Strength and weaknesses of three approaches to fast modelling of the spectral wave action balance

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INTRODUCTION

Wave models are essential for coastal engineering applications, because the (nearshore) wave conditions are required for the design of coastal structures, important drivers of coastal floodings and coastal erosion. Various spectral wave models exist to model the wave propagation, wind growth and energy transfer within a spectrum (Booij, et al. 1996, Günther, et al., 1992). These models have been improved over the last years (e.g. Rogers et al., 2015) and are able to accurately be applied for various applications.

As a consequence the wave models became rather slower in terms of computational time than faster. This constricts the use of state-of-the-art wave models in probablistic flooding forecasts or continental scale wave climate downscaling for forcing shoreline modelling. Both these types of applications demand ensemble mode simulations achieved running all simulations in parallel on a super computer. Given that uncertainties in forecasting outcomes are not only a result of model uncertainty but also forcing uncertainty (e.g. hurricane track, storm intensity), this study investigates two alternative approaches to modelling the spectral wave action balance. The first model aims to downscale climate models to the water depths and regions relevant for shoreline modelling. The second alternative model approach aims to provide input to continental scale probabilistic (hurricane-driven) wave forecasts.
This study investigated accuracy and computational time of the two alternative wave models and compared to the state-of-the-art wave model SWAN through model-data comparison on 5 test cases.

MODEL FORMULATIONS

The implicit SWAN model is applied as reference state-of-the-art spectral wave model. It resolves the stationary or instationary action balance given by:

$$\frac{\partial N(\sigma,\theta)}{\partial t} + \nabla[(c_g + u)N(\sigma,\theta)] + \frac{\partial c_g N(\sigma,\theta)}{\partial \sigma} + \frac{\partial c_\sigma N(\sigma,\theta)}{\partial \sigma} + \frac{\partial c_\theta N(\sigma,\theta)}{\partial \theta} = \frac{S_{wind}}{\sigma} + \frac{S_{diss.}}{\sigma} + \frac{S_{nl3}}{\sigma} + \frac{S_{nl4}}{\sigma} \quad (1)$$

Where $N(\sigma,\theta)$ action density as a function of frequency σ and direction θ, c_g is the group velocity, u the mean current, c_θ propagation speed in theta direction, c_σ propagation speed in frequency space (due to currents), S_{wind} the wind input, $S_{diss.}$ the dissipation term, S_{nl3} the triad formulation and S_{nl4} the quadruplet formulations.

One way of simplifying is modelling a stationary, frequency integrated wave action balance. With this assumption, the model problem size is reduced because it is 3-dimensional (x,y,θ) instead of 4-dimensional (x, y, θ, σ) and non-linear wave-wave interactions do not have to be solved for explicitly. Thus, the set of coupled equations that is solved for is:

$$\nabla[c_g N(\theta)] + \frac{\partial c_\theta N(\theta)}{\partial \theta} = \frac{S_{wind}}{\sigma} + \frac{S_{diss.}}{\sigma} \quad (2)$$

$$\nabla[c_g E(\theta)] + \frac{\partial c_\theta E(\theta)}{\partial \theta} = S_{wind} + S_{diss.} \quad (3)$$

$$T = 2\pi \frac{E}{N} \quad (4)$$

The wind input is given by the formulation of Van der Lugt et al. (2017) based on the growth curves of Young-Verhagen. The foreseen domain of application of this *implicit frequency integrated model (IFIM)* is downscaling global climate models to the nearshore over arbitrary bathymetry.
Next to the *implicit frequency integrated model* a second approach is verified. This approach focusses solely on the deeper water physics, such that ensemble storm forecasts can be obtained in reasonable time. This model adopts an explicit numerical scheme and In addition, triads are unaccounted for and effects of currents are not included. This *explicit spectral model (ESM)* solves the following equation:

$$\frac{\partial N(\sigma,\theta)}{\partial t} + \nabla[c_g N(\sigma,\theta)] + \frac{\partial c_g N(\sigma,\theta)}{\partial \sigma} + \frac{\partial c_\theta N(\sigma,\theta)}{\partial \theta} = \frac{S_{wind}}{\sigma} + \frac{S_{diss.}}{\sigma} + \frac{S_{nl4}}{\sigma} \quad (5)$$

METHOD

To quantify the accuracy of the three model approaches and the same time investigate the computational gain the *IFIM* and *ESM* model, five test cases (Table 1) are modelled. For each test case the significant wave height (H_{m0}), peak period (T_p) and when available the wave direction (D) are compared with observations through computing Root Mean Square Error (RMSE). Furthermore, the computational time is compared.

RESULTS

The test cases are modelled and the results for the five test cases are shown in Table 2. The results show that the computational time and accuracy differs significantly depending on the test case.

