CHAPTER 89

BUOYANCY SPREAD OF WASTE WATER IN COASTAL REGIONS

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ABSTRACT

Waste water may due to buoyancy effects spread horizontally on top of the recipient. The spreading is chiefly determined by the buoyancy flux and to a large extent unaffected by dilution and diffusion. This phenomenon is of great practical importance for larger outfalls of waste or cooling water in coastal regions.

INTRODUCTION

The discharge of sewage water or cooling water into a recipient normally represents a buoyancy flux: The product of the discharge and the density difference between the receiving water and the waste water due to differences in salinity or temperature. In a homogeneous recipient this product is obviously indifferent to dilution.

The rising of the waste water to the surface if discharged below surface has been described in literature and will not be treated here.

The object of this paper is to describe what may be expected to happen when the diluted waste water at the rate of Q arrives at the water surface - or is trapped at an intermediate level - with almost the same horizontal velocity, U_0, as the main current. Due to the relative difference in density, Δ , the diluted waste water will try to spread in all directions.

BUOYANCY SPREAD WITHOUT VERTICAL TURBULENT DIFFUSION

The sheet of waste water formed on top of the main current is supposed to have a uniform thickness, h, in the direction perpendicular to the main current. According to ABBOTT (ref. 1) a density front, h in thickness, will advance with a velocity $v = \sqrt{\Delta gh}$ relative to the main current, g being the acceleration of gravity. With the notations of fig. 1 the front condition states

$$U_o \sin\theta = \sqrt{\Delta gh} \tag{1}$$

When turbulent diffusion is neglected, the continuity equation is

$$2\ B\ h\ U_o = Q \tag{2}$$

Integration yields

$$\frac{B}{B_o} = 1.31 \left\{\frac{x}{B_o}\right\}^{2/3} + 1 \tag{3}$$

where B_o is the initial half width of the sewage field, chosen so that the front velocity at this section equals the velocity of the main current:

$$B_o = \frac{\Delta\ g\ Q}{2\ U_o^3} \tag{4}$$

Only the product of Δ and Q, being equivalent to the buoyancy flux appears in Eq (3) and (4). For a homogeneous recipient the spread is evidently not dependent on the initial dilution at x = o.

Normally B_o is larger than the diameter of the rising jet of waste water or the width of the outlet of cooling water. If not the equation of spreading, Eq (3), will have to be modified but the dependence of the buoyancy flux will remain.

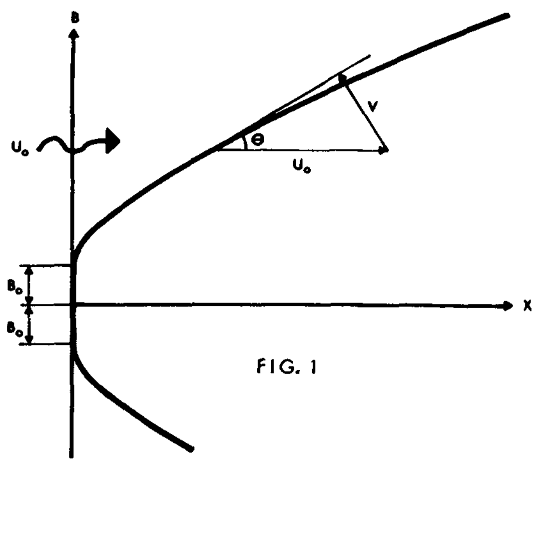

FIG. 1

TURBULENT DIFFUSION

Turbulent diffusion between two layers of slightly different density has been discussed by LARSEN (ref. 2). When turbulence is produced mainly outside the transition region it was by dimensional reasoning found that the velocity of exchange, W_e, between the layers could be expressed by

$$\frac{W_e}{\sqrt[4]{\nu\ w}} = f\left\{\frac{\rho\ w}{\Delta'\rho\ g\ \sqrt[4]{\nu\ w}}\right\} \tag{5}$$

where Δ' is the relative difference in density, ν is the kinematic viscosity, ρ the density of water and w the dissipation of energy into heat per unit mass of water. The function f $\{ - -\}$ would depend on the ratio between the rates of dissipation in the two layers assuming homogeneous turbulence. When the respective values of w in the two layers are equal, W_e represents the transfer in both directions but in case the level of turbulence in one layer is much lower than in the other the transfer expressed by W_e will only be from that layer to the other. It was estimated that the function f $\{ - -\}$ in both cases would be rather linear, the coefficient of proportionality, α , being of the order of magnitude 0.2 in agreement with experiments reported by ELLISON and TURNER (ref. 3). The dissipation of energy in the surface layer of a uniform current with a velocity U_o and a depth d is of the order of magnitude

$$w \sim \frac{U_o^3}{1000\ d} \tag{6}$$

and the rate of diffusion may then be estimated by

$$W_e \cdot \Delta' \cdot g \sim \frac{\alpha\ U_o^3}{1000\ d} \tag{7}$$

BUOYANCY SPREAD WITH DIFFUSION

The equation of continuity is

$$\frac{d}{dx}\left\{\Delta'\ B \cdot h \cdot U_o\right\} = -\ W_e \cdot \Delta' \cdot B \tag{8}$$

as well in the case of one sided vertical diffusion from the top layer to the underflowing recipient as in the case of symmetrical turbulent exchange between these layers. According to Eq (7) the right hand side of Eq (8) varies only with B. If $\Delta' \cdot h$ and B are considered as the dependent variables, the solutions to Eq (8) and Eq (1) are common for the two cases considered. In the former case Δ' is constant, Δ , and h varies with x. In the latter both Δ' and h vary with x but the total flow $2B \cdot h \cdot U_o$ is constant. (h is here an equivalent thickness of the surface layer).

