

Investigation of Hyperbolic Paraboloid Face Profile Efficacy for Free-Surface Breakwaters

Hamid ElDarwich, Princeton University, hse@princeton.edu
Krisna Adi Pawitan, Princeton University, krisna.pawitan@princeton.edu
Maria M. Garlock, Princeton University, mgarlock@princeton.edu

INTRODUCTION

Coastal regions worldwide face escalating challenges from intensified storm events and rising sea levels (Vardy et al., 2017). With rapid urbanization and increase in coastal land demand, there's a pressing need for innovative and sustainable coastal defense solutions (Sahavacharin et al., 2022). Located near the water's surface, Free-Surface Breakwaters (FSBW) stand out as a compelling solution to protect coastal infrastructure due to its ability to be deployed in deeper waters, high mobility, relatively low production and installation cost, and reduced environmental impact (Teh, 2013). Furthermore, the FSBW can be fabricated on land and deployed to the intended sea area relatively easily by towing it, regardless of the soil foundation or complexity of the bathymetry. FSBW can be either be fixed or floating, with reinforced concrete box-type with flat faces being the most common design (Teh, 2013). The current study examines both the hydrodynamic and structural performance of fixed FSBW with hyperbolic paraboloid (hypar) faces, which is a novel approach (Fig. 1).

The hypar can be characterized by its rise R , defined as the distance from the vertex to its projection on the plan area of the hypar (Fig. 1b). For a square plan area, the rise can be normalized by the side $s = \sqrt{Area}$ to obtain the normalized rise R_n . Owing to their negative Gaussian curvature, hypar surfaces are recognized for their structural efficiency, as evidenced by their use in applications like roofs and seawalls (Wang et al., 2022).

METHODOLOGY

To mitigate the high cost often associated with physical testing, while still maintaining the accuracy of interactions between waves and complex structures, a numerical approach is selected for the hydrodynamic study. A Lagrangian approach using the meshless Smoothed Particle Hydrodynamics (SPH) method was selected. In SPH, the fluid continuum is modeled as particles with fluid characteristics attached to it. The movement trajectory of each particle is then computed based on density driven Navier-Stokes equation for each time step. Pressure, velocity, and force is calculated based on the neighboring particles within the support region defined by smoothing length h of the kernel function W (Crespo., 2022).

Using the open-source SPH code DualSPHysics, this study explores the performance of hypar and vertical-face breakwaters using regular waves in deep-to-intermediate water scenario (Fig. 2a and Fig. 2b). A sensitivity study indicated that $H/15$ is suitable value for interparticle distance d_p . In addition, it was shown that Dynamic Boundary Condition (DBC) leads to reduced pressure fluctuations on the FSBW surface compared to modified DBC (mDBC), thereby making DBC the preferred choice for this study.

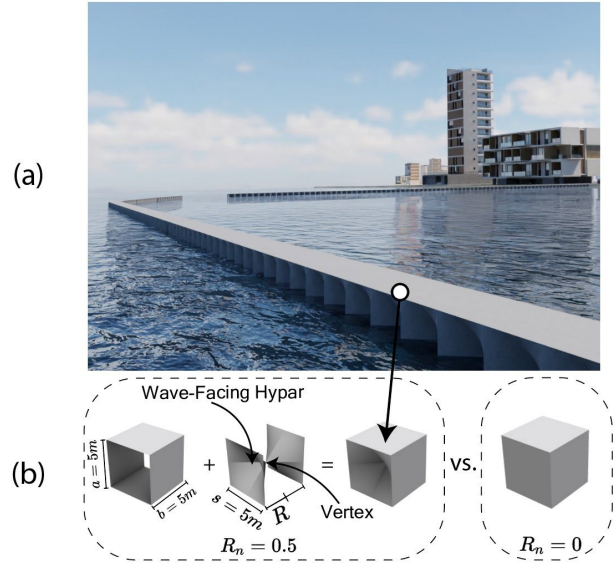


Figure 1 - (a) Artistic rendering of deployed hypar FSBW; (b) generating the proposed hypar-face FSBW (on the left), with vertical-face FSBW (on the right) for comparison.

