

GENERATION OF NONLINEAR RANDOM WAVES WITH BOTTOM WAVE MAKER USING EXTENDED BOUSSINESQ EQUATIONS: EFFICIENT METHOD TO ABSORB REFLECTED WAVE ENERGY

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INTRODUCTION

When waves are generated in a flume by the conventional piston or flap wave maker, waves reflected from structures will arrive at the wave maker and experience re-reflection. These re-reflected waves will propagate to the structures so that the target wave energy cannot be accurately obtained. Since 1980s, methods of internally generating waves inside a computational domain and putting sponge layers at the domain end have been developed in numerical modelling. Internally generated waves will propagate both directions from the wave generation zone and waves reflected from the structures will pass through the wave generation zone without any reflection by the wave maker and be absorbed in the sponge layers. Jung et al. (2023) developed linear wave makers in a flume which have paddles moving up and down from the bottom.

WAVE GENERATION METHOD

In this study, we generate nonlinear random waves with the bottom wave maker using the extended Boussinesq equations. The extended Boussinesq equations in one dimensional form are given by

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} [(h + \zeta)u] + \frac{1}{\cosh^2(kh)} \frac{\partial h}{\partial t} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial \zeta}{\partial x} + \frac{h^2}{6} \frac{\partial^3 u}{\partial x^2 \partial t} - \left(\frac{1}{2} + \gamma\right) h \frac{\partial^2}{\partial x^2} \left(h \frac{\partial u}{\partial t}\right) - \gamma g h \frac{\partial^2}{\partial x^2} \left(h \frac{\partial \zeta}{\partial x}\right) - \frac{1}{\cosh(kh)} \frac{h}{2} \frac{\partial^3 h}{\partial x \partial t^2} = 0 \quad (2)$$

where ζ is the water surface elevation, u is the depth-averaged velocity, h is still water depth, g is the gravity, $\gamma (= 1/15)$ is the tuning parameter. The terms with $\partial h / \partial t$ and $h[\partial^3 h / (\partial x \partial t^2)]$ in Eqs. (1) and (2) express the change of bottom in time. The terms with $\partial h / \partial t$ and $h[\partial^3 h / (\partial x \partial t^2)]$ are multiplied by $1/\cosh^2(kh)$ and $1/\cosh(kh)$, respectively, in order to generate waves in whole range of water depth, i.e., from shallow to deep water. The values of $1/\cosh(kh)$ and $1/\cosh^2(kh)$ are equal to unity in shallow water and thus the effect of time-varying bottom variation effect is considered. On the contrary, the values of $1/\cosh(kh)$ and $1/\cosh^2(kh)$ are equal to zero in deep water and thus the effect of time-varying bottom effect is not considered.

An Adam-Bashforth-Moulton predictor-corrector scheme is used to discretize the extended Boussinesq equations in time. The spatial derivative terms are discretized with 5-point symmetric formula for nondispersive terms and with 3-point symmetric formula for dispersive terms. Initially the water surface elevations and the velocity potentials are set to be zero, i.e., $\zeta = u = 0$. Triangular bottom wave maker

is used to generate waves (See Fig. 1).

ANALYSIS OF WAVE GENERATION RESULTS

First, we generate nonlinear waves in different relative water depths of $h/L = 1, 0.25, 0.025$. As water depth changes from deep to shallow, maximum wave amplitudes increase in both magnitude and steepness causing asymmetric wave profiles. Fig. 2 shows power spectral densities of linear and nonlinear water surface elevations measured at position of $x = 2L$. In shallow water, high frequency components are clearly seen because nonlinear wave interaction happens significantly, while, in deep water, high frequency components are not seen because nonlinear interaction happens very little.

Second, we compare nonlinear water waves generated by the bottom wave maker and the conventional piston-type wave maker in shallow water. After simulating linear extended Boussinesq equations for the bottom wave maker, we got the ratio of wave amplitude to bottom wave maker amplitude $a/\Delta h_0$ with conditions of b/L and h_0/L where Δh_0 is the wave maker amplitude, b is the wave maker length, and L is the generated wave length. Using these results, we generate nonlinear waves with target amplitudes. For the piston-type wave maker, we generate nonlinear waves by specifying particle velocities at the wave maker with target wave amplitudes using the linear wave theory. At a position $x = 2L$ (from center wave maker), the primary frequency component is significantly large while high frequency components are smaller because nonlinear wave interaction does not occur near the bottom wave maker. The spectral densities of the primary component are similar to each other between the bottom wave maker and the piston wave maker. At a position away from the wave maker ($x = 7L$), the primary frequency component is reduced while the high frequency components become larger because nonlinear wave interaction occurs at locations away from the wave maker.

