# AN ANALYTICAL APPROACH TO PRELIMINARY ASSESS THE FAR FIELD EVOLUTION OF PLUMES IN CONFINED WATERS

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## INTRODUCTION

Future development must be linked to environmental sustainability and better resource utilization is one of the key points of global political guidelines. Indeed, the management and improvement of water quality is one of the 17 Sustainable Development Goals of the 2030 agenda.

Therefore, exploring the evolution of released pollutants and contaminants poses a significant challenge in the fields of marine sciences and environmental hydraulics, with implications for both natural and industrial environments. Examples of potential release sources that could have a detrimental impact on the water quality include dredging operations, deep-sea mining, wastewater discharge in oceans or rivers, and unintended spills of pollutant substances. A detailed description of the plume's evolution can be achieved using numerical tools. However, there are situations where a preliminary and fast assessment is necessary. In these cases, the use of a numerical model is not cost-effective in terms of computational time. It has been shown in these cases that it is possible to employ analytical solutions (e.g., Di Risio et al., 2017). In particular, Di Risio et al. (2017) demonstrated that the plume fate resulting from dredging activities can be modeled by employing the advection-diffusion equation and applying the theory of linear dynamic systems. In particular, the solution of the advection-diffusion equation can be considered as an instantaneous response function of the dynamic system to a local, instantaneous, and unit sediment resuspension source. This evaluation allows for the assessment of concentration evolution (in both space and time) due to any source term, achieved by superposing a given number of instantaneous sources (i.e., through the application of the convolution integral). The theory of linear dynamic systems has been indeed successfully employed in modeling various other engineering applications (e.g., Pasquali et al., 2015, 2019).

Nevertheless, the solution obtained by Di Risio et al. (2017) is valid only for unbounded domains. This assumption holds for scenarios involving the open sea or when the boundaries are distant from the release area. However, it makes the model unsuitable for all other cases where the boundaries significantly influence the system, such as in small rivers, channel ports, etc.

The aim of this work is to overcome those limitations extending the model in the case of the presence of fixed and impermeable boundaries.

## THE PROPOSED MODEL FRAMEWORK

The evolution of the plume's fate in the far field can be modeled using the depth-averaged advection-diffusion equation (e.g., Fisher et al., 1979; Shao, 2015 et al., 2015, Di Risio, 2017):

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} + V\frac{\partial C}{\partial y} + D_x\frac{\partial^2 C}{\partial x^2} + D_y\frac{\partial^2 C}{\partial y^2} = q(x,y,t) - \frac{w_s}{h}\mathrm{C}$$

In the given system, *x* and *y* represent the horizontal coordinates, while *t* serves as the time variable, *C(x,y,t)* stands for the depth-integrated sediment concentration, *U* and *V* represent the x- and y-components of the ambient current respectively, $D_x$ and $D_y$ are the turbulent diffusion coefficients, while $w_s$ denotes the settling velocity, *h* represents the water depth, and *q(x,y,t)* refers to the source term. It's worth noting that the settling velocity $w_s$ is applicable only when the tracer is particulate; in all other cases, $w_s$ must be set to zero.

This equation can be used if the effects of vertical flow stratification are negligible and U and V are constant in space, as have also been considered as constants the turbulent diffusion coefficients, the settling velocity, and the water depth.

The solution for an instantaneous impulse localized source at the origin and for an unbounded domain (e.g., Mei, 1997), is:

$$\psi(x',y',t) = \frac{1}{4\pi t\sqrt{D_x D_y}}\exp\left[-\frac{(x'-U_0\lambda_u)^2}{4D_x t}\right]\times$$
$$\times\exp\left[-\frac{(y'-V_0\lambda_v)^2}{4D_y t}\right]\exp\left(-\frac{w_s}{h}t\right)$$

where $\lambda_\mathrm{u}(\mathrm{t}) = \int \mathrm{u}(\mathrm{t})\mathrm{dt}$, $\lambda_\mathrm{v}(\mathrm{t}) = \int \mathrm{v}(\mathrm{t})\mathrm{dt}$ and the cartesian reference frame $(x',y')$ is centered at the source location.

However, considering a source term occurring in a finite domain and the position of the source not coincident with the origin the source term becomes $q(x,y,t) = q_{imp}(x,y)\,\delta(t)$ with

$$q_{imp}(x,y) = \frac{1}{\Delta x\Delta y}[H(x+\Delta x/2) - H(x-\Delta x/2)]\times$$
$$\times[H(y+\Delta y/2) - H(x-\Delta y/2)]$$

where $H(x)$ is the Heaviside step function. Relying on the theory of linear dynamic systems the evolution of the plume's concentration can be obtained using the convolution between $q_{imp}(\xi,\varepsilon)$ and $\psi(x-\xi,y-\varepsilon,t)$ where $\psi$ can be viewed as the unit response function and the integral cannot be further manipulate (Di Risio et al, 2008).

To extend that solution to a confined domain a possible strategy is to apply the *image method* (e.g. Fisher, 1979), a well-known mathematical tool applied to many problems in mathematical physics.

