CHAPTER 11

NON-STATIONARY SPECTRUM ANALYSIS OF OCEAN WAVES

by

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ABSTRACT

Priestley's method [1, 2, 3]** of non-stationary spectral analysis is extended to the case of digitally sampled records. A computation procedure previously proposed in an earlier work [4] is further investigated. This procedure compensates for the inherent difficulties in the theory and computes time-dependent spectra from a sample in an iterative manner. Validities of the theory and the iterative procedure are tested with a simulated non-stationary process. Results establish a general agreement with theory, especially when spectra vary smoothly with time and frequency. Consequently, the procedure is applied to estimate non-stationary spectra for two wave records. A comparison is made between the non-stationary estimates and the stationary results derived from the same set of wave records.

1. INTRODUCTION

Since the pioneering work of Pierson and Marks [5], stochastic spectral analysis of wave records is well accepted. Among many available published works on the subject, references [6, 7, 8, 9, 10, 11, 12] are excellent sources of information on the analysis, significance and the current state of the wave spectrum approach.

The most important constraint of the conventional spectral analysis is the assumption of stationarity, i.e., the fundamental character of the wave field does not change with time. Unfortunately, in many realistic situations, this

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** Brackets refer to references in the Appendix.

is an unsatisfactory restriction imposed by the lack of a physically meaningful and mathematically rigorous non-stationary spectrum concept. A few analysts tried an intuitive approach to compensate for time-dependency. For example, Wilson [13] computes time-variant spectra from overlapping segments of a wave record. Such attempts are useful but are still unsatisfactory because they do not have a formal basis for guidance and a clear design rationale for practical application.

Recently, Priestley has made a major effort to define a non-stationary spectrum as a smooth extension of the classical concepts and discussed a method of estimating time-dependent spectra from a single realization. This method is a generalization of the conventional analog approach. Therefore, it involves all the difficulties of the stationary analysis as well as new ones imposed by time-dependency. Specifically, the optimal design of a spectral computation requires a prior knowledge of frequency- and time-domain characteristics of the spectrum of the underlying process. These are normally unavailable in a realistic situation. Tayfun, et al [4], suggested a practical procedure to compute these parameters and time-dependent spectra in an iterative manner. The procedure was successfully tested on an artificially generated process in which spectral components have the same time-history, which is called a uniformly modulated process.

Processes that arise in applications such as ocean waves are realistically non-uniformly modulated. The purpose of this study is, therefore, to investigate the validities of the estimation method and the iterative procedure in the more general case of a non-uniformly modulated process, and, consequently, to be able to apply the iterative procedure to estimate time-dependent spectra for ocean wave records.

The discussion will mostly be limited to the basic ideas and the operational results in the case digitally sampled records. The concepts are simple extensions of those of the continuous case given in references [1, 2, 3]. The reader is referred to these references for details and elegant mathematical analysis of the underlying theory.

2. DEFINITIONS AND ASSUMPTIONS

A zero-mean non-stationary process $x(t)$ is represented in the frequency-domain as [1].

$$x(t) = \int_{-\infty}^{\infty} A(t,\omega)\, e^{i\omega t}\, dZ(\omega) \tag{2.1}$$

where $Z(\omega)$ is a process with uncorrelated increments, and $A(t,\omega)$ is a deterministic modulating function. We see that, if $A(t,\omega)$ equals one, the natural representation of a stationary process is recovered in terms of its generalized Fourier transform process, $Z(\omega)$.

The above representation (2.1) provides a spectral decomposition of the process $x(t)$ in terms of harmonic components with different frequencies and time-variant random amplitudes $A(t,\omega)dZ(\omega)$. The mean-square function of $x(t)$ is

$$E|x(t)|^2 = \int_{-\infty}^{\infty} |A(t,\omega)|^2 \; E|dZ(\omega)|^2 \tag{2.2}$$

where $E\{\cdot\}$ denotes an ensemble-average. Note that $E|x(t)|^2$ depends on time t, as expected, by virtue of the modulating function $A(t,\omega)$. From the definition that the spectral density is the frequency distribution of the mean-square, it follows that the spectral density at time t is

$$f(t,\omega) = |A(t,\omega)|^2 \; f(\omega) \tag{2.3}$$

where

$$f(\omega) \; d\omega = E|dZ(\omega)|^2,$$

with $f(\omega)$ regarded as the spectral density, say, at time t_o if we assume that $A(t_o,\omega) = 1$. Hence, the function $|A(t,\omega)|^2$ describes the change or evolution of that density at subsequent times.

Assume that a sea-surface record collected at a fixed spacial reference as a continuous function of time is a sample of a zero-mean process which admits a representation of the form (2.1). If the record is digitally sampled at a periodic sampling interval of Δt sec., then we form the sequence $x_1, x_2, \ldots, x_N$, where $x_t = x(t\Delta t)$, and $N = T/\Delta t$ with T the total record length in seconds. Furthermore, assume that the interval Δt is so chosen that

$$f(t\Delta t,\omega) = 0 \;, \; |\omega| \geq \pi/\Delta t, \text{ all } t \tag{2.4}$$

so as to make sure that no error will be introduced due to the well-known aliasing effects.

