

PROBABILITY DENSITY FUNCTIONS AND THEIR POSITIVE TAILS TO ANY ORDER IN NONLINEARITY FOR THE SURFACE ELEVATION IN IRREGULAR SEAS

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INTRODUCTION

Until very recently, the probability density function (p.d.f.) arising from the cumulant generating function for the surface elevation in irregular seas - free of assumptions involving narrow-bandedness and/or small directionality - was only known exactly to first order, where it is Gaussian. At higher orders, this p.d.f. had only been represented approximately (Longuet-Higgins 1963) e.g. as a Gram-Charlier series. In a recent development, the present authors (Fuhrman, Klahn & Zhai 2023, henceforth referred to as FKZ) have shown that to second-order this p.d.f. may, in fact, be represented exactly in terms of the Airy function. Notably, this new theoretical result shows that the p.d.f. has an inherently heavier positive tail (i.e. increased probability density of large surface elevations, typical of e.g. rogue waves) than had been explained previously, and that at second order it is governed solely by the skewness.

RESULTS

In the present work, we extend our theoretical investigation on the p.d.f. of the surface elevation in irregular seas yet further. We develop a novel methodology enabling first-time numerical determination of the theoretical p.d.f. to any desired order in nonlinearity. For this purpose, a new ordinary differential equation (o.d.e.) is first derived, governing the p.d.f. to any order in nonlinearity. Asymptotic solutions to this o.d.e. are then found analytically in the limit of large surface elevations, newly providing the theoretical form of the positive tail of the p.d.f. beyond second order. These are important in their own right, as they make theoretically clear how higher-order cumulants (involving high-order statistical moments such as the kurtosis, hyperskewness and hyperkurtosis) may affect the positive tail, which is shown to get heavier at each successive order. The asymptotic solutions are finally utilized to provide necessary boundary conditions, such that the governing o.d.e. may be solved numerically. This enables novel determination of the full theoretical p.d.f. to effectively any desired order.

Successful comparisons with challenging (highly nonlinear) data sets are made, confirming accuracy of obtained higher-order p.d.f.s. An example result is shown in Figure 1, with the p.d.f. computed up to fifth order in nonlinearity. The p.d.f. is compared against laboratory data from Trulsen et al. (2020), which is known to be very challenging (see also e.g. Zhang & Benoit 2021). The case selected is extremely nonlinear, where the surface elevation data has skewness of 0.79, kurtosis of 4.2, hyperskewness of 10 and hyperkurtosis of 45. Additional comparisons will be made from data sets involving nonlinear, directionally-spread irregular wave fields generated both theoretically (Madsen & Fuhrman 2012), as well as numerically (based on the fully-nonlinear wave model of Klahn et al. 2021).

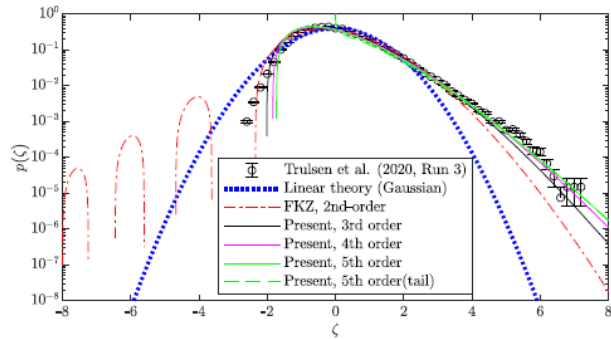


Figure 1 - Comparison of the new theoretically-based p.d.f.s up to fifth order in nonlinearity with a highly nonlinear experimental data set of Trulsen et al. (2020).

ACKNOWLEDGEMENT

This research has been financially supported by the Independent Research Fund Denmark project STORM: STatistics and fOrces on stRuctures from extreMe water waves in finite depth (grant ID: 10.46540/2035-00064B). This support is greatly appreciated.

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