NONLINEAR WAVE GROUP INTERACTION IN THE LONG -TIME WAVE EVOLUTION PROCESS

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INTRODUCTION

Freak waves are a sudden nonlinear extreme wave disaster that seriously threatens people's lives and the safety of offshore structures. The occurrence of freak waves is generally accompanied by wave groups (Trulsen,2000; Slunyaev et al., 2006; Tang et al., 2019). In order to investigate the characteristics of wave groups and freak waves during the long time wave evolution process, the High-Order Spectral method (HOS) is adopted, and the modulated Stokes wave train is used as the initial conditions. Characteristics of wave groups are analyzed under different evolution stages and generation mechanisms. Furthermore, largest Lyapunov Exponent, correlation dimension and Kolmogorov entropy are used to calculate the nonlinearity levels at different time scales during long time wave evolution process. This provides a theoretical basis for the generation and prediction of freak waves.

EVOLUTION OF WAVE SURFACE MAXIMUM

Taking the initial wave steepness ε_0=0.06 as an example, Figure 1 shows the evolution of the wave surface maximum during the long time wave evolution process. The horizontal axis of the figure represents the time scale of evolution, while the vertical axis represents the maximum value of the spatial wave surface at any time of evolution. It is evident that the maximum peak height in the early stage undergoes a process of increasing and then decreasing, indicating modulation instability. The first maximum modulation occurs at $t/T_0 \approx O(\varepsilon_0^{-2})$, followed by demodulation. The recurrence phenomenon of modulation and demodulation continues until $t/T_0 \approx O(\varepsilon_0^{-3})$. However, after $t/T_0 \approx O(\varepsilon_0^{-3})$, The periodic recurrence phenomenon disappears, and modulation instability no longer plays a dominant role. And a new dominant nonlinear mechanism has emerged, that is nonlinear wave group interaction.

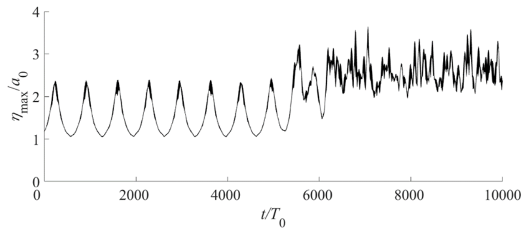

Figure 1 - Evolution of spatial wave surface maximum under the condition of ε_0=0.06

EVOLUTION OF WAVE GROUPS

As shown in Figure 2, it reveals that after $t/T_0 \approx O(\varepsilon_0^{-3})$, nonlinear wave group interactions have emerged, and the occurrence of freak waves is accompanied by the aggregation and separation process of wave groups. In the stage of modulation instability, the number of wave groups remains unchanged, there is only one type of wave group at each moment. While in the stage of nonlinear wave group interaction, the number of wave groups changes, and there is interaction between the wave groups, there are different wave groups at each moment.

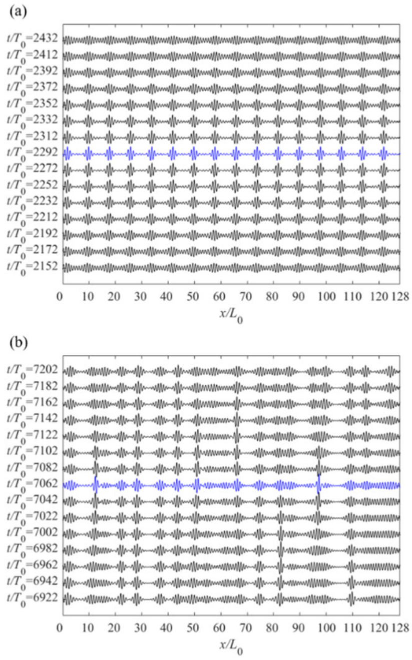

Figure 2 - The evolution of wave groups in the long time wave evolution process of modulated wave trains under the condition of ε_0=0.06 ((a) is the stage of modulation instability ($t/T_0 < O(\varepsilon_0^{-3})$), (b) is the stage of nonlinear wave group interaction ($t/T_0 > O(\varepsilon_0^{-3})$)).

THE DEFINITION OF CHAOS

Chaos is a seemingly irregular motion that refers to the occurrence of random behavior in a deterministic nonlinear system without the need for any additional random factors. Haykin (1997) suggests that a chaotic system must satisfy the following necessary conditions:

(1) The process is nonlinear;
(2) It has a finite correlation dimension(CD);
(3) The largest Lyapunov exponent (LLE) is positive;
(4) The dynamical system is dissipative;
(5) It has a finite Kolmogorov entropy.