At this point it is convenient to regard the sequence x_t as if it consisted of prints at unit time intervals. This is equivalent to transforming the original frequency scale into a standardized dimensionless frequency, say ω', which is defined in $(-\pi, \pi)$ and such that $\omega' = \omega\Delta t$. Consequently, the spectral density, say $f_d(t,\omega')$, of the sequence x_t and that of the actual wave record $x(t\Delta t)$ relate to each other in the form

$$f(t\Delta t,\omega'/\Delta t) = \Delta t \cdot f_d(t,\omega') \;, \; |\omega'| < \pi \tag{2.5}$$

Associated with the discrete process from which the sequence x_t is assumed to be sampled, three dimensionless parameters, B_x, B_o and B_f are respectively defined as the "characteristic width," the time and frequency domain bandwidths of the spectral density f_d as a function of t and ω'. The characteristic width, B_x, is a measure of the maximum interval over which the statistical properties of the process are approximately stationary. B_o and B_f are of the order of magnitudes of the physical bandwidths based on the "half-power points" of the major peaks of $f_d(t,\omega')$ as a function of only t, and then ω', respectively. Both B_x and B_o are imposed by time dependency introduced through the modulating function $A(t,\omega')$. They relate to the inverse time-rate of change of $A(t, \omega)$, and therefore, to one another in the sense that if one is large, so is the other, and conversely. The explicit definitions of these parameters are given in [1, 2, 3].

3. ESTIMATION OF NON-STATIONARY SPECTRAL DENSITY

The "raw" and "smooth" spectral estimates of $f_d(t,\omega')$ in the neighborhood of t and ω' are given respectively by

$$|U_t(\omega')|^2 = \left|\sum_{u=t-N}^{t} g_u \, X_{t-u} \, e^{-i\omega'(t-u)}\right|^2 \tag{3.1}$$

and

$$\hat{f}_d(t,\omega') = \sum_{t-N}^{t} W_{T',u} \, |U_{t-u}(\omega')|^2 \tag{3.2}$$

where g_u is a digital "filter," and $W_{T',u}$ is a weight-sequence which depends on a parameter T'. Associated with the filter g_u and its Fourier transform $\Gamma(\omega')$, two parameters, B_g and B_Γ are respectively defined as measures of bandwidths in time and frequency. These parameters indicate the concentration of the functions g_u and $|\Gamma(\omega')|^2$ near their origins. We likewise define a parameter B_w as a measure of bandwidth for the weight-sequence $W_{T',u}$. Explicit definitions of the parameters B_g, B_Γ and B_w as well as certain suitability conditions which g_u and $W_{T',u}$ must satisfy are given in [1, 2, 3].

The raw estimate, $|U|^2$, is essentially a digital filtering of the sequence x_t through a filter g_u with a frequency response, $\Gamma(\omega')$, centered on frequency ω'. Under the conditions, $B_g \ll B_x$ and $B_\Gamma \ll B_f$, Priestley [1] shows that

$$E|U_t(\omega')|^2 \simeq f_d(t,\omega') \tag{3.3}$$

Hence, $|U|^2$ is an approximately unbiased estimate of $f_d(t,\omega')$. The condition $B_g \ll B_x$ implies that this filtering is to be done through a filter with a narrow bandwidth so that over the effective range of the filter the statistical properties are approximately constant. The condition $B_\Gamma \ll B_f$ requires that the spectral window, $|\Gamma(\omega')|^2$, be narrow enough just like a slit through which the value $f_d(t,\omega')$ can be observed without any contamination from neighboring values. As is well-known, a small bandwidth, B_g, in the time-domain corresponds to a large bandwidth, B_Γ, in the frequency-domain. Consequently, the estimation would be accurate for processes with spectra changing slowly over time and frequency (i.e., large B_x, B_o and B_f), so that a fulfillment of one of the conditions, $B_g \ll B_x$ and $B_\Gamma \ll B_f$, would not delete the other.

Note that the raw estimate is derived through a linear transformation of the random sequence x_t. Hence, it also is random in character, and not a useful estimate for practical purposes. It may be recalled that, in the conventional analog approach when x_t is in fact stationary, a stationary spectral estimate is obtained simply by averaging the raw estimates of the form (3.1) over the total length of the record. This enables one to approximate an ensemble-average as well as to obtain more stable estimates. Such an approach is not valid in the non-stationary case, since one cannot replace an ensemble-average with a time-average. Therefore, in order to account for the unstable nature of the raw estimate, $|U_t|^2$, it is smoothed over neighboring values of t, as implied by (3.2), resulting in a more stable "smooth" estimate, $\hat{f}_t$. This smoothing is done through a weight-sequence $W_{T',u}$ depending on the parameter T' which satisfies the condition $B_g << T' << \hat{N}$. The last condition implies that smoothing with $W_{T',u}$ should be over a range that is substantially larger than the effective width of g_u to achieve satisfactory stability. On the other hand, T' should be small enough so as to prevent excessive "smudging" of the temporal characteristics of the raw estimates.

Finally, note that, once $\hat{f}_d(t,\omega')$ is computed, we can obtain the smooth estimate, say $\hat{f}(t\Delta t,\omega)$, corresponding to the actual wave process through the transformation (2.5), i.e.,

$$\hat{f}(t\Delta t,\omega'/\Delta t) = \Delta t \cdot \hat{f}_d(t,\omega') \ , \ |\omega'| < \pi \tag{3.4}$$

is a smooth estimate of $f(t\Delta t,\omega)$ at time $t \cdot \Delta t$ (sec) and frequency $\omega = \omega'/\Delta t$ (rad $\cdot$ sec^{-1}).