To evaluate the hydrodynamic performance of the FSBW, the transmission coefficient k_t is computed for each case, defined as the ratio of transmitted wave height to the incoming wave height [Liu, 2020]. The research varies 3 parameters representing the operating sea conditions: wave period (T), wave height (H), and freeboard (F), where F is defined as the distance between still water level (SWL) and the top of the FSBW (Table 1).

Due to constraints in computational resources, mostly 2D SPH hydrodynamic analyses were performed to determine k_t . However, since the structural analyses necessitated inputting hydro-pressures in a 3D space, some SPH simulations needed to be done in 3D. For preliminary studies, 3D simulations for both $R_n = 0.5$ and $R = 0$ were performed for only the most "critical scenario". Structurally speaking, a critical scenario refers to that which causes largest total force on the FSBW. Hence, for the selected range of parameters, the critical scenario corresponded to longest period ($T = 5sec$) and largest wave height ($H = 1.8m$). The F value of 2.5 m was selected as that value where the hypar-face demonstrated the best improvement to k_t compared to the flat-face.

Once the critical case was identified, 900 pressure data points were extracted from 3D SPH simulations of the FSBW into the finite element model (FEM) for structural analysis. The data point captures the combined hydrostatic and wave pressures (as a function of time) on the front-facing surface (Fig. 1b). For the time step

corresponding to total maximum force (Fig. 2c), pressures were then mapped into the FEM constructed in Grasshopper, a visual and parametric programming tool within the Rhinoceros 3D software. The surface was discretized into 120×120 shell elements based on previous studies (EIDarwich et al., 2023). The shell is assumed to be elastic and made of concrete with a modulus of elasticity (E) of 2486 kN/cm^2 and a thickness of 12 cm . Pressure values were converted to normal forces by multiplying them with the tributary area associated with each FEM force node. The support condition was assumed to be fixed around the surface's perimeter. Analysis was conducted using Karamba3D, a structural analysis plugin within Grasshopper, from which maximum principal stresses were determined for both hypar and vertical-face FSBW.

RESULTS AND ANALYSIS

As detailed in Table 1, the 2D SPH simulations indicate that the design featuring a hypar-face, exhibits better wave attenuation performance (evidenced by lower k_t value) compared to its vertical-face counterpart irrespective of the freeboard value. The analysis covers two freeboard scenarios: $F = 0 \text{ m}$, where SWL at the top of the FSBW, and $F = 2.5 \text{ m}$, where SWL at the mid-height of the FSBW.

For 2D SPH simulations, the hypar FSBW demonstrates a more pronounced improvement over the vertical-face FSBW at a freeboard of $F = 2.5 \text{ m}$ compared to $F = 0 \text{ m}$. Thus, the "critical scenario" is determined to be $T = 5 \text{ sec}$, $H = 1.8 \text{ m}$, and $F = 2.5 \text{ m}$. For this "critical scenario", a comparison of 2D and 3D SPH simulations reveals two key insights: firstly, since the vertical-face FSBW maintains a constant cross-section along the y - axis (width) its 2D and 3D behaviors are expectedly similar, with k_t values of 0.60 and 0.56, respectively. Conversely, the hypar FSBW, having a varying cross-section along the width, shows a discrepancy in k_t values (e.g. 0.25 in 2D versus 0.47 in 3D). This discrepancy highlights the need for 3D simulations to accurately capture the hypar FSBW's hydrodynamics. Secondly, while the hypar FSBW has better wave attenuation performance than the vertical design in both 2D and 3D simulations, the margin of improvement in 3D is more modest, changing from 0.56 to 0.47 as R_n changes from 0 (vertical face) to 0.5 (hypar).

