CHAPTER 123

AN EVALUATION OF MIXING IN THE TAY ESTUARY

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ABSTRACT

The authors consider the derivation of an sxprsssion for ealinity distribution in an sstuary, from an instantaneoue mass balance equation for a solute in a continuum. Ths exprsssion is compatible with existing field instrumentation and limited economic resources. Details are given of a prototype survsy which included continuous monitoring of the salinity distribution of the Firth of Tay, an estuary subject to a wide range of fluvial and marine influencee. The survey results enabled an apparent dispersion coefficient to be svaluated and an estimate to bs made of the net mixing.

INTRODUCTION

An important feature of estuarine management and research is a knowledge of the temporal and spatial distribution of salinity with respect to fluvial and marine influences.

At present there exists an incomplete understanding of the solute transport aesociated with turbulent flow in the prssence of density gradients, that is characteristic of many partially and apparently well mixed estuaries. Study of this phenomenon is hindered by the difficulty of directly monitoring turbulent fluctuations of fluid velocity and soluts concentration, and the wide range of the tsmporal and spatial variation of theee variables for given influences sxternal to the estuarine system. A further difficulty is that in many estuariee, the components of these marine and fluvial influences exhibit wids variations in both magnitude and time ecale.

A mathematical representation of solute distribution may be formulated from the fundamental principle of mass balance. Its practical application in the absence of a priori relationehips for the variables describing mass traneport, can only bs achieved by the application of statistical averaging, which leads to the assumption of empirical functions that requirs evaluation through the comparison of thsory with prototype data.

The results predicted with such a modsl must be treated with caution unless ths dependence of ths empirical functions is wsll understood for ths relevant input conditions to ths system. The expense and difficulty of collecting prototype data requires the use of the simplest form of model compatible with an adequate return of information.

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In thie papsr, a description ie given of the use of a ons dimsneional form of the solute mass balance squation in ordsr to obtain a mathematical repressntation of salinity dietribution in the Tay Estuary. Refersnce ie made to the proper reduction of dimensionality and time averaging of the inetantaneous three dimeneional soluts mass balance equation. The conditione of significant tidal variation of both channel cross eectional area and solute concentration are recognieed. An apparent coeffioient of diepereion and a net mixing term are defined, svaluatad and correlatad to fluvial discharge and distance along the estuary.

THE TAY ESTUARY

The estuary links a catchment having an area of approximately 7,500 km^2 through 50 km of tidal watere to the North Ssa. The River Tay and ite main tributary, the River Earn, have mean discharges of 160 cumecs and 20 cumecs (1) reepectively. The combined monthly mean dischargee ehows an annual variation ranging from 60 cumecs to 250 cumece (calculated from 15 yeare data).

The mean tidal ranges of epring and neap tides are 4.7 m and 2.3 m at the bar (2). Ths amplitude of the tidal wave ie similar at the mouth of the eetuary and at Dundee, and thence it undergoes a gradual reduction until the relevant tidal limit is reached.

The crose eectional area of flow at mean water level is fairly conetant as far as Dundee and then decreasee approximately linsarly with longitudinal dietance to Newburgh where the channel becomee of the more uniform nature characterietic of a fluvial regime. Betwsen Newburgh and Balmerino, a main channel, bordered by tidal flats, closely follows the southern shore. It then aesumee a more central poeition through the moving sand bank complex upstream of Broughty Ferry, whereafter it reachee the mouth of the setuary in a well defined channel bordered by ehoale and tidal flate. A maximum depth of 30 m occurs near Broughty Ferry, but generally depths range from 20 m to 2 m relative to low elack water levels. A datailed description of the bathymetry and eedimsntological featuree of the eetuary ie given slsewhere (3).

The ealinity intrueion extends to the Newburgh region during low river flowe. Generally the difference between ealinity near the free surface and near the bed rangee from leee than 0.1 ppt to the order of 2 - 3 ppt. For particular conditions of wind and tide, or near slack water, coneiderably greater differences may occur.