4. SAMPLING PROPERTIES AND DESIGN RELATIONS

The overall sampling quality of the estimate $\hat{f}_d(t,\omega')$ is measured by its relative mean-square error at time t and frequency ω', i.e.,

$$M(h,T') = \frac{E|\hat{f}_d(t,\omega') - f_d(t,\omega')|^2}{f_d^{\,2}(t,\omega')} \sim \frac{1}{4}\left\{\frac{B_w^{\,2}}{B_o^{\,2}} + \frac{B_T^{\,2}}{B_f^{\,2}}\right\}^2 + \frac{C}{T'}\int_{-\pi}^{\pi} |\Gamma(\omega')|^4 d\omega' \tag{4.1a}$$

where h is a parameter related to the functional form of g_u, and C is a constant determined from the weight-sequence $W_{T',u}$.

Design of a spectral estimate involves determining the functions g_u and $W_{T',u}$. Functional form of these may be chosen from a standard collection of filter windows that already exists in the classical analysis. For example, in terms of the parameters h and T', we may choose a rectangular filter g_u and a discrete Daniell window $W_{T',u}$ of the forms

$$g_u = \begin{cases} \dfrac{1}{\sqrt{2\pi(2h+1)}} \ , & u = 0, \pm 1, \ldots, \pm h \\ 0 \ , & |u| > h \end{cases} \tag{4.2}$$

and

$$W_{T',u} = \begin{cases} \frac{1}{T'+1}, & u = 0, \pm 1, \ldots, \frac{1}{2}T' \\ 0, & |u| > \frac{1}{2}T' \end{cases} \tag{4.3}$$

The above functional forms will be adopted in the subsequent discussion and applications for illustrative purposes. On the basis of (4.2), (4.3) and from (4.1a), we may show that, for large values of h and T'

$$M(h,T') \simeq \frac{1}{4}\left\{\frac{B_W^2}{B_o^2} + \frac{B_\Gamma^2}{B_f^2}\right\}^2 + \frac{4h}{3T'} \tag{4.1b}$$

where $B_W = T'/\sqrt{12}$ and $B_\Gamma \sim \pi/h$, an approximation based on half-power points of the major peak of $|\Gamma(\omega')|^2$.

The parameters h and T' may now be determined as follows:

(a) Fixed Frequency-Domain Resolution: For each t, $\hat{f}_d(t,\omega')$ is required to have a prescribed degree of resolution in frequency-domain, i.e., we require $B_\Gamma/B_f = \lambda$, where λ is a small prescribed constant. Here, $B_\Gamma \sim \pi/h$. Hence, given B_f, h is the largest integer smaller than

$$\frac{\pi}{\lambda B_f} \tag{4.4}$$

Now, T' is chosen so as to minimize the relative mean-square error, M, now a function of only T'. Subject to the condition $B_g < T' < N$, the optimum value of T' is easily obtained from (4.1b) as the largest even integer smaller than

$$[192\ h\ B_o^4]^{1/5} \tag{4.5}$$

(b) Fixed Resolution in Time and Frequency-Domains: In this case, we set $B_\Gamma/B_f = \lambda$ and $B_W/B_o = \mu$, where μ is likewise a small prescribed constant. Assuming B_o and B_f are available, h and T' are determined from

$$h \sim \frac{\pi}{\lambda B_f}, \text{ and } T' \sim \sqrt{12}\, B_o \mu \tag{4.6}$$

The sampling quality of the estimates is now determined by the above conditions and from (4.1b), the relative mean-square error.

5. FEATURES AND INHERENT DIFFICULTIES

The estimation method described above is a smooth generalization of the conventional analog approach to non-stationary cases. Especially with the particular choice of $W_{T',u}$ here, the technique is roughly equivalent to estimating spectra from overlapping segments of a record. This is achieved through a formal consideration of the temporal characteristics (i.e., B_x and B_o) of the underlying process. In the limit case of stationarity, B_x and B_o both become infinite and smoothly disappear from the formulations as the analysis reduces to the conventional results. In that case, a stationary spectral estimate, say $\hat{f}(\omega)$, is obtained by

$$\hat{f}(\omega'/\Delta t) = \frac{\Delta t}{N-2h} \sum_{t=h+1}^{N-h} |U_t(\omega')|^2, \tag{5.1}$$

where $|U_t|^2$ is given by (3.1) and limits are chosen over the range of t in (1, N) for which $|U_t|^2$ can be evaluated from (3.1) by using a filter g_u of the form (4.2). This simply corresponds to an averaging of the squared filter output, $|U_t|^2$, over the complete record length, just as in the conventional analog approach.

The method involves all the difficulties associated with the classical analysis as well as new ones imposed by time-dependency. A rational application requires a knowledge or a rough estimate of the parameters B_x, B_o and B_f as a background information for design. These are unavailable a priori on the basis of a given sample record in a realistic situation such as ocean waves, and therefore, form the major source of difficulty in application.

6. A PRACTICAL PROCEDURE

Consider a practical situation where it is required to estimate the non-stationary spectral density, $f(t\Delta t,\omega)$, of a wave process on the basis of a digitally sampled record $x(t\Delta t)$, where $t = 1, \ldots, N$.