Obviously, for the long-time evolution of wave trains, conditions (1) and (4) are met. Therefore, to prove whether the evolution of the wave trains in the long-time evolution process enters a chaotic state, in addition to the qualitative analysis based on the reconstruction of the phase space, we also need to further calculate LLE (Strogatz, 2015), CD, and the Kolmogorov entropy.

QUANTITATIVE ANALYSIS

The quantitative analysis is conducted through the reconstruction of phase space, as shown in Figure 3, taking the initial wave steepness of 0.06 and 0.08 as an example, the results of the reconstructed phase space are presented.

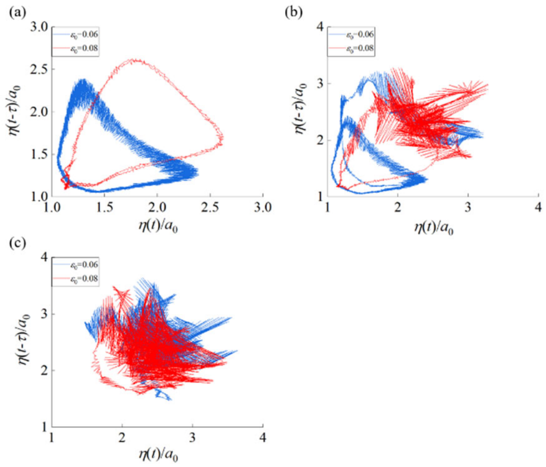

Figure 3 - Phase space reconstruction trajectory under the condition of ε_0=0.06, 0.08 ((a) is the stage of modulation instability, (b) is the transition stage, (c) is the stage of nonlinear wave group interaction).

According to the Figure 3, it is clear that the reconstructed trajectories throughout the entire long-time evolution process can be divided into two parts. One is where the trajectories are concentrated within a closed triangular trajectory in the lower-left corner (Figure 3(a)), and the other part is concentrated in an irregular area in the upper right corner (Figure 3(c)). They correspond to two important nonlinear mechanisms during the long-time evolution of the wave trains. The triangular area corresponds to the stage of modulation instability, where the wave train, after a finite period of evolution, forms a closed area in phase space. Such closed trajectories usually indicate that the system's behavior is periodic, corresponding to the periodic recurrence of modulation-demodulation, where the system is in a stable state. As the evolution progresses, it enters the stage of nonlinear wave group interaction, where the trajectories are no longer regular and orderly but chaotic and mainly concentrated within an irregular, finite area, forming a complex, strange attractor. This indicates that the system's behavior is chaotic at this stage.

QUANTITATIVE ANALYSIS

Taking the initial wave steepness of 0.06 and 0.08 as examples, the quantitative analysis results are presented in Table 1.

Table 1 Chaos parameters for ε_0= 0.06, 0.08

ε_0	LLE	CD	Kolmogorov Entropy
0.06	-0.000086	1.98	0.0032
0.08	0.0012	2.22	0.012

According to qualitative and quantitative analysis, we found that when the wave steepness is small, the nonlinearity is weak, and the dynamic behavior of the wave train evolution may be in a transition between stability and chaos. The system exhibits certain characteristics of chaos (strange attractor, LLE>0, CD>0), but at the same time, there are also stable characteristics (LLE<0). When the initial wave steepness is large, the nonlinearity is stronger and the system becomes more unstable. After modulation instability, the evolution of wave trains gradually evolves into a chaotic state. And the larger the wave steepness ε_0, the more significant the chaotic behavior is.

CONCLUSIONS

In our study, the generation process of freak waves and the characteristics of wave groups under the two different dominant mechanisms of modulation instability and nonlinear wave group interaction in the long-time evolution of modulated wave trains are investigated. And through qualitative and quantitative analysis of the state changes and nonlinear levels in the process of the long-time evolution. The results indicate that in the stage of modulation instability, the wave train evolution system is stable, and as the evolution progresses and nonlinearity strengthens, the evolution gradually enters a chaotic state. When the initial wave steepness is small, the nonlinearity is weak, and the evolution of the wave train is in a transition between stability and chaos. And larger the wave steepness ε_0, the more significant the chaotic behavior becomes.

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