ONE DIMENSIONAL REPRESENTATION OF SOLUTE DISTRIBUTION

A eolute mase balance equation bassd on fundamental physical principles may be derived for turbulent flow in eetuariss. Aseuming the variation in fluid daneity to bs small compared with that of the other variablee, and that the effects of molecular diffusion may be neglected, then the equation for a/

conservative solute may be written as

$$\frac{\partial C}{\partial t} + \frac{\partial}{\partial x_i}(U_i C) = 0 \qquad (1)$$

where i = 1, 2 or 3, refers to a cartesian coordinate system

C = Solute concentration

U_i = Component of fluid velocity

t = Time

The reduction of dimensionality of equation (1) has been shown by Pritchard (4), with the "turbulent" terms introduced by a temporal average over a small time interval, to give

$$\frac{\partial}{\partial t}(C_A A) + \frac{\partial}{\partial x}(A U_A C_A) + \frac{\partial}{\partial x}(A \langle\langle u'C' \rangle_{\Delta t} + \langle u'_A C'_A \rangle_A) = 0 \qquad (2)$$

where x = Distance from mouth of estuary

A = Lateral cross sectional area of channel

Δt = A small time interval

$f_d = \langle f \rangle_d = \frac{1}{d}\int_d f\, dd$ & $f = f_d + f'_d$

f = A function dependent on d

Further temporal averaging over a tidal period has been considered by Okubo (5) and modified by Williams and West (6) for the case where there exists a significant tidal variation of cross sectional area and of the spatial mean value of solute concentration. Assuming steady state conditions gives

$$\frac{\partial}{\partial x}(C_{AT} Q_T) + \frac{\partial}{\partial x}(\langle A(u'_{AT} C'_{AT} + \langle\langle u'C' \rangle_{\Delta t} + \langle u'_A C'_A \rangle_A + A'_T C'_{AT} u_{AT} \rangle_T) = 0 \qquad (3)$$

where $f_{AT} = \frac{1}{T}\int_T f_A\, dt$

f_A = Function dependent on t

T = Tidal period

Q = Fluvial discharge

An inspection of the terms in equation (3) is useful at this stage. A value of the term Q_T may be obtained by standard river gauging techniques and a good estimate of C_{AT} for salinity at any section may usually be calculated from data acquired using a small fast craft and in situ salinity measurement procedures.

To the authors knowledge, the useful measurement of the turbulent perturbations U_i and C_i in estuarine flow is impractical at present. The measurement of turbulent mean velocity components ($\overline{U}_i$) is technically feasible, but gathering detailed data is generally economically prohibitive in most natural tidal channels having an irregular cross section.

Thus in practice it is convenient to rewrite equation (3) using a/

coefficient $D_{x_{AT}}$ to give

$$\frac{\partial}{\partial x}\left(C_{AT}Q_T\right) - \frac{\partial}{\partial x}\left(A_T D_{x_{AT}} \frac{\partial c_{AT}}{\partial x}\right) = 0 \qquad (4)$$

where $D_{x_{AT}}$ is dsfined as

$$D_{x_{AT}} = -\frac{1}{A_T \frac{\partial c_{AT}}{\partial x}}\left(\langle A\left(u'_{AT}c'_{AT} + \ll u'c' \gg_{\Delta t} + u'_A c'_A\rangle_A\right) + A'_T C'_{AT} u_{AT}\rangle_T\right) \qquad (5)$$

The precise physical meaning of $D_{x_{AT}}$ is not easily understood. Howsver, its use introducss uniformity and simplicity into the algebra and psrmits practical rssults to be achieved. The intuitive uss of a turbulent diffusion coefficient (D_{ij}) has besn discusssd by many authors (for example Okubo (7)) but further progress to a ussful end, analytically or numerically of necsssity requirss the condition.

$$D_{ij} = 0 \qquad i \neq j$$

where D refers to elements of the turbulent diffusion

and i, j = 1, 2 or 3

This assumption is difficult to justify in the light of the present understanding of estuarins flow, but is often accspted in the processs of model development. On this basis the tsrm $D_{x_{AT}}$ is proposed. It is a function of mean (A, T) concentration gradient, the spatial and temporal variation of fluid velocity and solute concentration, and the temporal variation of cross sectional area. Herein ths term $D_{x_{AT}}$ is loossly termsd an apparent dispersion cosfficisnt. It should be noted that it is necsssary to include a convective term with the non-convective terms.

DATA ACQUISITION AND ANALYSIS

Turbulent mean values of salinity at a point are likely to bs ths combined result of marine and fluvial inputs to the estuarinssystem over a psriod of time of the order of weeks. To establish the magnitude and tims scale of these effects the continuous monitoring of salinity, with respect to tidal range and fluvial discharge, was arranged at a point approximatsly equidistant bstwsen the mouth of the estuary and the upstream limit of salinity intrusion. Ths tidal exc ursion in the Tay is such that at a point so placed, the mean (T) salinity is a function of conditions ovsr 75% of the intrusion length.

Water sampling instrumentation and a water levsl recordsr wers installed at Newport Pier. The sits had the advantages of being accsssibls in all anticipated weather conditions and was for most of a tidal cycls subject to a strong tidal current. Hourly water samplss having a volume of 600 - 800 ml were

collectsd and analysed for salinity using a salinity/temperaturs bridge. The automatic sampler (North Hants. Eng. Co. MK IV) consists essentially of twenty four evacuated bottles with clockwork actuated valves. To be independent of mains electricity on an sxposed site was considersd to be important. The suotion head of 7 m at low tide made ths use of a battery powered sampler unattractive.

