# SPECTRAL MODELLING OF COASTAL WAVES USING QUADWAVE1D


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## INTRODUCTION

QuadWave1D is a fully dispersive and weakly nonlinear model for coastal wave prediction in one dimension. This model is based on the quadratic modelling approach (often referred to as frequency-domain models, weakly nonlinear mild-slope models, amplitude models, etc.), allowing efficient wave predication over large spatial domains. The efficiency of this approach stems from a significant modelling reduction of the original governing equations (e.g., Euler equations). Most significantly, the description of wave nonlinearity essentially collapses into a single mode coupling term determined by the quadratic interaction coefficients (QIC). As a result, it is expected that the efficiency achieved is accompanied by a decrease in model accuracy. This unfavorable consequence has motivated the development of QuadWave1D, with the aim of minimizing the error associated with the modelling reduction introduced by the quadratic approach.

The quadratic approach was initially formulated on the basis of the time-domain weakly nonlinear and weakly dispersive Boussinesq models (e.g., Freilich and Guza, 1984, Madsen and Sørensen, 1993). Without loss of efficiency, this modelling approach was developed further to allow for full linear dispersion properties (e.g., Kaihatu and Kirby, 1995) and exact second-order transfer (i.e., bound wave solutions exactly match the solutions according to the second-order Stokes theory as achieved by the model of Bredmose at al., 2005). While these developments have improved linear behavior and bound wave response in deeper waters, they also led to unfavorable modifications of the embedded nonlinear balance of propagating waves in shallower waters. Specifically, it seems that the fully dispersive formulations tend to overestimate the so-called amplitude dispersion over water depths that characterizes the coastal environment (e.g., Bredmose et al., 2005, Akrish et al., 2023). Not only does this causes the development of phase errors (Bredmose et al., 2005), it may also lead to unexpected evolution of energy spectra due to false impact of the modulational instability mechanism (Akrish et al., 2023). Additionally, modelling experience with fully dispersive formulations indicates that these model may present unfavorable evolution of coastal waves also for cases where modulational instability cannot emerge, due to triad interactions involving high frequencies (implying model dependence on the maximum considered frequency). As a result, the evolution of both the primary wave field and the secondary components (i.e., the forced higher harmonics and infragravity band) may predicted inadequately over coastal waters, despite the accurate implementation of linear wave properties.

This study proposes a new quadratic formulation – QuadWave1D, that preserves full linear dispersion, but attempts to minimize the error associated with the truncation in nonlinearity. In other words, QuadWave1D aims to optimize nonlinear model description based on the quadratic term, under the constraint of full linear dispersion.

## MODEL FORMULATION

QuadWave1D is derived based on a heuristic parametrized formulation with the constraint of full linear dispersion. The derivation proposes a parametrized QIC which are based on the QIC proposed by Bredmose et al. (2005). This parametrization aims to remove unfavorable nonlinear mechanisms (e.g., modulational instability) and to optimize the nonlinear evolution of waves in coastal waters. The QIC of QuadWave1D are formulated using three tuning parameters (i.e., α1, α3, α3). The optimal values of these parameters are found based on three examples of monochromic wave evolution in a flume of constant depth (i.e., E1, E2, E3). These examples were chosen such that a wide range of QIC values can be examined. The QIC generated by these examples govern the wave amplitudes predicted by QuadWave1D. Therefore, the optimal parameter values are found by minimizing the errors of the predicted amplitudes. The amplitude errors for examples E1 and E2 are determined based on results of the SWASH model (Zijlema et al., 2011) and the amplitude errors for example E2 are determined based on a laboratory experiment conducted by Chapalain et al. (1992). The resulted QIC of QuadWave1D and the sample of QIC based on which the optimal parameters were found are illustrated in Figure 1.

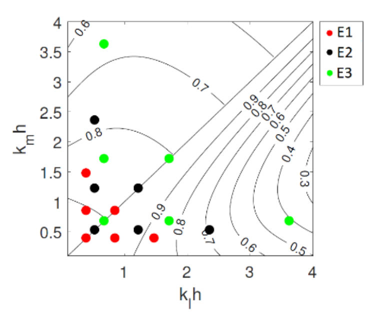


Figure 1 – The QIC of QuadWave1D normalized by the QIC according to second-order Stokes theory due to bichromatic interactions between waves with wavenumbers $k_l$ and $k_m$, where h is the water depth. The dots represent the sampled QIC values that correspond to the leading order terms govern the quadratic solution of the different considered examples.

