally. The broad range of sea-states and bed slopes explored in this study establishes a wide scope for the findings of this work.

METHODOLOGY

The present study explores the evolution of unidirectional random waves propagating over two bed slopes of uniform gradient 1:15 and 1:50. The 6 sea-states investigated are provided in Table 1, which are of the same peak period, T_p, but increasing offshore significant wave height, $H_{s,0}$ (and increasing offshore steepness, $S_{p,0}$). The sea-states are simulated experimentally in the Coastal Flume at the Hydrodynamics Laboratory of Imperial College London by Bellos (2023). A high resolution of 40 and 80 gauges (measuring the free surface elevation at a sampling frequency of 128 Hz) are positioned in the Coastal Flume for the 1:15 and 1:50 bed slope, respectively. The location of these gauges is such that measurements are taken within the effective water depth, $k_p d$, range outlined in Table 1, where k_p is the peak wavenumber and d is the water depth. The experimental data is generated using a JONSWAP spectrum with a peak enhancement factor of $\gamma = 2.5$ to reflect realistic storm conditions in intermediate waters (Jonathan and Taylor, 1997). 20 random realisations (seeds) of each sea-state are simulated, each corresponding to approximately 3 hours in field scale. Hence, approximately 18,000 waves are recorded for each random simulation which is far greater than most coastal studies involving in the order of 1000 waves (Suzuki et al., 2017), thus reducing statistical uncertainties.

Equivalent numerical simulations are conducted using SWASH. A unique feature of SWASH is its ability to discretise the vertical domain into a number of layers to improve the frequency dispersion of the model without an increase in the order of the dissipation term (Zijlema and Stelling, 2008). In this study, 10 vertical layers are used following the recommendations of Smit et al. (2013), with a finer resolution closer to still water level. A linear time series generated using the aforementioned JONSWAP spectrum is utilised in conjunction with a weakly reflective boundary condition at the inflow boundary. At the outflow boundary, a numerical beach is utilised alongside the Sommerfeld radiation condition to accurately capture wave run up effects on the slope. Given the negligible influence of the horizontal eddy viscosity in coastal seas, which are dominated by bottom friction, only vertical mixing is considered numerically using the $k-\varepsilon$ model of Spalding et al. (1974). The breaking parameters recommended by The SWASH team (2022) of $\alpha = 0.6$ and $\beta = 0.3$ are utilised throughout this study. In addition, to accurately capture frictional effects, the Manning formulation is adopted with $n = 0.019\, m^{-1/3} s$.

Table 1. Test cases (expressed in laboratory scale)

Sea-state	$T_p\ [s]$	$H_{s,0} [m]$	$S_{p,0} = \frac{2\pi H_{s,0}}{g T_p^2}\ [-]$	$k_p d\ [-]$
A1	1.4	0.029	0.01	1.22 - 0.47 for slope 1:15, 1.22 - 0.52 for slope 1:50.
A2		0.057	0.02	
A3		0.086	0.03	
A4		0.115	0.04	
A5		0.139	0.05	
A6		0.156	0.05	

RESULTS AND DISCUSSION

This work is concerned with modelling the evolution of wave heights across a range of uniform bed slopes numerically and comparing these predictions to experimental measurements. As such, the numerically simulated time series of the water surface elevation is extracted at the location of each experimental gauge. Subsequently, for each sea-state, a zero-crossing analysis is performed on both the numerically predicted and experimentally measured surface elevation to extract the properties of individual waves. The results for each sea-state across the 20 seeds are concatenated. As such, the wave heights may be reordered, and their probability of exceedance determined. Given the length of this concatenated dataset, probabilities of exceedance up to $Q = 10^{-4}$ are determined, thus obtaining information on extreme wave statistics.

Figure 1 shows the spatial evolution of the normalised wave height, H/H_s, corresponding to a probability of exceedance of $Q = 10^{-3}$ for bed slopes of 1:15 and 1:50 in panels (a) and (b), respectively. The results presented correspond to sea-states A1, A3 and A5 shown in blue, red and black, respectively. Wave heights corresponding to numerical predictions and experimental measurements are shown in a solid and dashed line, respectively.