Table 1 - Overview of test cases.

Test case	Features
North Sea	- Large domain - Wind growth - Whitecapping

Amelander Zeegat	- Complex shallow bathymetry - Wind growth - Refraction
Haringvliet	- Small domain - Onshore wave propagation - Breaker bar
Lake George	- Small domain - Local wind generation - Shallow water
Gulf of Mexico	- Very large domain - Hurricane Florence

For the simulations with a large course grid (North Sea model and Gulf of Mexico model) the *ESM* model is significant faster without losing accuracy. When modelling deep water condition to the nearshore (Ameland Zeegat and Haringvliet), the *IFM* also reasonably results compared to the much slower SWAN model. Even for the Amelander Zeegat with a complex bathymetry and the less complex lake George case, the *IFM* shows reasonable accurate results.

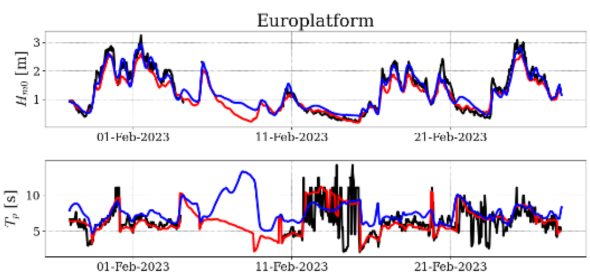

Figure 1 – Wave height (upper panel) and peak period (lower panel) for computations with SWAN (blue) and *ESM* (red) for location Europlatform in the North Sea case.

Table 2 – RMSE and computation time for the different wave models and the five test cases.

Test case		RMSE			Comp. time [s]
		H_{m0} [m]	T_p [s]	D [°]	
North Sea model	*SWAN*	0.28	2.42	27	21600
	IFIM	-	-	-	-
	ESM	0.27	2.47	31	4212
Amelander Zeegat	*SWAN*	0.25	0.59	10	887
	IFIM	0.43	1.32	20	6
	ESM	0.17	1.09	9.0	960
Haringvliet	*SWAN*	0.29	1.85	-	136
	IFIM	0.3	1.49	-	6
	ESM	0.33	2.45	-	650
Lake George	*SWAN*	0.04	0.35	-	98
	IFIM	0.13	0.60	-	4
	ESM	0.11	0.30	-	196
Gulf of Mexico	*SWAN*	0.83	3.70	53	7326
	IFIM	-	-	-	-
	ESM	0.66	2.5	52	274

The results of the SWAN and *ESM* model for the North Sea case (Figure 1) demonstrate the accuracy of the *ESM* model. This time series show that the both the SWAN and *ESM* model show similar results for the wave height and peak period. Only for milder conditions some deviations are visible related to the applied physics. The added value of the *IFIM* model is shown in Figure 2 for the Amelander Zeegat case, where the spatial results of the *IFIM* model are similar to the SWAN simulation.

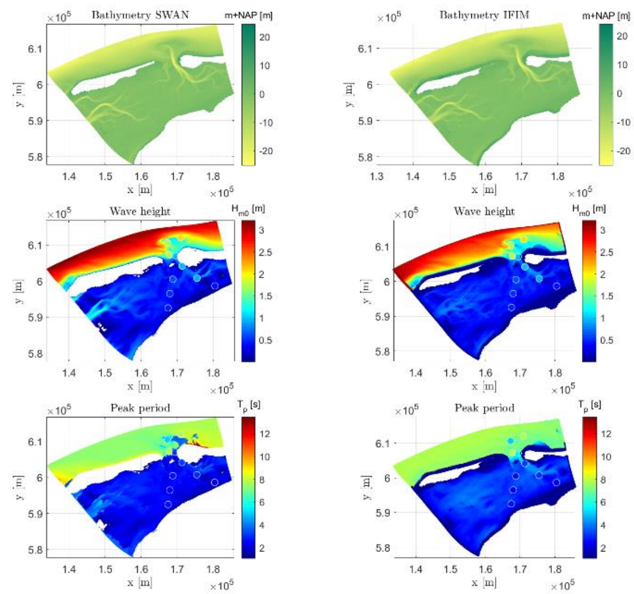

Figure 2 – Wave height (upper panel) and peak period (lower panel) for computation with SWAN (left) and IFIM (right).

CONCLUSION

When the results of the three wave models are compared with observations for the various test cases it is concluded that it is possible to speed-up the calculation significantly at limited expense of model accuracy. Depending on the type of application, *implicit frequency integrated model (IFIM)* or *explicit spectrum model* (*ESM*) approaches can speed up computational time. The *IFIM* model is a factor 25 faster than the *SWAN* model without losing much accuracy when applied in the nearshore with a small domain. The larger domain models, benefit with the explicit scheme in the *ESM*.

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