

# 3D DEPTH-LIMITED BREAKING WAVES OVER VARIABLE BATHYMETRY IN FULLY NON-LINEAR POTENTIAL FLOW

Jeffrey Harris, LHSV, Ecole des Ponts, EDF R&D, Chatou, France, [jeffrey.harris@enpc.fr](mailto:jeffrey.harris@enpc.fr)

Sunil Mohanlal, LHSV, Ecole des Ponts, EDF R&D, Chatou, France, [sunil.mohanlal@enpc.fr](mailto:sunil.mohanlal@enpc.fr)

Marissa Yates, LHSV, Ecole des Ponts, EDF R&D, Chatou, France, [marissa.yates@enpc.fr](mailto:marissa.yates@enpc.fr)

Stephan Grilli, Department of Ocean Engineering, University of Rhode Island, Narragansett, RI, USA, [grilli@uri.edu](mailto:grilli@uri.edu)

## INTRODUCTION

A new method of modeling 3D depth-limited breaking waves is proposed and implemented in a fully non-linear potential flow model based on the boundary element method. The method is implemented in three steps: (1) identification of breaking onset using the kinematic  $B$  criterion; (2) application of damping pressure in the dynamic free surface condition, function of crest kinematics; and (3) a breaking termination criterion to stop this dissipation. A comparison with an experimental study of breaking waves over variable bathymetry is further studied.

## NUMERICAL MODEL

Fully non-linear potential flow model (FNPF) assumes the fluid flow to be irrotational and inviscid, therefore the Laplacian of the velocity potential is equal to zero. These models are computationally more efficient than Navier-Stokes models, but dissipation processes have to be explicitly modelled, for example, when breaking waves are present. Following Grilli et al. (2001), here, the model of Harris et al. (2022) is used, in which the Laplace's equation is solved as a boundary integral equation, discretized with a higher-order BEM.

## BREAKING DISSIPATION

Implementing wave breaking in a fully nonlinear potential flow model is carried out in three steps. First, the breaking onset, or instant at which wave breaking starts, is identified. The recently proposed universal criterion  $B = u/c$  is used, which suggests that an evolving crest whose ratio of horizontal particle velocity at the crest,  $u$ , to crest velocity,  $c$ , has not yet, but will break when  $B$  exceeds 0.85 (Barthelemy et al., 2018). Second, the magnitude of the energy dissipated by breaking is determined by estimating the non-dimensional breaking strength parameter  $b$  (defined such that wave energy dissipation rate, or power, per unit length of the breaking crest  $\epsilon = b\rho g^{-1}c^5$ ). This instantaneous power to be dissipated, once estimated, is then modelled by applying a damping pressure across the free surface of the breaking wave (as in Grilli et al., 2020), extended here to a 2D surface around the breaking crest (Fig. 1). Lastly, a breaking termination criterion is specified.

Mohanlal et al. (2023) used a similar 2D model to simulate 2D depth-limited breaking waves (dissipating energy with a hydraulic jump analogy) and compared their results with experiments for a range of regular and irregular wave cases. They found that waves with a wave height nearly equal to the local water depth at the onset of breaking,  $H/d \approx 1$ , would dissipate with  $b \approx 0.1$ . This dissipation resulted in the decay of the wave height across the surfzone, until the termination of breaking occurred for  $H/d \approx 0.4$  with  $b \approx 0.01$ . With this rationale, a constant value  $\bar{b} = 0.05$  was selected, and simulations showed that this provided satisfactory results. With a constant  $b$  value, the estimation of the power dissipated in breaking waves is simplified and becomes independent of the wave parameters, which are challenging to estimate for irregular waves, particularly in 3D. Therefore, in the applications of the 3D model presented next, which all fall within the same shallow water breaking regime, a constant value of  $\bar{b} = 0.05$  is used.

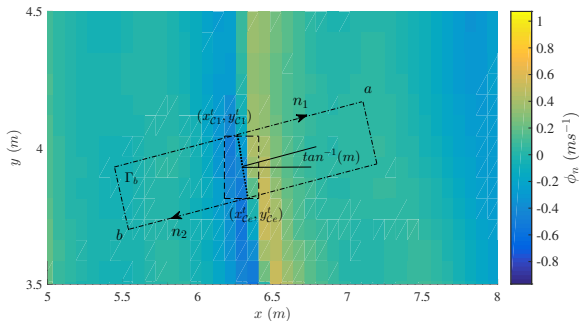


Figure 1: For a given 3-by-3 surface unit (vertical, dashed line rectangle), with a linear crest unit (dotted line) identified as breaking, the damping pressure  $P_b$  is applied in the breaking area  $\Gamma_b$  (large dash-dotted rectangle). The color scale corresponds to the normal velocity at the free surface.

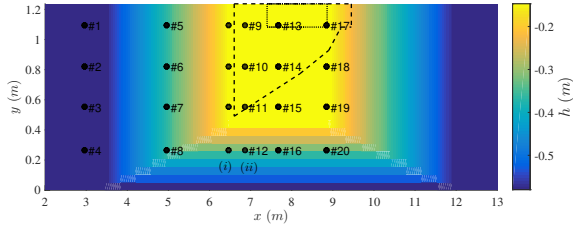


Figure 2: Bathymetry and wave gauge location in the simulations and experiments of Kamath et al. (2022). The array of gauges #9-#12 was placed at (i) for test cases  $C1$  and  $C6$ , and at (ii) for  $C4$  to measure the waves before they became unstable during breaking events. The flap wavemaker is located at  $x = 0$ . The approximate breaking regions in the NWT (from onset to termination) are shown for  $C4$  (dotted lines) and  $C6$  (dashed lines).