The water level recorder produced quarter hourly readings on water rssistance computer compatible punched tape. Economic and site conditions limited the stilling well size to 100 mm and float diameter to 80 mm. Calibration (8) indicated an accuracy of $\pm$ 13 mm though during quiet surface wave conditions the effect of interference betwesn float and countsrweight can be observed in figure (3).

River discharge data was supplied by the Dept. of Agriculture, Fisheries and Food for Scotland as mean daily discharges. Hydrographic data was taken from charts prepared by the Dundee Harbour Trust and by the Admiralty.

The salinity record was examined with respect to salinity in the main channel at that section and for ths sffects of tidal range and river flow.

Confirmation that the autosampler data reflected conditions elsswhere on the cross section was sought by comparing the autosampler data with spatial mean valuss of vertical profiles measured in the main channel, whsre the lateral salinity variation was known to be small. The results shown in fig (2) ars typical of the three periods monitorsd. The lag of the point values on the flood is more likely to be the effect of the main flood current tending to flow to the north of ths ebb current (10) rather than vertical salinity gradients.

The tidal dependence of the salinity time series was demonstrated through the use of the least squares criteria to approximate to the data a function of the form (9)

$$y = a_0 + a_1 \sin \omega t + a_2 \cos \omega t \qquad (6)$$

where

$$\omega = \frac{2\pi}{T}$$

a_0, a_1, a_2 = constants

$\sqrt{a_1^2 + a_2^2}$ = amplitude

The period was varied from 11.90 hr to 13.00 hr in steps of 0.01 hr. This revealed for both the water level and salinity data the existence of large values of amplitude for the semi-diurnal (12.42 hr) and 14 day (12.00 hr) components.

Insight into the salinity river flow relationship was achieved by filtering a major part of the tidal effects by approximating equation (6) to various intervals of data and taking a_0 as parametric representation of salinity in that interval. An interval of 24 hr was found to be a satisfactory compromise between loss of detail of the record and of the smoothing of the semi-diurnal tidal/

effects. During psriods of steady fluvial discharge when the tidal effects on mean (T) salinity might be expectsd to bs apparent, the changes in value of a_o suggest a steady trsnd towards an equilibruim value for that flow. However, the tidal range was found to be related to the salinity range, to a first approximation by a linear function.

The values of a_o wsre plotted against an arithmetic average of the msan daily fluvial flow for the precsding 7 days. Study of the records of several floods indicated that 7 days was approximately the average time between peak discharge at the Ballathie (R. Tay) gauging station and ths corresponding minimum value of mean (24 hr) salinity at Newport Pier.

A linear relationship could be approximated to the salinity river flow data. Such a rslationship must of necessity bs of an approximate nature bscause of the dynamic features of the system and of ths variability in the maximum flow, volume and duration of fluvial floods. Data for effectively steady state conditions is givsn in figure (4).

Having sstablished that salinity data might be detected and charactsrissd, salinity rsadings were taken at fivs stations in the main channel (approx 4 km apart) for several tidal rangss and river flows, and examined for a similar relationship. At each station vertical profiles of salinity distribution were measured at intervals of about 20 mins for complete flood and ebb tides (i.e. bstween slack water). Field msasurements showed that cross sectional area msan values of salinity could often be estimated to better than 1 ppt from channel station data. An sstimats of the spatial msan was obtainsd by assuming a linear variation between measured values and thsn taking a spatial avsrage. The spatial mean values of salinity were then approximated to a sixth order polynomial and the mean (A, T) salinity evaluated from this function by temporal avsraging.

At Pool, Broughty Ferry, Newport and Flisk, the rslationship betwesn mean (A, T) salinity and mean (7 day) fluvial discharge closely approximated to a linsar function. The regression linss are shown in figure (4). The lack of variation in river flow on the occasions that the station at Balmerino was monitored serves to indicate reproducibility of results for those conditions at that station. As the slopes of the mean salinity/fluvial discharge functions are similar at Newport and at Flisk, a like value of slope was assumed for the function at Balmerino.

Plotting of the mean (A, T) longitudinal salinity distribution for various flow conditions, figure (5), enabled the abstraction of the data necessary in order to obtain estimates of the value of the apparent dispersion cosfficient.