## RESULTS

To demonstrate the prediction capabilities of QuadWave1D, the well-known experiment of monochromatic wave propagation over a submerged bar (conducted first by Beji and Battjes, 1993 and later by

Dingemans, 1994) is considered. Model results are compared with measurements and with results obtained by alternative quadratic formulations in Figure 2. Generally speaking, the comparison of the computed and measured results suggests that all the formulations capture the expected physical phenomena emerging in this example. Namely, the permanent Stokes behavior over the incoming zone, the harmonics' growth over the bar and the decoupling of the harmonics in deeper water beyond the bar where they are essentially propagate as linear waves. However, the main modelling challenge of this example is to correctly describe the development of the harmonics outside the validity range of second-order Stokes theory, i.e., over the region $10.5 \leq x \leq 14.8$. As shown in Figure 2, the fully dispersive models (i.e., the models by Kaihatu and Kirby, 1995 and Bredmose et al., 2005) describe excessive energy exchanges between the harmonics and thus inaccurately describe the development of the different amplitudes. As a result, these models mispredict the output spectrum. In addition, these models also describe rapid oscillations attributed to the sensitivity of these models to the presence of very high frequencies. On the other hand, the prediction of the Boussinesq model by Madsen and Sørensen (1993) seems much more adequate and shows better agreement with the measurements. Nevertheless, some deviations are demonstrated by this model as well, shown by the under-prediction of the higher harmonics. Finally, QuadWave1D demonstrates the most adequate results and accurately agrees with the measurements.

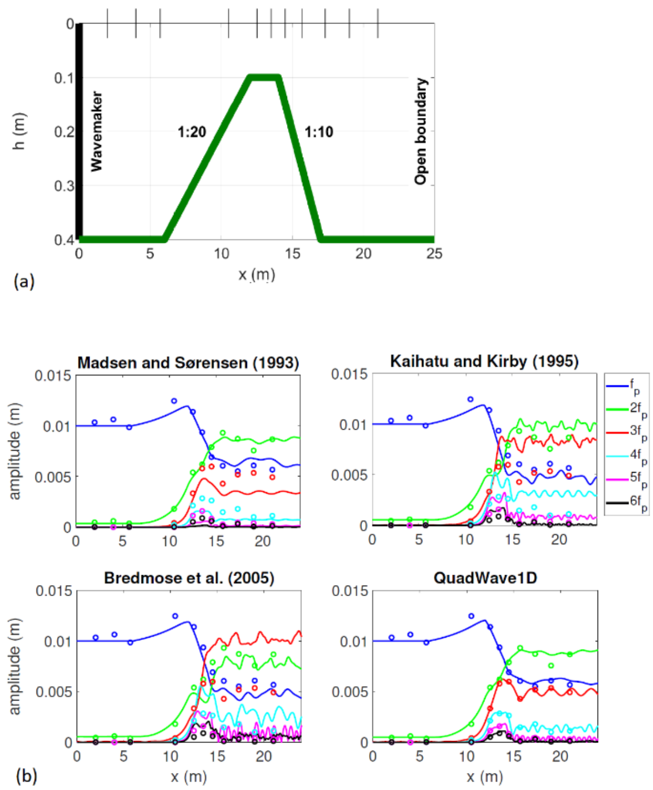


Figure 2 – (a) Schematic illustration of the experiment conducted by Dingemans (1994). The structure of the bathymetry is described by the thick green line. The thin vertical lines, plotted along the still water level (h=0), indicating measurement locations. (b) Amplitude evolution of the first six harmonics as obtained by the different quadratic formulations (lines) and the laboratory results (circles).

## CONCLUSIONS AND OUTLOOK

This study presents an attempt to find the quadratic formulation which describes most adequately nonlinear wave developments over water depths and bathymetrical structures which characterize the coastal environment. To this end, an optimization process was put forward to search for the quadratic formulation that minimizes wave evolution errors comparing to experimental data (Chapalain et al., 1992) and data obtained based on the SWASH model (Zijlema et al., 2011). The outcome is the model QuadWave1D: a fully dispersive quadratic model for coastal wave prediction in one-dimension. The validation study of QuadWave1D consists of different cases involving different incoming wave conditions and bottom topographies (presented partially here). Based on the considered examples and comparing to other quadratic formulations, it is found that QuadWave1D presents superior predictive capabilities of both the sea-swell components and the infragravity field. Finally, QuadWave1D can be potentially generalized to allow for wave breaking and directional wave propagation over two-dimensional bottom topography.

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