## RESULTS

Kamath et al. (2022) studied the lateral energy transfer across the wavefront occurring for non-breaking and breaking periodic waves propagating over a 3D submerged bar with a  $1 : 0.963$  lateral slope and  $1 : 6.933$  front and back slopes. They performed 18 experiments in the 18.3 m long, 1.237 m wide, and 1.2 m tall tank equipped with a flap wavemaker, (Fig. 2), for 3 water depths (0.52, 0.55, and 0.58 m) and a range of wave heights (0.019-0.1 m) and periods (1.21-3.93 s). In each experiment, free surface elevation time series were measured at 20 locations (Fig. 2).

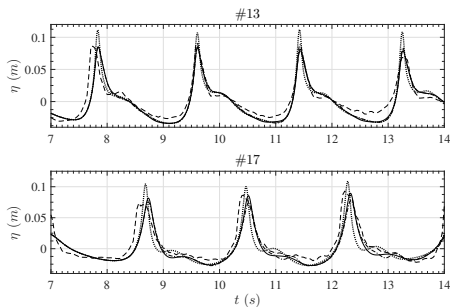


Figure 3: Free surface time series at gauges 13 and 17 (located on top of the bar, Fig 2) for the spilling breaking test case  $C4$  (Kamath et al., 2022). The numerical simulations with the breaking model deactivated ( $b = 0$ ; dotted line), and activated ( $b = 0.05$ ) with  $B_{\text{off}} = 0.3$  (solid line) and  $B_{\text{off}} = 0.4$  (dash-dot line) are compared to the experimental measurements (dashed line).

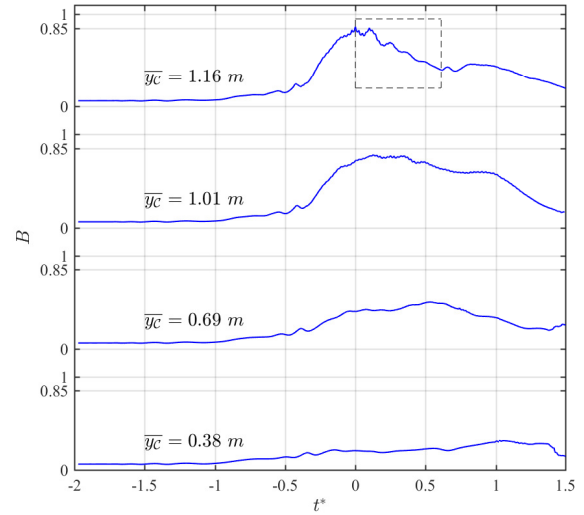


Figure 4: Evolution of the breaking onset criterion  $B$ , as a function of  $t^* = (t - t_O)/T_b$  along four cross-tank transects (y-axis), where  $t_O$  is defined as the breaking onset of the crest at  $\bar{y}_c = 1.16$  m for the spilling- breaking test case  $C4$ . The dashed rectangle encloses the breaking region.

## CONCLUSION

The NWT is able to simulate three-dimensional non-linear waves accurately until breaking onset. A new method is implemented to simulate 3D depth-limited breaking waves, where the dissipation strength is assumed constant,  $b = 0.05$ , and the model is then able to simulate the breaking wave with reasonable accuracy in comparison to experiments of regular breaking waves. Work in progress includes studying a wider range of wave conditions and analyzing the sensitivity of the results to the breaking termination criterion.

## REFERENCES

- Barthelemy, X., Banner, M., Peirson, W., Fedele, F., Allis, M., Dias, F., 2018. On a unified breaking onset threshold for gravity waves in deep and intermediate depth water. *J. Fluid Mech.* 841, 463–488.
- Grilli, S.T., Guyenne, P., Dias, F., 2001. A fully non-linear model for three-dimensional overturning waves over an arbitrary bottom. *Intl. J. Numer. Meth. Fluids* 35, 829–867.
- Grilli, S.T., Horrillo, J., Guignard, S., 2020. Fully nonlinear potential flow simulations of wave shoaling over slopes: Spilling breaker model and integral wave properties. *Water Waves* 2, 263–297.
- Harris, J.C., Dombre, E., Benoit, M., Grilli, S.T., Kuznetsov, K.I., 2022. Nonlinear time-domain wave-structure interaction: a parallel fast integral equation approach. *Intl. J. Numer. Meth. Fluids* 94, 188–222.
- Kamath, A., Roy, T., Seiffert, B.R., Bihs, H., 2022. Experimental and numerical study of waves breaking over a submerged three-dimensional bar. *J. of Waterway, Port, Coastal, and Ocean Engng.* 148, 04021052. doi:10.1061/(ASCE)WW.1943-5460.0000697.
- Mohanlal, S., Harris, J.C., Yates, M.L., Grilli, S.T., 2023. Unified depth-limited wave breaking detection and dissipation in fully nonlinear potential flow models. *Coastal Engineering* 183, 104316. doi:10.1016/j.coastaleng.2023.104316.