SIMULATION OF ROGUE WAVE FORMATION IN FINITE DEPTH IRREGULAR WAVES WITH A FULLY NONLINEAR MODEL

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INTRODUCTION

The formation of rogue waves in coastal areas can be triggered by modulational instability (MI, known also as Benjamin-Feir instability), especially in deep water. The analysis of rogue wave generation and statistical characteristics of irregular waves in finite depth has predominantly focused on low-order (third-order) nonlinear interactions. In the present study, we conduct numerical simulations using a fully nonlinear, spectrally accurate water wave model (Klahn et al. 2021) to explore the statistical properties of irregular, uni-directional wave fields initially described by a TMA spectrum (Holthuijsen 2010). The numerical model is initially validated through the simulation of modulational instability in nonlinear plane wave trains in deep water. Subsequently, a series of random uni-directional wave fields, spanning a broad spectrum of water depths, is examined. This investigation encompasses various aspects of wave statistics, and discusses the relationship between the occurrence probability of extreme waves (rogue waves) with full nonlinearity. The significance of fully nonlinear behavior, as opposed to third-order nonlinearity, is also assessed.

RESULTS

In the present paper, we proceed to validate the numerical model using 1D MI in deep water conditions. We compare the simulated spatial evolution of amplitudes with experimental data from Tulin & Waseda (1999). We also verify the exponential growth rate of sidebands against the theoretical prediction of McLean (1982), as shown in Figure 1. Here it is seen that the model accurately produces the initial growth rate of the unstable sidebands, closely matching both the theoretical prediction, as well as the experimental observations. The model likewise captures the observed frequency downshift to the lower side band reasonably, following the initial unstable growth.

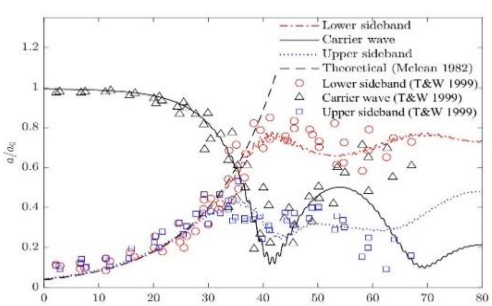

Figure 1 – Spatial evolution of the amplitudes of the carrier wave and the two sideband waves. The theoretical sideband growth rate corresponding to the evolution predicted by Mclean's analysis (McLean 1982), and T&W 1999 represents the experimental data obtained by Tulin & Waseda (1999).

We now investigate the exceedance probability of the wave crest, defined as the highest elevation of each individual wave with respect to the mean water level using zero-up crossing analysis. In Figure 2, the theoretical Rayleigh and second-order Tayfun (1980) distributions, along with the simulated results presented by Liu et al. (2022), are compared against the present fully-nonlinear numerical results at $t/T_p = 50.$, where t is time and T_p is the peak period.

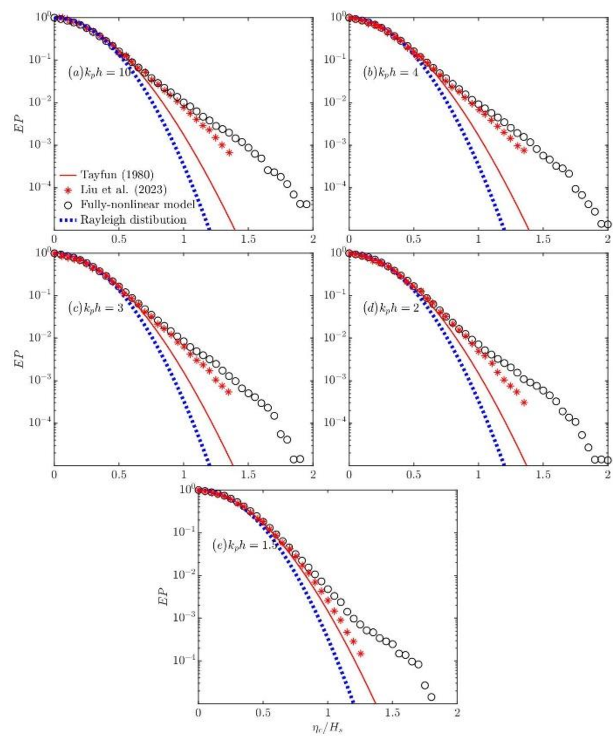

Figure 2 – Exceedance probability of wave crest for different depths: Rayleigh distribution(dot lines),second-order Tayfun distribution (full lines), simulated results with third-order HOS method from Liu et al. (2022) (asterisks) and the fully-nonlinear simulations(circles).