The first objective is to determine the process parameters B_x, B_o, and B_f in a rational way. At present, there seems to be no way of estimating a lower bound for the characteristic width B_x on the basis of a sample record. However, we recall that the parameters B_x and B_o are related to each other in the sense that if one is large, so is the other, and conversely. Hence, if we could estimate B_o, then it could be used as a measure of the temporal characteristics of the wave process. Furthermore, design relations are dependent on B_o and B_f. A rational design for the parameters h and T′ requires, therefore, a background information about B_o and B_f. An iteration scheme may be suggested to compute the parameters B_o and B_f, and consequently, h and T′ in the following manner.

We perform a pilot-estimation by using "trial values" for h and T' such that $h < T' < N$. Accordingly, a value h can be chosen arbitrarily. Then, we may choose $T' \sim rh$, where r is a constant larger than unity, and $f_d(t,\omega')$ is "pilot-estimated" for several values of t. Recall, from (4.1b), that the dimensionless variance of the estimates is given by

$$\frac{\operatorname{var} \hat{f}_d(t,\omega')}{f_d^{\,2}(t,\omega')} \sim \frac{4h}{3T'} \sim 0(\frac{1}{r}) \tag{6.1}$$

Therefore, the larger r is, the more stable the pilot-estimates will be. It now becomes possible to estimate B_o and B_f on the basis of the pilot-estimates of $f_d(t,\omega')$ and according to the definitions of these parameters conceptually illustrated in Figure 1. After B_o and B_f are computed, using one of the design criteria discussed in Section 4, we determine "refined values" h and T'. The same procedure is iterated for enough number of times until the parameters B_o and B_f (therefore h and T') simultaneously attain a convergent behavior. The spectral estimate $\hat{f}(t\Delta t,\omega)$ corresponding to the wave process is derived from the values $\hat{f}_d(t,\omega')$ in the convergent cycle through the transformation (3.4).

Intuitively, the success of the procedure would be affected by the degree of "smoothness" of the true spectral density $f(t\Delta t,\omega)$ as a function of t and ω. This is due to "blurring" and "smudging" effects of filtering and smoothing operations as discussed previously. The more detailed features of true $f(t\Delta t,\omega)$ as a function of t and ω are likely to be obscured by the estimation procedure.

7. APPLICATIONS

An Artificial Non-Uniformly Modulated Process

This example is a non-stationary process in discrete time with a frequency-domain representation of the following general form, i.e.,

$$x_t = \int_{-\pi}^{\pi} A(t,\omega)\, e^{it\omega}\, dZ(\omega) \tag{7.1}$$

In general, a process x_t of the form (7.1) has an equivalent interpretation in the time-domain in terms of a linear time-variant filter and a suitable stationary process (see Priestley [1]). For example, we may write

$$x_t = \sum_{u=-\infty}^{\infty} h_{t,u}\, V_{t-u}\ , \tag{7.2}$$

if we formally define

$$h_{t,u} = \frac{1}{2\pi} \int_{-\pi}^{\pi} A(t,\omega)\ e^{iu\omega}\ d\omega \tag{7.3}$$

as a time-variant filter, and specify V_t as a second-order stationary process of the form

$$V_t = \int_{-\pi}^{\pi} e^{it\omega}\ dZ(\omega) \tag{7.5}$$

in the frequency-domain. Note that $E|dZ(\omega)|^2 = f_V(\omega)d\omega$, with $f_V(\omega)$ defined as the spectral density corresponding to the stationary process V_t.

The theoretical spectral density, say $f_x(t,\omega)$, of the non-stationary process x_t is, by definition

$$f_x(t,\omega) = |A(t,\omega)|^2\ f_V(\omega) \tag{7.6}$$

In order to illustrate and test the estimation method and the iterative scheme on a non-uniformly modulated process of the form (7.1), we may now choose some explicit functional forms for the processes V_t and X_t such that

$$V_{t+2} + 0.5\ V_t = N_t \tag{7.7}$$

in which N_t are independent random variables, identically distributed as $N(0,3^2)$, and

$$A_{t,\omega} = 5\left(\frac{t}{500}\right)^2 \left\{\frac{e^{-0.2|\omega|}}{4}\left(\frac{t}{600}\right)^2 - e^{-0.1|\omega|}\ \frac{t}{600} + 1\right\} \tag{7.8}$$

where $|\omega| < \pi$, and $t = 0, 1, \ldots, 1400$. It can easily be shown that the process V_t given by (7.5) has a frequency-domain representation of the form (7.5) with

$$f_V(\omega) = \frac{3^2}{2\pi}\ \frac{1}{(0.25 + 2\cos^2\omega)}\ , \quad |\omega| < \pi \tag{7.9}$$

The above form of $A(t,\omega)$ is, for any frequency ω, basically a fourth-order polynomial of the form $at^4 + bt^3 + ct^2$. The coefficients as functions of the parameter ω are deliberately so chosen that the function $A(t,\omega)$ smoothly changes with respect to t, and its transform with respect to ω, i.e., $h_{t,u}$ given by

$$h_{t,u} = \frac{5}{2\pi} \left(\frac{t}{600}\right)^2 \left\{ \left(\frac{t}{600}\right)^2 \frac{0.1}{0.04+u^2} \left[(-1)^{u+1} e^{-.2\pi} + 1\right] - \left(\frac{t}{600}\right) \frac{0.2}{0.01+u^2} \left[(-1)^{u+1} e^{-.1\pi} + 1\right] + 2\pi\, \delta_{0,u} \right\} \quad (7.10)$$

falls off rapidly to zero as u becomes large. Therefore, these functional forms are of convenience in the simulation of a sample of x_t. This is done as follows: we first generate a sufficient number of independent normal variables with zero means and standard deviation 3 corresponding to N_t. A sample of V_t is obtained through (7.7) iteratively. In so doing, a sufficient number of initial V_t's are neglected to account for the transient. Finally, the sample x_t is obtained through (7.10) and (7.2), using only a finite number of terms in the summation. The sample simulated in the described manner is shown in Figure 2 for values t = 0, 10, ..., 1400, to give a general idea on the temporal behavior of the process studied.