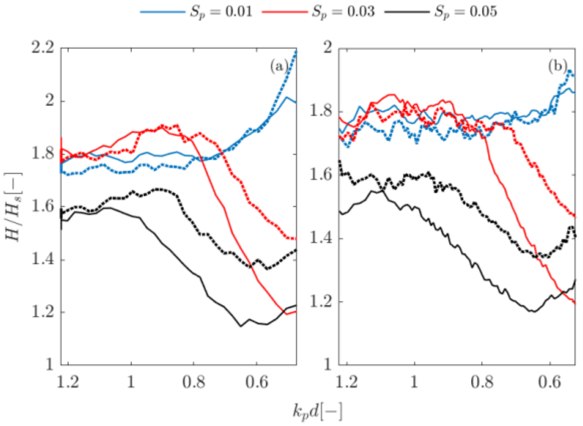

Figure 1 - Spatial evolution of the normalised wave height, H/H_s, corresponding to a probability of exceedance of $Q = 10^{-3}$ for (a) 1:15 slope and (b) 1:50 slope for sea-states A1 ($S_p = 0.01$), A3 ($S_p = 0.03$) and A5 ($S_p = 0.05$), shown in blue, red and black, respectively. Numerical and laboratory results are shown in a solid and dashed line, respectively.

The first aspect to analyse is SWASH's ability to capture the full extent of the nonlinear behaviour of very steep non-breaking waves observed experimentally. For both bathymetries considered, within the initial portion of the slope in deeper effective water depth prior to the onset of wave breaking ($1.02 \leq k_pd \leq 1.22$), the numerical model predicts experimentally observed wave heights with an accuracy of ±5%. This minor deviation is manifested as overpredictions in milder sea-states ($S_p \leq 0.03$) and underpredictions in steeper sea-states ($S_p > 0.03$). Hence, it can be stated that prior to the onset of wave breaking, the numerical model is able to accurately capture the effects associated with higher order nonlinearity with 5% accuracy even for very steep waves. In shallower water, for waves of small offshore wave steepness ($S_p = 0.01$), this level of accuracy continues to hold across the spatial domain ($0.47 \leq k_pd \leq 1.22$).

The second factor of interest is the accuracy of SWASH at modelling the breaking process in terms of predicting the breaking location and the subsequent energy dissipated. For steep seas ($S_p \geq 0.03$) experiencing significant breaking in shallower water ($k_pd < 1.02$), underpredictions in the normalised wave height are observed in this region in both steep (Figure 1 (a)) and mild (Figure 1 (b)) slopes. However, the agreement with experimental measurements is marginally better in milder slopes (Figure 1 (b)) which have a maximum discrepancy of 13% in the steepest sea ($S_p = 0.05$) compared to 16% for the corresponding sea-state propagating over a steeper slope (Figure 1 (a)). These discrepancies are driven by the treatment of breaking in the numerical model. Across both bathymetries, the numerical model is shown to break earlier (at deeper effective water depths) compared to experimental observations. This contrasts with the findings of Smit et al. (2013) which has shown the breaking parameters $\alpha = 0.6$ and $\beta = 0.3$ to accurately capture the experimental measurements of Ting and Kirby (1994). Although the breaking parameters in SWASH may be calibrated to delay or advance breaking onset, given the lack of a global breaking parameter that is applicable to all sea-states investigated, no such calibration is undertaken in the present study. In addition, it is important to consider the rate of energy dissipation following the initiation of wave breaking. Compared to experimental measurements for sea-states of moderate steepness ($S_p = 0.03$), the rate of reduction in the normalised wave height is more pronounced numerically, shown by a steeper gradient of the curves in Figure 1, across both bathymetries. Conversely, for very steep sea-states ($S_p = 0.05$), the rate of energy dissipation due to breaking numerically is comparable to experimental measurements.

CONCLUSION

The applicability of SWASH at modelling random waves propagating over a range of bathymetries is assessed through comparisons across a wide range of sea-states. The present study has highlighted the validity of SWASH at modelling extreme wave heights corresponding to a probability of exceedance of $Q = 10^{-3}$ in the shoaling zone ($1.02 \leq k_pd \leq 1.22$) for up to very steep waves ($S_p \leq 0.05$) with an accuracy of ±5% compared to experimental measurements. However, discrepancies in the surf zone ($k_pd < 1.02$) are more notable with underestimations up to 13% and 16% in milder (1:50 slope) and steeper (1:15 slope) bathymetries, respectively. These deviations are driven by the limitations in the numerical modelling of wave breaking which generally instigates an earlier breaking onset and a greater rate of subsequent energy dissipation.

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