The structural analysis results demonstrate that the hypar FSBW is structurally superior. Table 2 presents the maximum principal tension σ_t and compression σ_c stresses for the critical case. It is seen that the hypar surface ($R_n = 0.5$) exhibits around 11 times less

compression stresses and 5 times less tension stresses compared to the flat surface ($R_n = 0$).

CONCLUSION

This study explored the effectiveness of hypar surfaces as a viable design for free-surface breakwaters, by comparing both their hydrodynamic and structural performance against vertical free-surface breakwater. The results demonstrate the potential of leveraging hypar surfaces in novel coastal engineering applications.

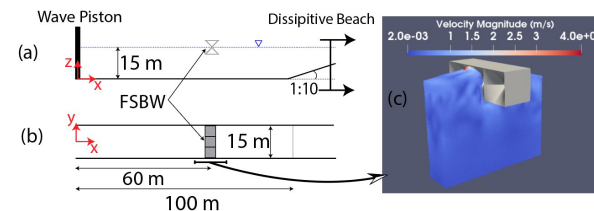


Figure 2 - Numerical flume setup depicting (a) 2D side view with a cutting plane at $y=0$ (b) top view, and (c) 3D SPH simulation at maximum total force time step, displaying a fluid slice for clarity.

Table 2 - FEM principal stresses result for $R_n = 0.5$ and (b) $R_n = 0$ under $T = 5 \text{ sec}$, $H = 1.8 \text{ m}$, and $F = 2.5 \text{ m}$

	$R_n = 0.5$	$R_n = 0$
$\sigma_t (\text{kN/cm}^2)$	2.06E-03	9.90E-01
$\sigma_c (\text{kN/cm}^2)$	-2.78E-02	-3.10E-01

REFERENCES

Vardy, Oppenheimer, Dubash, O'Reilly, and Jamieson, (2017). The intergovernmental panel on climate change: challenges and opportunities. *Annual Review of Environment and Resources*, 42, pp.55-75.

Teh (2013): Hydraulic performance of free surface breakwaters: A review, *Sains Malaysiana*, vol. 42.9, pp. 1301-1310.

Wang et al. (2022): Kinetic Umbrellas as flood barriers, *J. of Struct. Eng.*, vol. 148.5, pp. 04022047.

Crespo, Domínguez, Rogers, Gómez-Gesteira, Longshaw, Canelas, Vacondio, Barreiro, García-Feal (2015): DualSPHysics: Open-source parallel CFD solver based on Smoothed Particle Hydrodynamics (SPH), *Computer Physics Communications*, 187, pp.204-216.

Liu, Wang (2020): Numerical studies of submerged moored box-type floating breakwaters with different shapes of cross-sections using SPH, *Coastal Engineering*, ELSEVIER, vol. 158, pp. 103687.

EIDarwich, Mansouri, Garlock, Wang (2023): Predicting Maximum Deflection of N-Edged Thin-Shelled Hyperbolic-Paraboloid Umbrella Using Machine Learning Techniques. *Engineering Structures*, ELSEVIER. Under review. [doi: 10.13140/RG.2.2.11947.41766

Table 1 Wave attenuation coefficient k_t for vertical-face ($R_n = 0$) vs. hypar-face ($R_n = 0.5$) for 2D SPH (bold indicates 3D scenario).

H (m)	T (s)	$F = 0 \text{ m}$		$F = 2.5 \text{ m}$	
		$R_n = 0$	$R_n = 0.5$	$R_n = 0$	$R_n = 0.5$
0.6	3	0.14	0.08	0.07	0.05
	4	0.13	0.13	0.27	0.02
	5	0.38	0.38	0.68	0.20
1.8	3	0.36	0.30	0.11	0.07
	4	0.51	0.47	0.29	0.03
	5	0.62	0.54	0.60 (0.56 3D)	0.25 (0.47 3D)