The process parameters B_o (~210) and B_f (~0.70) are approximately estimated from the theoretical spectral density $f_x(t,\omega)$. For this example, a fixed frequency-time-domain resolution criterion with $\lambda = 1/2$, $\mu = 1/3$ is adopted. Therefore, from (4.6), we have h (~9) and T′(~240). The corresponding relative mean-square error is approximately 8%. Some of the theoretical spectra, $f_x(t,\omega)$, and the estimates $\hat{f}_x(t,\omega)$ at various times based on the simulated sample with the above values of h and T′ are illustrated in Figure 3 with solid and dashed lines (simulation 1), respectively. The estimates compare favorably with the theoretical ones, and indicate to the validity of the concepts and the estimation method.

Table 1 illustrates convergence of process and design parameters in the iterative procedure based on the same design criterion and two pairs of trial values h and T′. The convergent parameters h (~12), T′ (~210 and 220), B_o (~180) and B_f (~.5) compare fairly with those (i.e., h = 9, T′ = 240, B_o = 210, B_f = 0.70) obtained on the basis of a prior knowledge about the underlying process x_t. The estimates based on the iterated parameters h = 12 and T′ = 220 are also presented in Figure 3 by dotted solid lines (simulation 2) for comparison with the first estimates as well as the theoretical ones. In this case, again, the comparison is fairly satisfactory and indicates the validity of the iterative scheme.

Atlantic City Record

The example we consider here is a twenty-minute sample from the wave data collected at Atlantic City, New Jersey, on May 23, 1969 (14:00 ~ 14:20). The data was available in the form of punched cards, and sampled at intervals Δt = 0.25 sec. Table 2 shows the convergence of design and process parameters in interation based on the trial values h (~10 sec.) and T′ (~100 sec.), and a fixed frequency-domain resolution criterion with $\lambda = 1/2$. Some of the non-stationary estimates corresponding to the convergent parameter h (~24 sec.) and T′ (~270 sec.) are presented in Figure 4 at various times. The relative mean-square of an estimate at any time and frequency is approximately 13%.

For comparison, a stationary analysis was also performed on the same wave record. This is done using results of the non-stationary analysis for the special limit case of stationarity as described briefly in section 5. The stationary estimate, also shown in Figure 4, is similarly based on a frequency resolution of $\lambda = 1/2$, and a relative mean-square error of 6% with 60 equivalent degrees of freedom.

A comparison of the non-stationary spectral estimates at various times with the stationary estimate indicates that they generally differ in both magnitude and shape. The location of the major spectral peak and the magnitudes of the non-stationary estimates differ from the conventional in an unconservative manner, indicating that the wave field is in a general time-dependent state. We believe that the stationary analysis in this case, by smearing and smoothing out the temporal changes, may not provide all the critical spectral characteristics of waves.

	TRIAL VALUES		ITERATIONS				
			1	2	3	4	5
ATLANTIC CITY	h (sec)	10	16	21	22	24	24
	T' (sec)	100	172	216	260	270	270
	B_o (sec)		90	100	132	134	134
	B_f (sec^{-1})		.4	.3	.28	.26	.26
HURRICANE DORA	h (sec)	8	11	11	13	13	
	T' (sec)	120	176	220	280	284	
	B_o (sec)		90	120	160	165	
	B_f (sec^{-1})		.6	.56	.52	.5	

Table 2. Convergence of Parameters in Iteration For Atlantic City and Hurricane Dora Records.

Hurricane Dora

This is an approximately twenty-five minute sample from the wave data collected offshore Panama City, Florida, during Hurricane Dora while it crossed the northern part of Florida on September 9-10, 1965. The data was collected at a multi-legged tower (Stage I) in about 100 ft. of water at 11 nautical miles offshore. The record time of the waves examined here is 17:00 - 17:25 on September 10, the same data analyzed by J. I. Collins [14] through the conventional autocorrelation method of Blackman and Tukey.

On the basis of the trial values h (~8 sec.) and T′ (~120 sec.), and a fixed-frequency-domain criterion with $\lambda = 1/2$, the iterative scheme is carried out. The convergence of the parameters in iteration is presented in Table 2. Some of the non-stationary estimates corresponding to the convergent parameters h (~13 sec.) and T′ (~284 sec.) are presented in Figure 5 together with the stationary estimate obtained on the same record with respect to the same design criterion with $\lambda = 1/2$. The non-stationary estimates and the stationary one have 12% and 4% relative mean-square errors, respectively. The equivalent degrees of freedom corresponding to the stationary estimate is 150.

The non-stationary estimates generally differ from the stationary results. However, these differences in magnitude and shape are not very significant, indicating the waves are in a more or less steady state. In this case, we believe that the conventional analysis seems to provide the essential spectral characteristics of the wave field. This also confirms the fact that the results reduce to the conventional when stationarity dominates.

