CORRECTION OF THE SHIMOSAKO SLIDING DISTANCE ANALYTICAL SOLUTION OF A RIGID CAISSON UNDER A TRIANGULAR IMPULSIVE FORCE

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INTRODUCTION

Shimosako et al. (1994) presented a simplified model for the estimation of the sliding distance of a rigid caisson subjected to a triangular impulsive force in the hypotheses in which the ground can be considered non-deformable and the caisson is subject to only sliding. In these conditions, the displacement of the caisson can be expressed with an analytical formula which was provided by Shimosako et al. (1994) and is reported by Goda (2010).
The analytical formula provided by Shimosako et al. (1994) is incorrect cause slightly underestimates the displacement due to a single impulsive force: the error isn't negligible for a succession of impulsive forces over the service lifetime.
The objective of the present work is to provide the correct analytical solution obtained using the same hypotheses as the Shimosako et al. (1994) simplified model.

EQUATION OF MOTION OF CAISSON

Fig. 1 illustrates the forces acting on the caisson sliding along the x direction:

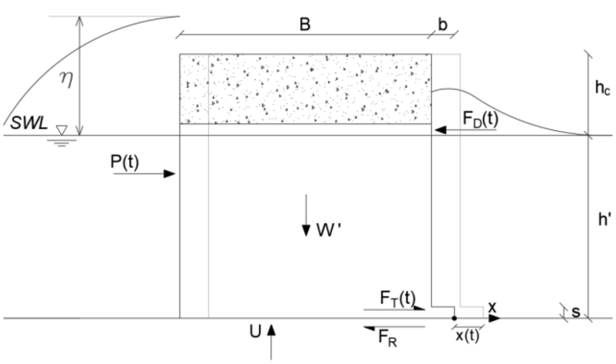

Fig.1 - Wave Forces acting on the caisson during its sliding motion.

- P(t): is the impulsive triangular force which may be generated by breaking waves or eg. C-CI (Castellino et al., 2018; 2021);
- $F_T(t)$: is the shear force acting at the caisson's bottom which is in general different with respect to P(t) because $F_T(t)$ account for the dynamic response of the caisson. If it is assumed that the caisson is non-deformable, $F_T(t)$ is equal to P(t);
- U: is the wave uplift force;
- W': is the caisson's weight in the water provided by $W' = W - W_b$, where W is the caisson's weight in the air and W_b is the caisson's buoyancy force;
- $F_D(t)$: is the wave-making resistance force related to the caisson sliding velocity;
- F_R: is the frictional resistance force.

In the case of a non-deformable caisson, resting directly on a rigid seabed, subjected to an impulsive force, the equation of motion representing caisson sliding is:

$$\left(\frac{W}{g} + M_a\right)\ddot{x}(t) = F_T(t) - F_R - F_D(t) \qquad (1)$$

where:

- $\ddot{x}(t)$ is the caisson acceleration;
- $F_R = \mu(W' - U)$;
- μ is the dynamic friction coefficient assumed constant before and during sliding;
- $M_a = 1.086\rho_w h'^2$ (Goda, 2010) is the added hydrodynamic mass.

Introducing the equivalent sliding force $F_S(t) = F_T(t) + \mu U$ and neglecting in favor of safety $F_D(t)$ and M_a, Eq.1 can be rewritten as follows:

$$\left(\frac{W}{g}\right)\ddot{x}(t) = F_S(t) - \mu W' \qquad (2)$$

DERIVATION OF THE ANALYTICAL SOLUTION FOR THE SLIDING DISTANCE

If we assume in Eq. (2) for $F_S(t)$ an impulsive force varying linearly in time in the shape of a triangle, defined as follow:

$$F_S(t) = \begin{cases} 2\dfrac{t}{\tau_0}F_{Smax} & for \quad 0 \le t < \dfrac{\tau_0}{2} \\ 2\left(1 - \dfrac{t}{\tau_0}\right)F_{Smax} & for \quad \dfrac{\tau_0}{2} \le t < \tau_0 \\ 0 & for \quad \tau_0 \le t \end{cases} \qquad (3)$$

where symbols are described in Fig. 2, it is possible to solve analytically Eq. (2) and calculate the sliding distance x(t).
In Fig. 2 the impulsive force $F_S(t)$ and the sliding distance x(t) are represented.
In the figure:

- τ_0 is the total application time of the impulsive force;
- $\frac{\tau_0}{2}$ is the instant in which Fs(t) is equal to its maximum value F_{Smax};
- t_1 is the instant of time in which $F_S(t)$ is increasing and becomes equal to μW' and the caisson starts to slide;
- t_2 is the instant of time in which $F_S(t)$ is decreasing and becomes equal to μW';
- t_3 is the instant of time in which the caisson stops.

For the evaluation of the total horizontal displacement S_{TOT} it is necessary to distinguish two cases: (i) $t_3 \le \tau_0$, (ii) $t_3 > \tau_0$.

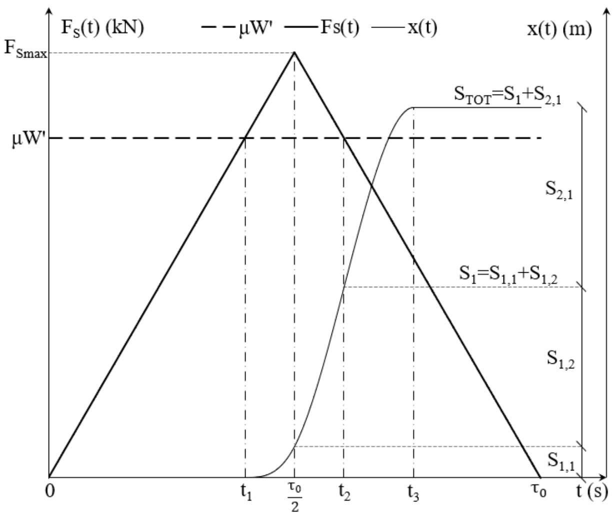

Fig.2 - Triangular shape of the impulsive force Fs(t) (left y-axis), characteristic time instants t_1, t_2, t_3, τ_0 and related caisson sliding distance $S_{1,1}$, $S_{1,2}$, $S_{2,1}$ (right y-axis).

If $t_3 \leq \tau_0$, S_{TOT}, is due to the following three contributions:

$$S_{1,1} = \frac{g\tau_0^2}{24WF_{Smax}^2}(F_{Smax} - \mu W')^3 \qquad \text{for } t_1 \leq t < \frac{\tau_0}{2} \quad (4)$$

$$S_{1,2} = \frac{5g\tau_0^2}{24WF_{Smax}^2}(F_{Smax} - \mu W')^3 \qquad \text{for } \frac{\tau_0}{2} \leq t < t_2 \quad (5)$$

$$S_{2,1} = \frac{g\tau_0^2}{3\sqrt{2}WF_{Smax}^2}(F_{Smax} - \mu W')^3 \qquad \text{for } t_2 \leq t < t_3 \quad (6)$$

The sum of $S_{1,1}$ and $S_{1,2}$ provides the displacement S_1 produced by the phase with positive acceleration in the time interval $t_1 \leq t < t_2$:

$$S_1 = \frac{g\tau_0^2}{4WF_{Smax}^2}(F_{Smax} - \mu W')^3 \quad (7)$$

The sum of S_1 with the displacement $S_{2,1}$, provides S_{TOT}:

$$S_{TOT} = \frac{(3+2\sqrt{2})g\tau_0^2}{12WF_{Smax}^2}(F_{Smax} - \mu W')^3 \quad (8)$$

Fig.2 shows the three displacements listed above.