The variation of the apparent dispersion coefficient with mean (7 day) river flow and distance along the estuary is shown in figures (6) and (7). The coefficient is sensitive to the effects of the variation of river flow. At the

three upstream sampling stations, the observed data indicates that the coefficient passes through a maximum value as the river flow increaees. A marked epatial variation is exhibited near the mouth of the estuary, elsewhere there is a comparatively gradual linear variation.

It is difficult to gain physical information from the apparent dispersion coefficient. An indication of the magnitude of the net effecte of the physical proceeees over a tidal cycle may be obtained from the net advective salt flux during a tidal cycle, here defined as the net mixing. The net mixing is a function of turbulent effects as well as epatial and longer term temporal variations. A good estimate can be obtained from the term $Q_T\ C_{AT}$ if the term $A'_T\ C'_{AT}\ U_{AT}$ may be assumed to be comparatively small, as is generally the caee for the Tay Estuary.

The linear relationship between the mean (A, T)salinity and mean (7 day) river flow,

$$C_{AT} = b_o + b_1 Q_T \qquad (7)$$

where b_o, b_1 = Constante at a point x
leads to a second order relationship for the net mixing F,

$$F = Q_T\ C_{AT} = b_o\ Q_7 + b_1 Q^2_7 \quad (8)$$

Equation (8) indicatee that the net mixing is also sensitive to the effects of mean river flow variation and that a maximum value is passed through as mean river flow increaeee (Figure (9)). The existence of this maxima are not confirmed for all the etations as some occur outwith the limits of conditions observed in the field. The field data indicates a nearly linear increaee in mixing along the estuary (Figure (8)).

CONCLUSION

The one dimensional solute mass transport equation in a tidally averaged form is a useful preliminary approach to the study of mixing in an estuary. While unsatisfactory from a physical point of view, the introduction of an apparent dispersion coefficient provides, along with other functions, an algebraic representation of the effects of processes that lead to what is here called the net mixing. The yield of physical information is limited to an estimateof the net effects of the physical processes of solute transport over a tidal cycle. In the Tay Estuary the apparent dispersion coefficient and net mixing are highly dependent upon distance along the eetuary and mean river flow.

ACKNOWLEDGEMENTS

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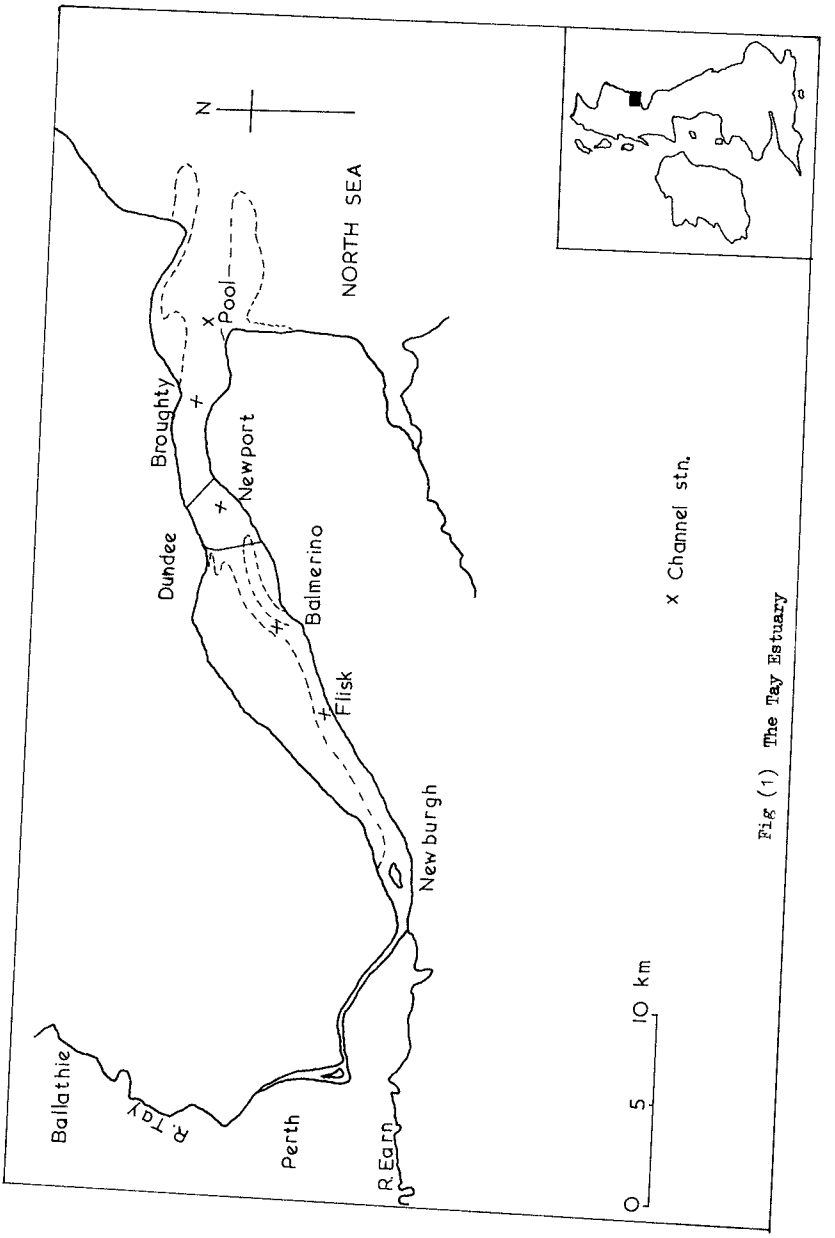