8. SUMMARY AND CONCLUSIONS

Priestley's estimation method and an approximate iterative procedure applicable to the realistic spectral analysis of ocean wave records with intrinsic non-stationarity have been examined. The validities of the concepts and the suggested iterative procedure have been tested with a simulated non-stationary process, and consequently applied to estimate time-dependent spectra for two wave records. Some characteristics results have been presented.

We may summarize the results of this study as follows: the results of the estimation method from a simulated non-stationary process compare favorably with the theoretical ones. This is especially true when the underlying spectrum changes smoothly with time and frequency. We believe that ocean wave records intrinsically satisfy these conditions. The subsequent application of the concepts to two actual wave records indicates that the time-dependent estimates generally differ in magnitude and form from the conventional stationary results. Differences in terms of magnitudes, shape and location of a major spectral peak are significant in one instance, but not so in the other. This may suggest that the stationary spectral analysis may not provide all the spectral characteristics of a wave process due to a smearing effect of the temporal changes. However, the method is generalized in the sense that the results reduce to the conventional if stationarity is dominant.

Note that the estimation method does not depend on an explicit knowledge of the modulating function $A(t,\omega)$. However, accuracy of the estimates directly relates to how "slowly" time-variant $A(t,\omega)$ is implicitly. During the conference presentation of the paper, J. I. Collins and L. E. Borgman raised the question whether one can determine the squared modulating function, $|A(t,\omega)|^2$. It seems that an approximate estimation of this function is possible via fitting a non-stationary time-series model with time-dependent parameters to the given sample record. For the details of such an approach, the reader is referred to the work of T. S. Rao [15].

It is hoped that this study, although limited in particular applications here, will provide some insight to the problem in general, as well as be a practical tool for application.

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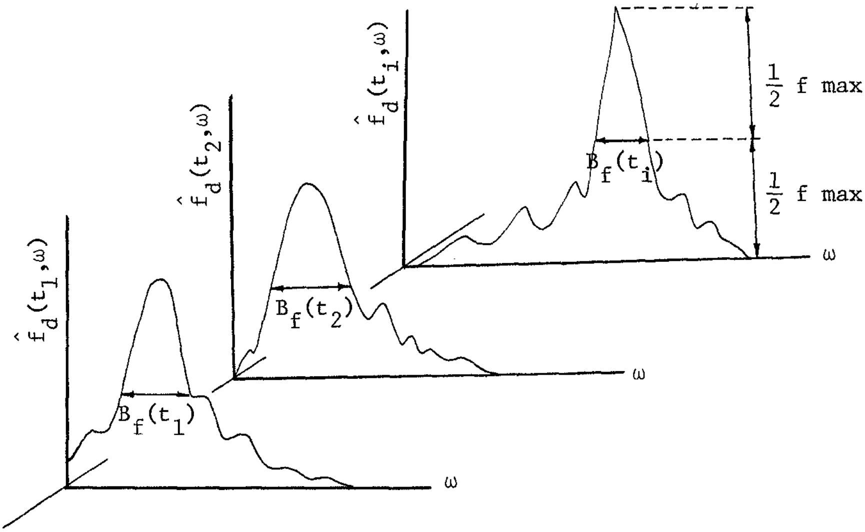
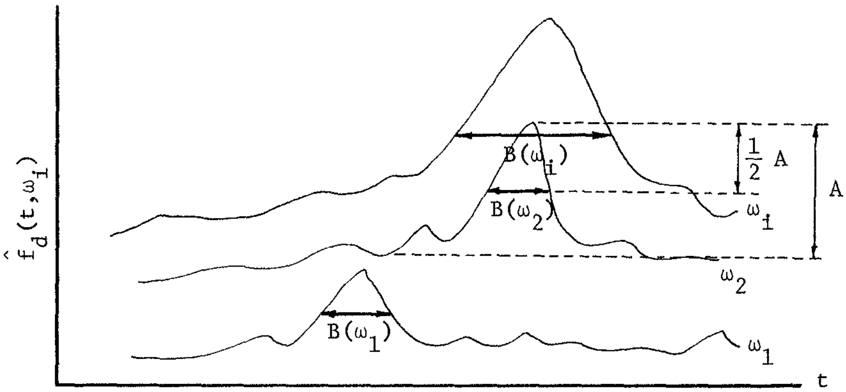


$$B_f = \min_{t_i} \; B_f(t_i)$$



$$B_o = \frac{1}{2} \min_{\omega_i} B(\omega_i)$$

Figure 1. Estimation of B_o and B_f.