The method involves imposing a zero-mass flow at the walls (see Figure 1). For this condition to be satisfied, it is necessary to add imaginary sources at y = $2L_1$ and y = $-2L_2$. The existence of these two imaginary sources resulted in a

positive gradient on $L_2$ and $L_1$ respectively. Therefore, to drop out that effect other imaginary sources are required (at y = $4L_1$, $6L_1$, $8L_1$ and y = $-2L_2$, $-6L_2$, $-8L_2$) inducing a recursive process. The number of imaginary sources to be reproduced in practice is correlated with the minimization of the positive gradients).
In this work, the case of a source surrounded by two fixed and impermeable boundaries is considered (see Figure 1).

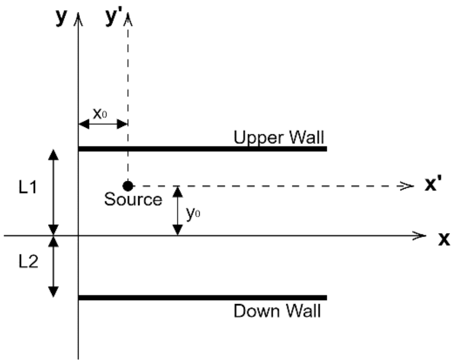


Figure 1 - Sketch of the proposed case study.

In that case, the *method of images* application reads as:

$$\Phi(x', y', t, x_0, y_0, \Delta x, \Delta y, \Delta t) =$$
$$= \frac{1}{\Delta t}\int_0^{\Delta t} [\eta(x', y', t-\tau, x_0, y_0, \Delta x, \Delta y) +$$
$$+\sum_{i=1}^{\infty} \eta(x', y' - 2jL_1 - 2jL_2 + 2L_2, t - -\tau, x_0, y_0, \Delta x, \Delta y)$$
$$+\sum_{i=1}^{\infty} \eta(x', y' - 2jL_1 - 2jL_2, t-\tau, x_0, y_0, \Delta x, \Delta y)$$
$$+\sum_{i=1}^{\infty} \eta(x', y' + 2jL_2 + 2jL_1 - 2L_1, t-\tau, x_0, y_0, \Delta x, \Delta y)$$
$$+\sum_{i=1}^{\infty} \eta(x', y' + 2jL_1 + 2jL_2, t-\tau, x_0, y_0, \Delta x, \Delta y)\, d\tau$$

with

$$\eta(x', y', t, x_0, y_0, \Delta x, \Delta y) = \frac{1}{4\Delta x \Delta y}\times$$
$$\times \left\{erf\left[\frac{x' - x_0 + \Delta x/2 - U_0\lambda_u}{\sqrt{4D_x t}}\right] - \right.$$
$$\left. -\mathrm{erf}\left[\frac{x' - x_0 - \Delta x/2 - U_0\lambda_u}{\sqrt{4D_x t}}\right]\right\}$$
$$-\left\{erf\left[\frac{y' - y_0 + \Delta y/2}{\sqrt{4D_y t}}\right] - \right.$$
$$\left. -\mathrm{erf}\left[\frac{y' - y_0 - \Delta y/2}{\sqrt{4D_y t}}\right]\right\}\times$$
$$\times \exp\left[-\frac{w_s}{h}t\right]$$

The evolution of the concentration can be found as the superposition of a series of source functions with constant Δt as:

$$C(x, y, t) = \sum_{j}^{I(t)\le I_0} q_j^* \Phi(x, y, t - j\Delta t, x_{0j}, y_{0j}, \Delta x, \Delta y, \Delta t)$$

Where $x_j$ and $y_j$ represent the average locations of the resuspension impulse, $t_j$ denotes the time at which the j-th resuspension impulse occurs, $I_0$ stands for the total number of resuspension impulses, and I(t) indicates the total number of resuspension impulses that have occurred up to time t.

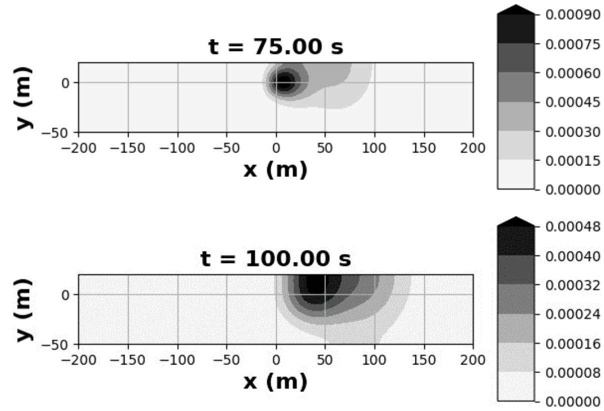


Figure 2 - Snapshots of the evolution of the concentration in the considered configuration (Δx = 1m, Δy = 1m, $w_s$=0, $D_x$ = 5 $m^2$/s, $D_y$ = 5 $m^2$/s, U = $U_0$=1 m/s, V=$V_0$=0 m/s, L1 = 20 m, L2 = 50 m, x0 = 0 m, y0 = 0 m).

Figure 2 shows a snapshot of the numerical implementation of the obtained solution in the analyzed case. The comprehensive findings of the research will be presented at the conference.

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