It has been known that the bottom wave makers can be used to efficiently generate linear waves in a flume without disturbances of reflected waves from structures (Jung et al., 2023). In the presentation, we will show whether the bottom wave makers can be used to efficiently generate nonlinear random waves in a flume without disturbances of reflected waves from structures. After simulating the extended Boussinesq model, we found that waves reflected from structures pass over the bottom wave maker with very little reflection and are absorbed in the sponge layer located at the domain end. Fig. 3 shows incident wave surface elevations at $t=72T$ and total wave surface elevations at $t=71.1T, 71.2T, \dots, 72T$ for the triangular bottom wave maker

with ratio of wave maker length to wavelength $b/L = 0.7$. In deep and intermediate waters, the standing wave heights on the right side are two times as the incident wave heights because of full reflection at the right boundary, the total waves on the left side are composed of the incident waves and the transmitting waves passing over the wave maker. The total wave heights are almost constant in space because wave reflection by the wave maker is negligibly small. In shallow water, due to nonlinearity, the total wave heights are not constant in space. We analyze the total water surface elevations by separating incident waves from the bottom wave maker and reflected waves from the right boundary which can be made by specifying particle velocity at the right boundary. As nonlinearity is significant, both incident and reflected waves are propagating unpredictably and thus have heights are not constant in space and time. However, even with high nonlinearity, the bottom wave maker is working well to pass through reflected waves because water surface elevations on the left side show a typical pattern of progressive waves with two components of incident and reflected waves propagating to the left.

Acknowledgements

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Reference

Jung, J.-S., Lee, C., Tran, M. T. and Park, Y. S., 2023: Generation of linear waves with bottom wave makers using analytical solutions and extended mild-slope equations, Journal of Waterway, Port, Coastal and Ocean Engineering, Volume 149 Issue 5

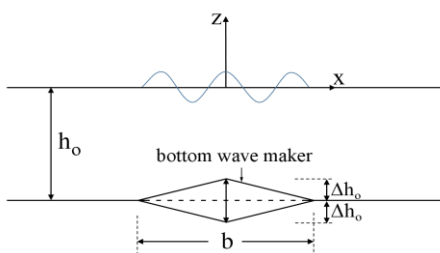


Figure 1 - Definition sketch for solution of wave generation with triangular bottom wave maker.

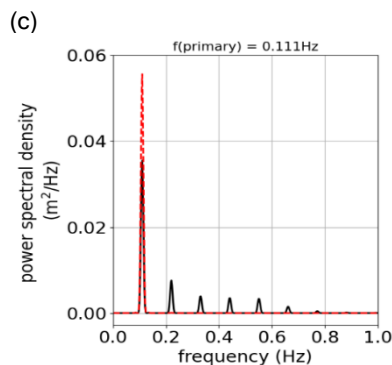
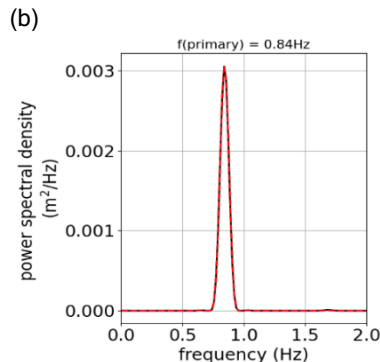
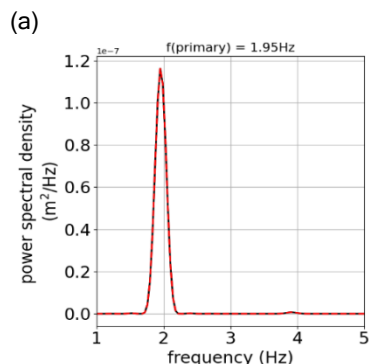


Figure 2 - Comparison of power spectral densities of linear and nonlinear water surface elevations generated by the bottom wave maker. (a) $h/L = 1$, (b) $h/L = 0.25$, (c) $h/L = 0.025$.

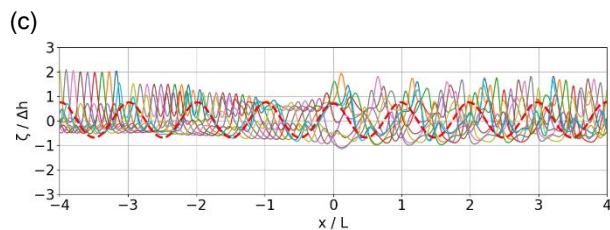
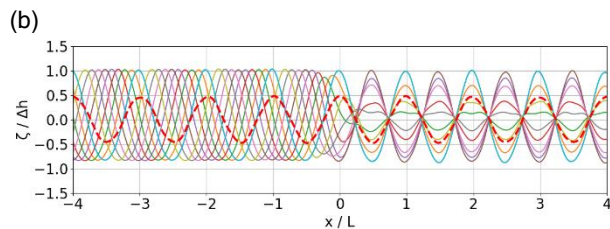
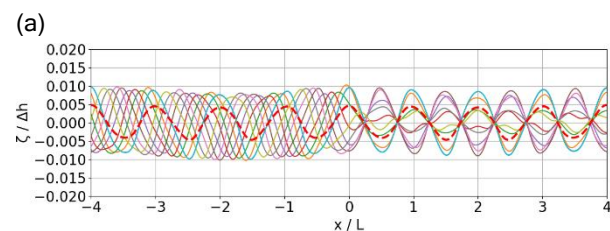


Figure 3 - Surface elevations of incident waves (dot) and surface elevations of total waves (solid line). (a) $h/L = 1$, (b) $h/L = 0.25$, (c) $h/L = 0.025$.