

# VARIABLE BATHYMETRY EFFECTS ON THE RESPONSE OF U-OWC WAVE ENERGY CONVERTERS

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## INTRODUCTION

U-Oscillating Water Columns are wave energy harvesters commonly embedded in port infrastructures. They allow transforming a passive infrastructure (the vertical breakwater) into an active structure serving simultaneously the purpose of creating a sheltered basin and harvesting the incident wave energy for producing electrical energy.

This technology has been tested through small scale models and employed in relevant full-scale applications (Arena et al., 2013). From a modelling perspective, the system has been investigated through a combination of numerical and analytical tools. The former have been used for conducting the time domain integration of the nonlinear integro-differential equation governing the motion of the system. The latter have been used for determining the associated hydrodynamic parameters. That is, added mass and radiation damping (Malara and Arena, 2019).

A critical limitation of current models is the assumption of constant water depth. Specifically, the analytical and numerical models available in the open literature consider U-OWCs installed in constant intermediate water depths, which are the typical working conditions of vertical breakwaters. This fact limits the current knowledge of the U-OWC behavior to a quite specific condition, which may not reflect the realistic working condition of the converter.

For overcoming this limitation, this article investigates the effect of bathymetry variability through the combination of a U-OWC dynamic model with a Boundary Element Method (BEM) - based solver accounting for irregular bathymetry. The BEM model used in this work has been applied to classical Oscillating Water Column systems (Belibassakis et al., 2020), which are commonly analyzed in the framework of linear water wave theory (Mei et al., 2005). Herein, its utilization with U-OWCs is pursued in combination with a nonlinear dynamic model accounting for the real flow phenomena (specifically, head losses) and large water column oscillations' effects.

## MATHEMATICAL MODELS

### U-OWC GOVERNING EQUATIONS

The U-OWC response is determined via the unsteady Bernoulli equation (applied to the water column) and the mass conservation principle (applied to the air chamber). Such a model has the disadvantage of describing the water column free surface as a rigid moving surface, but it allows accounting for large water column fluctuations and for minor and major head losses. Following the approach described by Scialò et al. (2022), water column oscillations are described by the unsteady Bernoulli equation,

$$M(x)\ddot{x} + C(x, \dot{x})\dot{x} + x + \frac{p_c - p_{atm}}{\rho g} + \int_{-\infty}^t K(t - \tau)\dot{x}(\tau)d\tau = \frac{\Delta p^{(D)}}{\rho g}, \quad (1)$$

where  $\rho$ ,  $g$  and  $p_{atm}$  are constants denoting water density, acceleration due to gravity and atmospheric pressure;  $x$  and  $p_c$  denote the water column oscillation measured from the mean water level (positive upwards) and the air chamber pressure, respectively;

$$M(x) = \frac{1+C_{in}}{g} \left( \frac{b_2}{b_1} l_i + l_i + h_o + x \right) + A(\infty), \quad (2)$$

$$C(x, \dot{x}) = \frac{C_{dg}}{2g} \left[ \frac{l_i}{R_{h1}} \left( \frac{b_2}{b_1} \right)^2 + \frac{l_i + h_o + x}{R_{h2}} \right] |\dot{x}| + \left[ 1 - \left( \frac{b_2}{b_1} \right)^2 \right] \frac{\dot{x}}{2g}, \quad (3)$$

$K(t)$  is the retardation function; and  $\Delta p^{(D)}$  is the wave pressure at the U-OWC inlet calculated in a diffracted wave field. The symbols shown in eq. (2) - (3) are shown in Fig. 1,  $R_{h1}$  and  $R_{h2}$  are the hydraulic radii of the vertical duct and of the inner chamber, while  $A(\infty)$  is the infinite frequency added mass, and  $C_{in}$  and  $C_{dg}$  are empirical coefficients used to account for the head losses in the U-OWC. Eq. (1) is coupled with the mass conservation equation applied to the air chamber volume under the assumption that the air thermodynamic process is isentropic. That is,

$$b_2 b_3 (h_c - x) \dot{p}_c - \gamma b_2 b_3 p_c \dot{x} + \gamma p_c \left( \frac{p_{atm}}{p_c} \right)^{\frac{1}{\gamma}} \frac{\dot{m}_{turb}}{\rho_{atm}} = 0, \quad (4)$$

$\gamma$  and  $\rho_{atm}$  being specific heat ratio and air density in atmosphere; and  $\dot{m}_{turb}$  is the air mass flow rate through the turbine according to the equation,

$$\dot{m}_{turb} = \frac{\Lambda D}{\Omega} (p_c - p_{atm}), \quad (5)$$

which is applicable to the case of Wells turbine having characteristic parameter  $\Lambda$ , outer diameter  $D$  and rotational speed  $\Omega$ .