A parameter of diffusion may be defined according to

$$M = \frac{W_e \cdot \Delta' \cdot g \cdot B_o}{U_o^3} = \frac{a\, B_o}{1000\ d} \tag{9}$$

The result of integration of Eq (8) and Eq (1) as shown in fig. (2) indicates that the variation of B with x is rather unaffected by

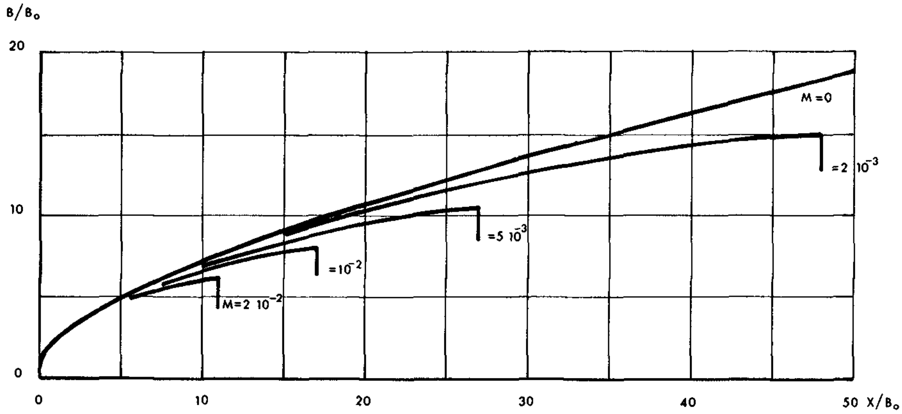

FIG. 2

the rate of turbulent diffusion expressed by the value of M as long as the surface layer exists. The width, B, of the sewage field may therefore be calculated according to Eq (3).

An evaluation of the distance x_m over which the waste water is able to spread due to density effects, may now be obtained by integration of Eq (8) using Eq (3). The result is shown graphically in Fig. 3.

In case of symmetrical turbulent exchange the dilution, S, of the waste water will be determined by

$$\frac{1}{S} = 1 - \frac{Mx}{B_o} \left\{ 0.78 \left(\frac{x}{B_o}\right)^{2/3} + 1 \right\} \tag{10}$$

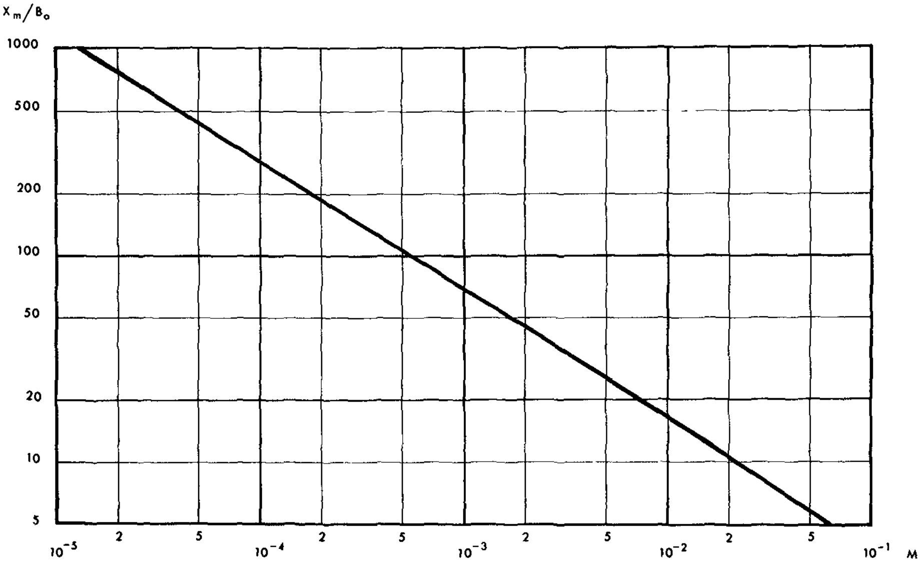

FIG. 3

NUMERICAL EXAMPLE

Waste water is discharged into a homogeneous recipient at the rate of $q = 10\ m^3/sec$. The relative difference in density between the undiluted waste water and the recipient is $\Delta_o = 25\ 10^{-3}$. The depth of the recipient is 10 m. One sided transport is considered and α is taken as 0.2. The dilution at the point of discharge is irrelevant, the buoyancy flux corresponds to $\Delta Q = \Delta_o\ q = 0.25\ m^3/sec$.

According to Eqs (3), (4) and Fig. 3.

U_o	m/sec	0.10	0.20	0.40
B_o	m	1250	156	19.5
M	-	0.025	0.0031	0.00039
x_m/B_o	-	9	35	120
x_m	m	11300	5500	2340
B_m/B_o	-	7	15	31
B_m	m	8700	2340	600

For smaller values of U_o the level of turbulence may no longer be determined by the bottom shear alone. This corresponds to larger values of M and the numerical results may be somewhat doubtful.

CONCLUSION

The theory presented above taking into account the vertical turbulent diffusion shows that the spreading of waste water due to density differences is determined chiefly by the buoyancy flux and to a large extent unaffected by dilution and diffusion. The theory is incorrect in many details but may be expected to give results of the correct order of magnitude. Calculations for outfalls from larger cities will show that places normally considered sheltered from the outfall zone by a region of considerable turbulent diffusion may in some cases be polluted by poorly diluted waste water. The influence of wind may add significantly to this effect.

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