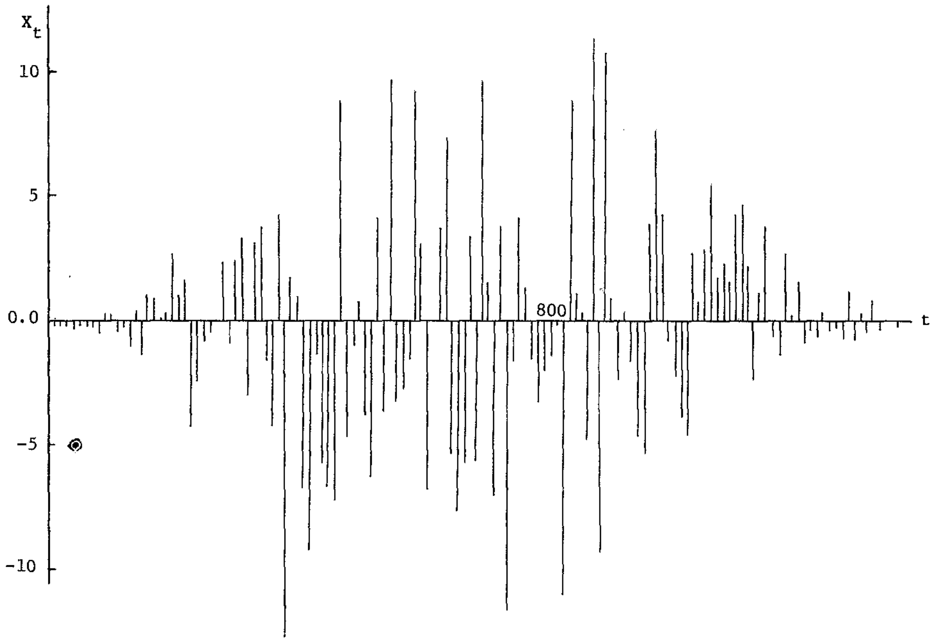


Figure 2. Simulated Non-Stationary Sample (t = 0, 10, ..., 1400).

TRIAL VALUES		ITERATIONS		
		1	2	3
h	5	9	12	12
T′	120	200	230	220
B_o		200	190	180
B_f		0.7	0.5	0.5
h	20	16	12	12
T′	350	230	210	210
B_o		200	180	180
B_f		0.4	0.5	0.5

Table 1. Convergence of Parameters in Iteration For the Simulated Example.

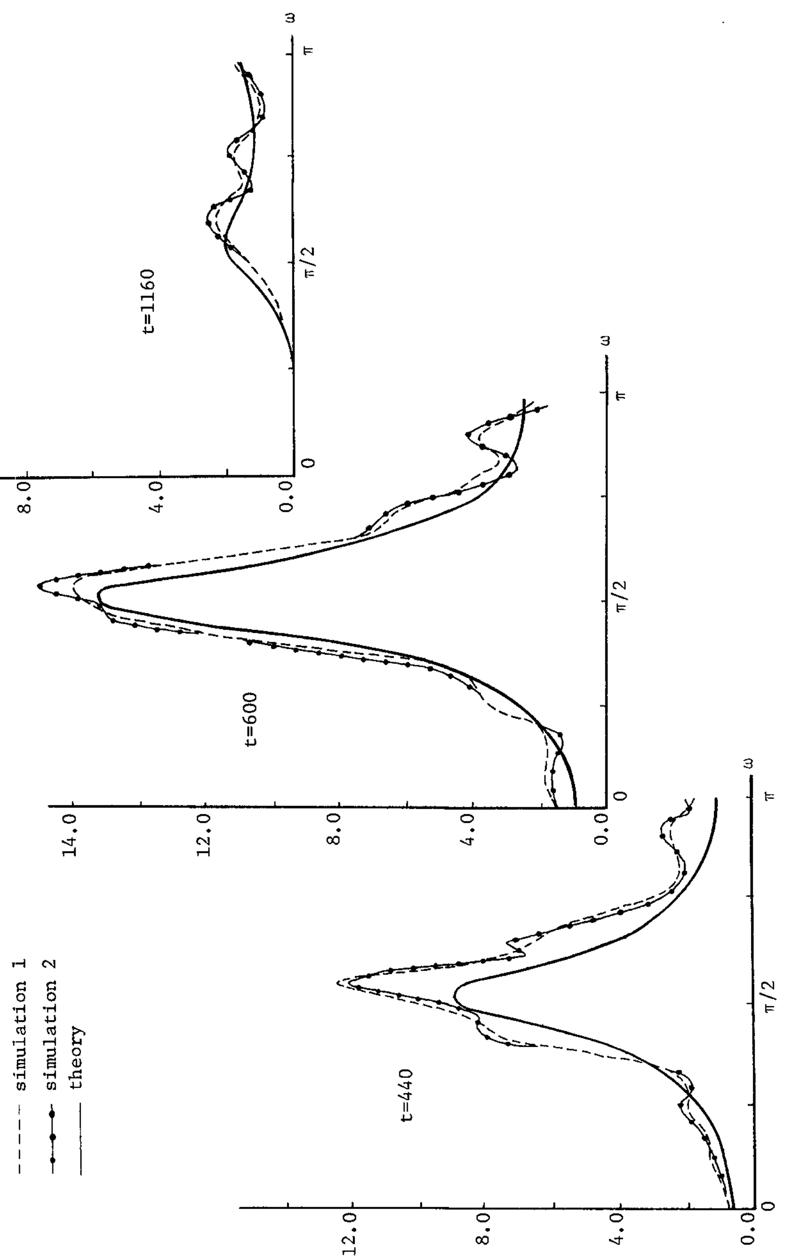


Figure 3. Non-Stationary Spectra for the Simulated Example.