### BEM FOR VARIABLE BATHYMETRY REGIONS

Assuming normally incident waves 2D diffraction and radiation problems are defined in the semi-infinite domain  $D = \{x_\alpha < x, -h(x) < z < 0\}$  (see Fig. 2), starting from  $x_\alpha = 0$ , where the U-OWC device is located. In order to study variable bathymetry effects on its performance, a low-order BEM is developed. Assuming time-harmonic dependence in the form  $\exp(-i\omega t)$ , complex wave potentials are represented by the equation,

$$\varphi(x) = \int_{\partial D} \sigma(x_0) G(x|x_0) dS(x_0), \quad (6)$$

where  $G(x|x_0) = \ln \|x - x_0\| / 2\pi$  denotes the fundamental solution of the Laplace equation, satisfying the following conditions on the various parts of the boundary:

$$\partial_n \varphi - \mu \varphi = 0, x \in FS, \mu = \omega^2 / g \quad (7)$$

$$\partial_n \varphi = 0, x \in O_1 \cup O_3 \cup BS \quad (8)$$

$$\partial_n \varphi = f_1, x \in O_2 \quad (f_1 = 0 \text{ diffraction}, f_1 = 1 \text{ radiation}) \quad (9)$$

$$\partial_n \varphi - ik\varphi = f_2, x \in RB, \quad (10)$$

where  $f_2 = 0$  for the radiation problem, and

$$f_2(x) = -2ik_2 \exp(-ik_2 x) \frac{\cosh[k_2(h_2+z)]}{\cosh(k_2 h_2)}, \quad (11)$$

for the diffraction problem,  $k_2 = k(\omega, h_2)$  being obtained as the root of the dispersion relation of water waves at the

depth  $h_2$  of the radiation boundary.

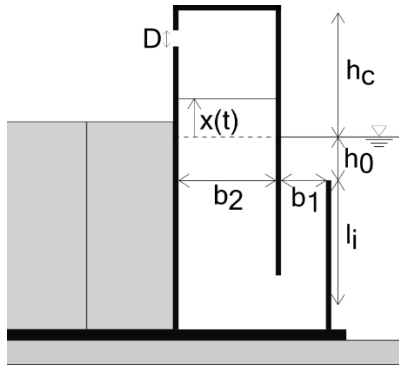


Figure 1 -U-OWC schema and geometrical parameters.

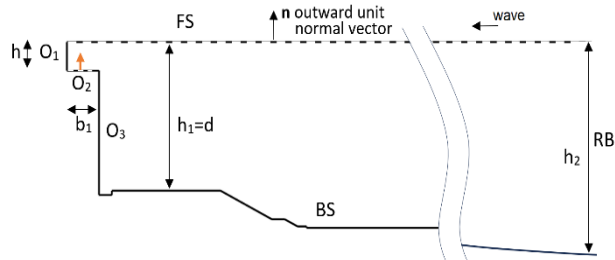


Figure 2 - Variable bathymetry domain.

The boundary is discretized by a closed polygonal line, formed by distributing nodes across its length. In particular, the free surface nodes are determined by specifying a number of nodes per characteristic wavelength in the domain that is usually larger than 30-40. The z-coordinates of the free surface's nodes are equal to  $z_i = 0, i = 1, M_1 + 1$ , where  $M_1$  denotes the number of elements on the free surface. The bottom and rest parts of the boundary are accordingly discretized. The node spacing is finer on the horizontal boundaries near the corners of the domain, while it is uniform on the vertical sides. The latter is selected to match the fine spacing of the horizontal sides at the corners. Based on the above discretization the total number of elements is  $M = M_1 + M_2 + \dots + M_6$ , where the index denotes the corresponding part of the boundary numbered counterclockwise. By assuming unit source-sink singularity strengths on each element, the induced potential and velocity matrices  $\varphi_{kj}, \mathbf{U}_{kj}, k, j = 1, \dots, M$ , from each element to each collocation point is analytically calculated; see also Magkouris et al (2020). Using the latter quantities in the boundary conditions, the matrix coefficient  $A_{kj}$  of the discrete system is obtained, as explained in more detail in the sequel. Denoting by  $\sigma_j$  the strength of the source/sink of the  $j$ -element and by  $b_k$  the boundary data that applies on the control point defined as the center of the  $k$ -element, the resulting linear system is:

$$\sum_{j=1}^M A_{kj} \sigma_j = b_k, k = 1, 2, \dots, M, \quad (12)$$

After the solution is obtained, the calculated discrete singularities are used in conjunction with the representation (6) to define the diffraction/radiation potential in the domain.

## RESULTS AND DISCUSSION

The BEM model is used for calculating the diffraction forces and the hydrodynamic coefficients in constant water depth conditions (flat domain  $h=10.4\text{m}$ ) and for the bathymetry available in the location of Salerno (Italy), where a U-OWC plant has been installed recently (Arena et al., 2012). Next, the U-OWC dynamic model is used for computing system response and power output. Herein, comparison is pursued in terms of mechanical power available to the turbine by assuming that the system is exposed to orthogonal regular incident waves of unitary amplitude and varying frequency. Fig. 3 shows that designing a U-OWC without accounting for bathymetry effects may lead to power underestimations and, thus, to an underestimation of the average annual produced energy. This difference is totally due to the differences between the computed hydrodynamic parameters and the diffractive excitations and will be further deepened and discussed in future work.

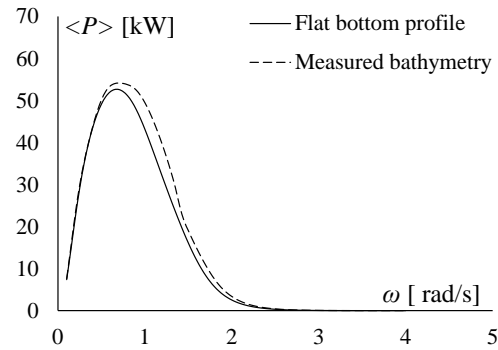


Figure 3. Average wave power available to the turbine.

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