The wave crest distributions accumulated from the fully-nonlinear simulations exhibit a substantial departure from the Tayfun distribution in the tails for cases having larger water depth (see Figure 2a-d). These deviations are primarily influenced by the nonlinear dynamics of free waves, which dominate the statistical characteristics of the wave crests. As we move towards smaller water depth (see Figure 2e), the crest amplitude attenuates, leading to a reduction in the deviation from second-order theory. Comparing the present fully-nonlinear results (black circles) with those from the third-order simulations of Liu et al. (2022, red asterisks), it is clear that the present results predict much greater exceedance probabilities in the extreme positive tail. This disparity can be attributed to the impact of higher-order nonlinear interactions among free waves, which will result in and enhance the modulational instability. Note that with $k_p h = 1.5$, Liu et al. (2022) showed that the difference between their results and the second-order Tayfun distribution almost disappears, which means the statistical properties of the wave fields are dominated by bound wave effects. Conversely, the results from the present model deviate significantly from the Tayfun distribution. This seemingly indicates that the nonlinear dynamics of free waves remain important in determining the statistical properties of the wave fields.

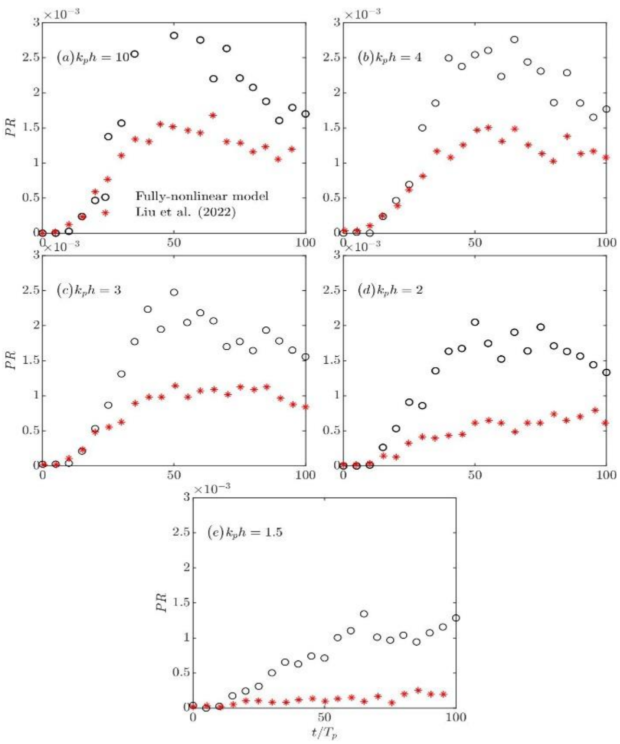

Figure 3 – Comparison of the probability of rogue wave occurrences from the fully-nonlinear model(circles) with which reported in Liu et al. (2022) (asterisks).

The simplest rogue wave definition identifies a rogue wave as one with a crest height exceeding 1.25 times the significant wave height ($\eta_c > 1.25H_s$). Figure 3 illustrates the temporal evolution of the probability of rogue wave occurrence (PR) at different dimensionless depths. As can be observed, PR initially increases in time, owing to the strong modulational instability of the wave field, and then decreases to become relatively stationary. With decreasing $k_p h$, the PR is reduced, but its maximum value from the fully-nonlinear model is still significantly larger than the third-order results of Liu et al. (2022). Furthermore, a strong relationship exists between the PR and kurtosis according to Liu et al. (2022). The value of kurtosis is closely associated with the modulational instability, which serves as a determining factor for the occurrence of rogue waves. The fully-nonlinear results suggest that the high-order nonlinear interactions play a significant role in promoting modulational instability, and consequently, in increasing the probability of rogue wave formation.

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REFERENCES

Klahn, M., Madsen, P. A. & Fuhrman, D. R. (2021): Simulation of three-dimensional nonlinear water waves using a pseudospectral volumetric method with an artificial boundary condition. Int. J. Numer. Meth. Fluids, vol. 93, 1843-1870.

Holthuijsen, L.H. (2010): Waves in oceanic and coastal waters. Cambridge university press.

Tulin, M. P. & Waseda, T. (1999): Laboratory observations of wave group evolution, including breaking effects. J. Fluid Mech., vol. 378, 197-232.

McLean, J. W. (1982): Instabilities of finite-amplitude water waves. J. Fluid Mech., vol. 114, 315-330.

Liu, S., Zhang, X. S., Yang, J. C. & Yao, J. Y. (2022): Modulational instability and statistical properties of irregular waves in finite water depth. Appl. Ocean Res., vol. 120, 103031.

Tayfun, M. A. (1980): Narrow-band nonlinear sea waves. J. Geophys. Res. , vol. 85, 1548-1552.