Fig (1) The Tay Estuary

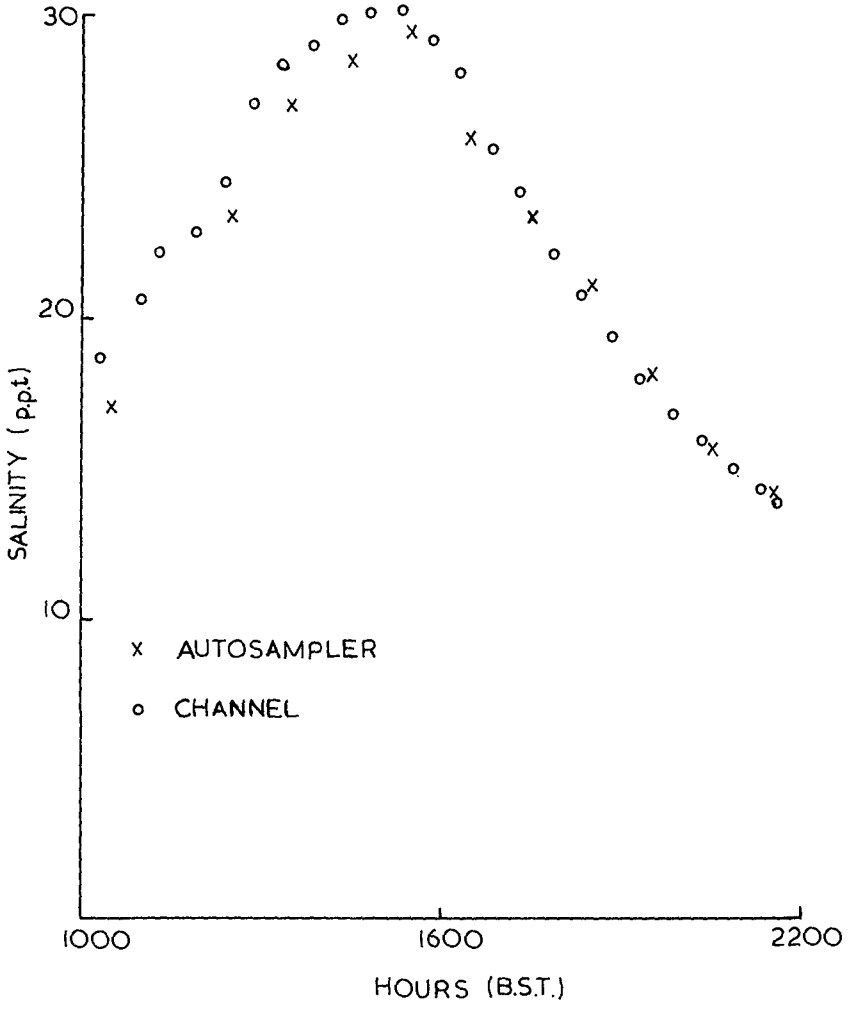