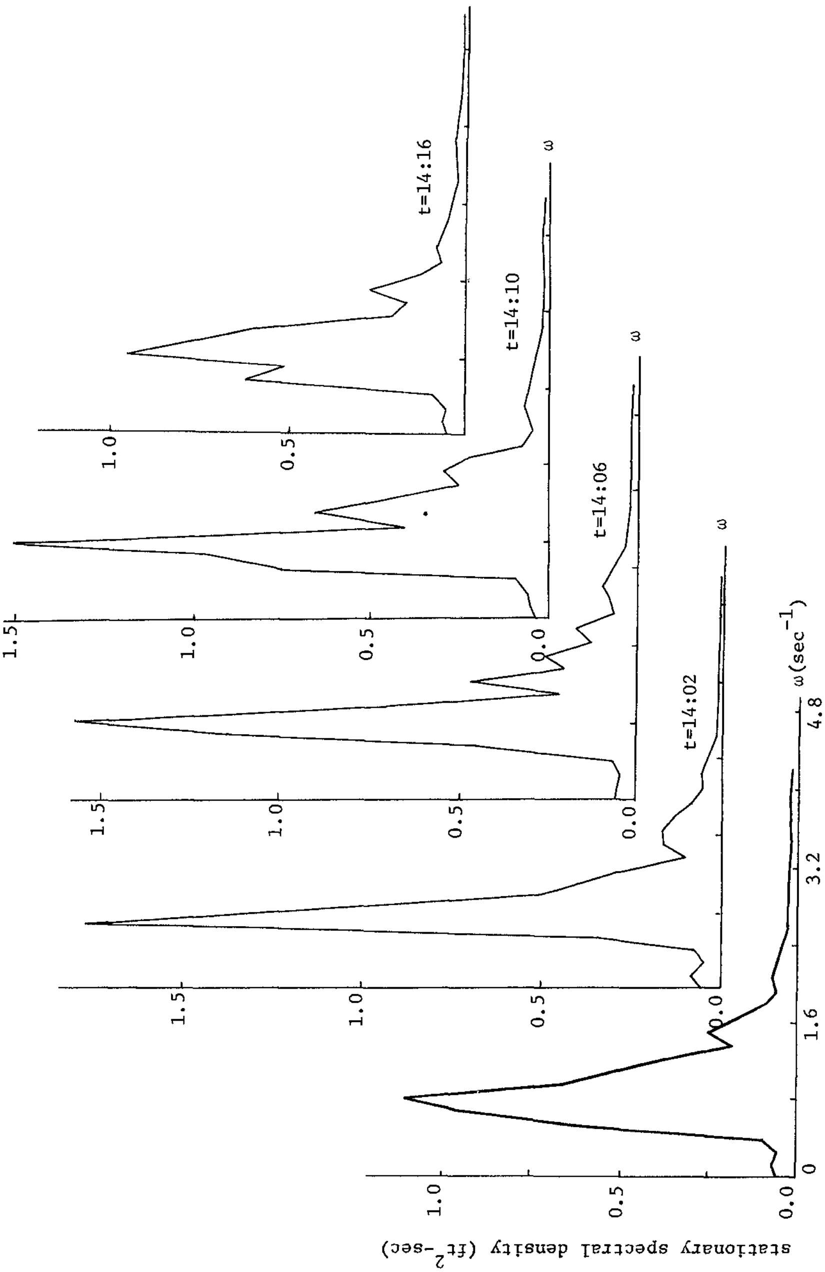


Figure 4. Stationary and Non-Stationary Estimates for Atlantic City.

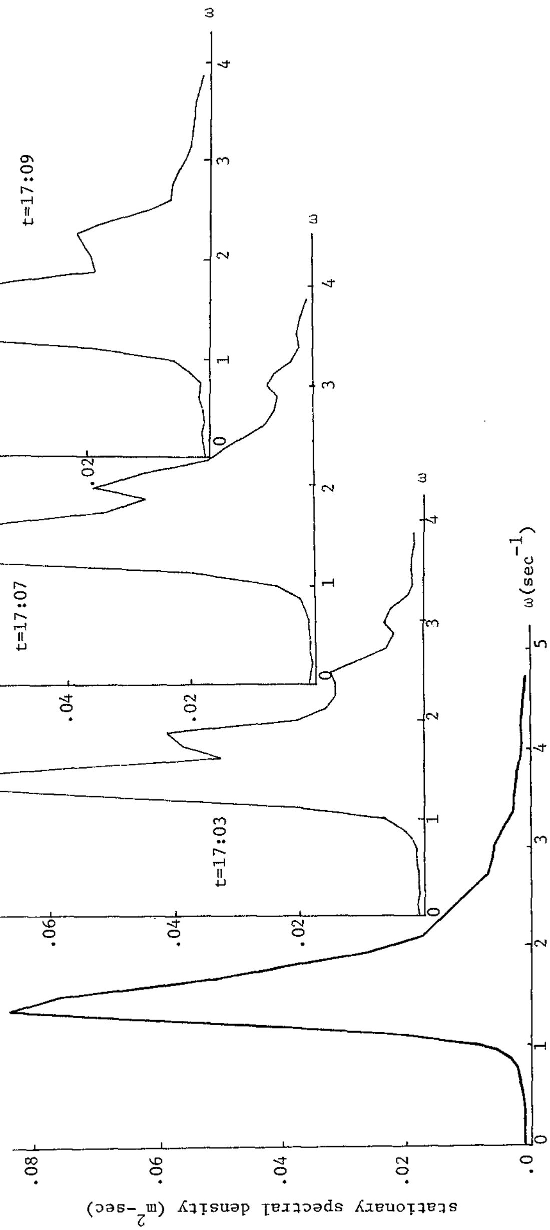


Figure 5. Stationary and Non-Stationary Estimates for Hurricane Dora.