If $t_3 > \tau_0$, S_{TOT}, is due to the following four contributions:

$$S_{1,1} = \frac{g\tau_0^2}{24WF_{Smax}^2}(F_{Smax} - \mu W')^3 \qquad \text{for } t_1 \leq t < \frac{\tau_0}{2} \quad (4)$$

$$S_{1,2} = \frac{5g\tau_0^2}{24WF_{Smax}^2}(F_{Smax} - \mu W')^3 \qquad \text{for } \frac{\tau_0}{2} \leq t < t_2 \quad (5)$$

$$S_{2,1} = \frac{g\tau_0^2\mu W'\left[F_{Smax}^2 - 2F_{Smax}\mu W' + \frac{5}{6}(\mu W')^2\right]}{4WF_{Smax}^2} \qquad \text{for } t_2 \leq t < t_0 \quad (9)$$

$$S_{2,2} = \frac{g\tau_0^2\left[F_{Smax}^2 - 2F_{Smax}\mu W' + \frac{(\mu W')^2}{2}\right]^2}{8\mu W' WF_{Smax}^2} \qquad \text{for } t_0 \leq t < t_3 \quad (10)$$

The sum of $S_{1,1}$ and $S_{1,2}$ provides the displacement S_1 Eq.(7), while the sum of $S_{2,1}$ and $S_{2,2}$ provides the displacement S_2 Eq.(11) produced by the phase with negative acceleration in the time interval $t_2 \leq t < t_3$:

$$S_2 = \frac{g\tau_0^2\left[F_{Smax}^4 - 4F_{Smax}^3\mu W' + 7(F_{Smax}\mu W')^2 - 6F_{Smax}(\mu W')^3 + \frac{23(\mu W')^4}{12}\right]}{8\mu W' WF_{Smax}^2}$$

The sum of S_1 with the displacement S_2, provides S_{TOT}:

$$S_{TOT} = \frac{g\tau_0^2\left[F_{Smax}^4 - 2F_{Smax}^3\mu W' + (F_{Smax}\mu W')^2 - \frac{(\mu W')^4}{12}\right]}{8\mu W' WF_{Smax}^2} \quad (12)$$

Fig.3 shows the four displacements listed above.

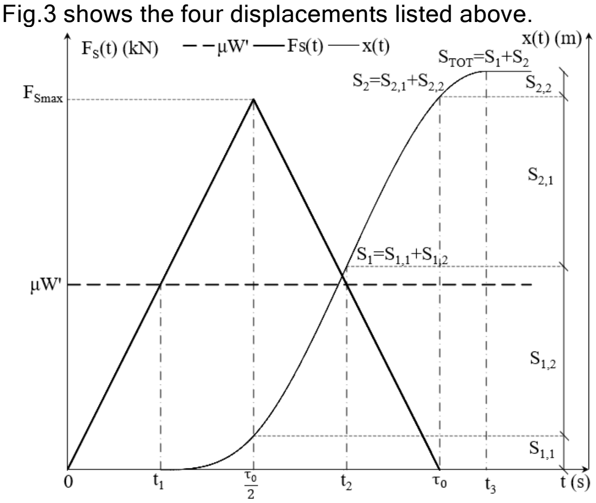

Fig.3 - Triangular shape of the impulsive force Fs(t) (left y-axis), characteristic time instants t_1, t_2, t_3, τ_0 and related caisson sliding distance $S_{1,1}$, $S_{1,2}$, $S_{2,1}$ and $S_{2,2}$ (right y-axis).

CORRECTION OF THE SHIMOSAKO ANALYTICAL SOLUTION FOR THE SLIDING DISTANCE

The analytical solution in terms of sliding distance obtained by Shimosako et al. (1994), is incorrect and differs from the new one obtained in the present paper. The difference between the two formulae lies in the term S_2. The total sliding distance calculated by Shimosako et al. (1994), here indicated as $S_{TOT,S}$, is equal to:

$$S_{TOT,S} = \frac{g\tau_0^2}{8\mu W' WF_{Smax}^2}(F_{Smax} - \mu W')^3(F_{Smax} + \mu W') \quad (13)$$

Comparing the total displacement at time t_3, provided by Eq.(8), Eq.(12) and Eq.(9), for different values of $\alpha = \frac{F_{Smax}}{\mu W'}$, it's noticeable that the Shimosako's analytical solution slightly underestimates the total horizontal displacement. The analytical solution calculated in the present research work was verified using: (i) the program Wolfram Mathematics and (ii) numerically solving the equation of motion of the caisson Eq. (2).
Some application of the new formula will be provided at the conference.

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