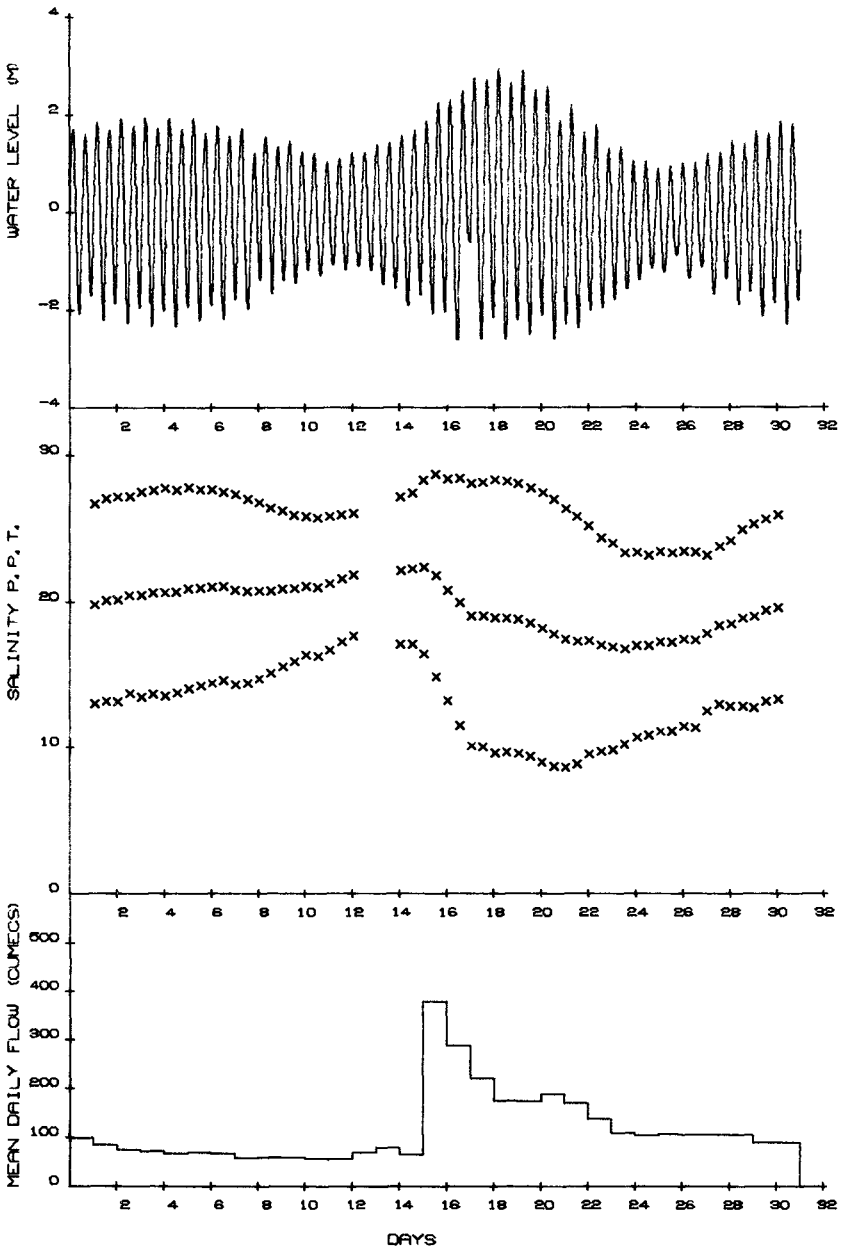


Fig. (2) Comparison of autosampler data with channel mean salinity data at Newport



MONTH 8 YEAR 70

FIG 3 MAX, MEAN & MIN SALINITY (NEWPORT), WATER LEVEL (NEWPORT) & RIVER DISCHARGE (TAY & EARN) VARIATION WITH TIME.

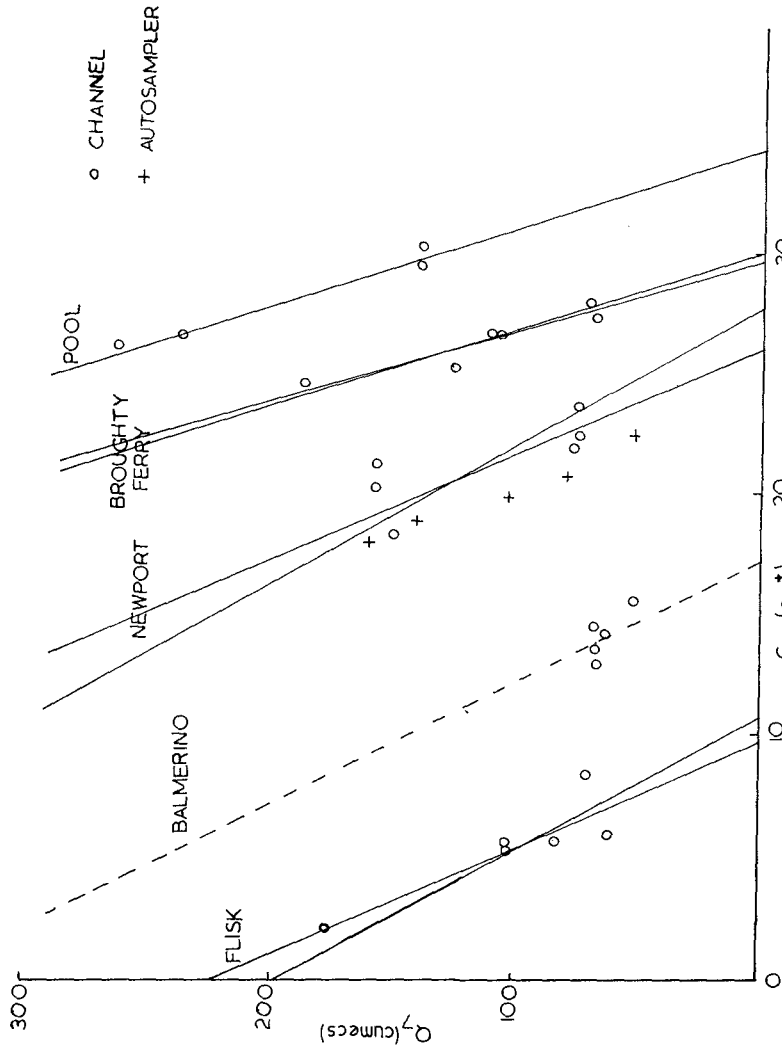


Fig. (4) Variation of mean (A,T) salinity with mean (7 day) fluvial discharge

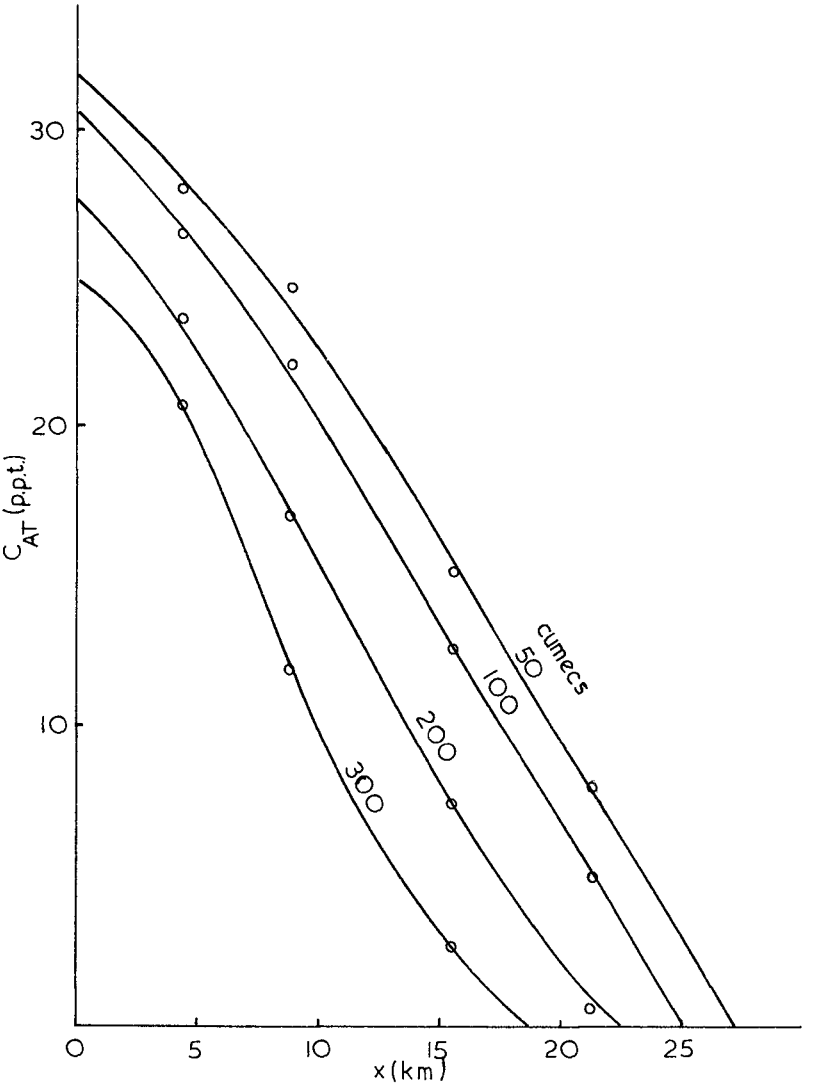


Fig. (5) Longitudinal mean (A,T) salinity distribution as a function of mean (7 day) river flow

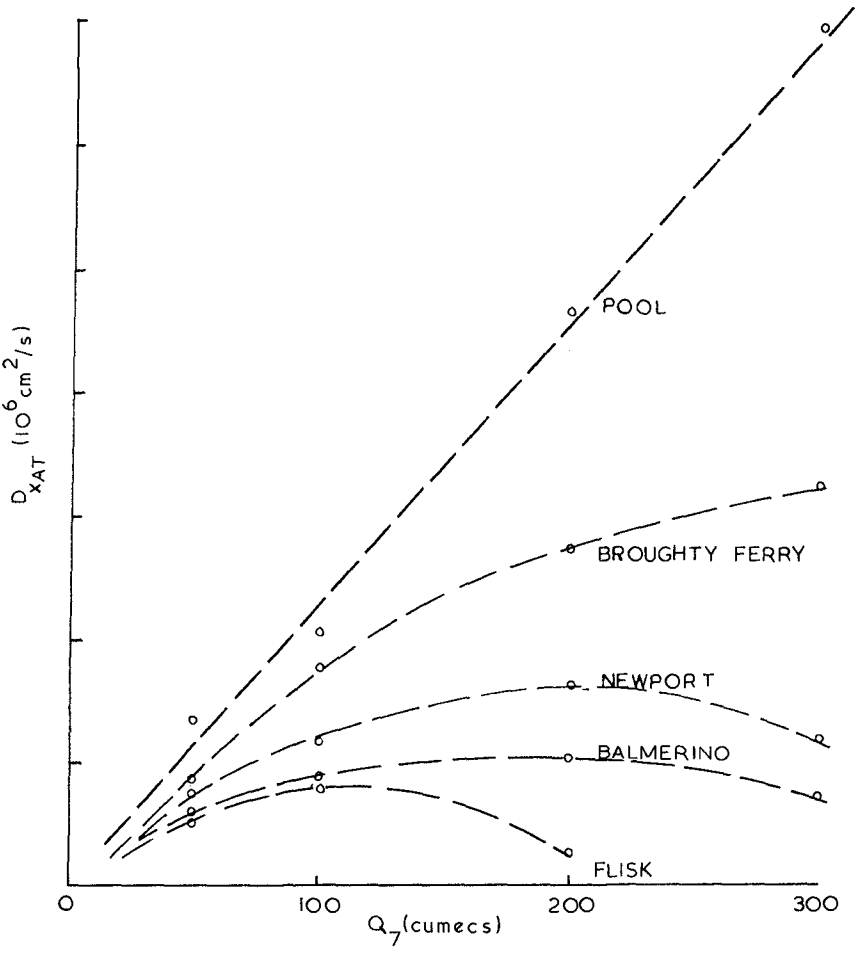


Fig. (6) Variation of apparent dispersion coefficient $D_{x_{AT}}$ with mean (7 day) river flow Q_7

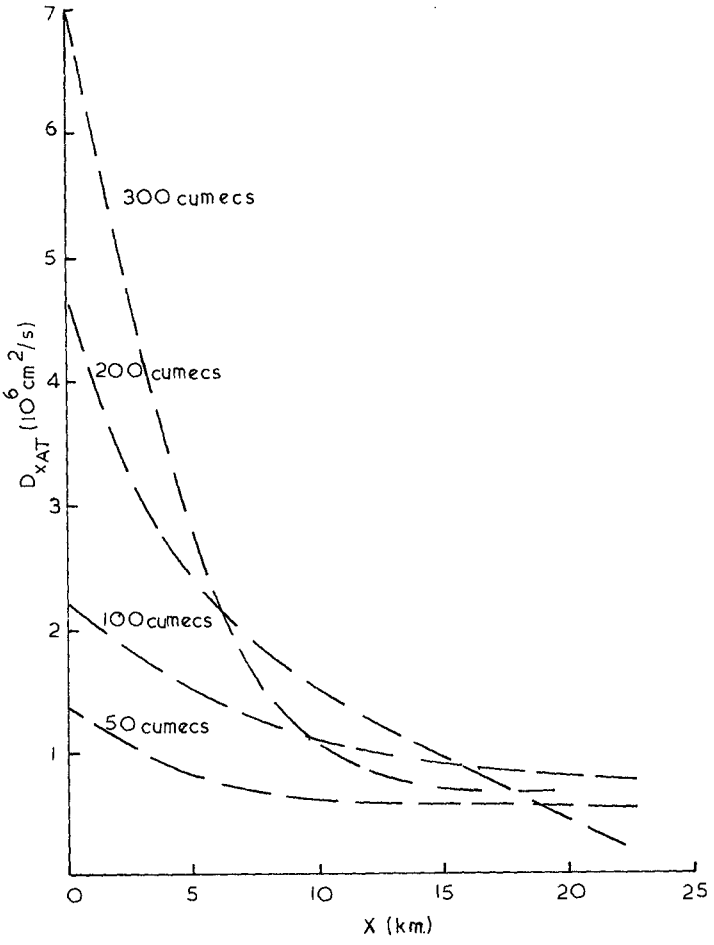


Fig. (7) Apparent dispersion coefficisnt $D_{x_{AT}}$ as a function of distance x from estuary mouth

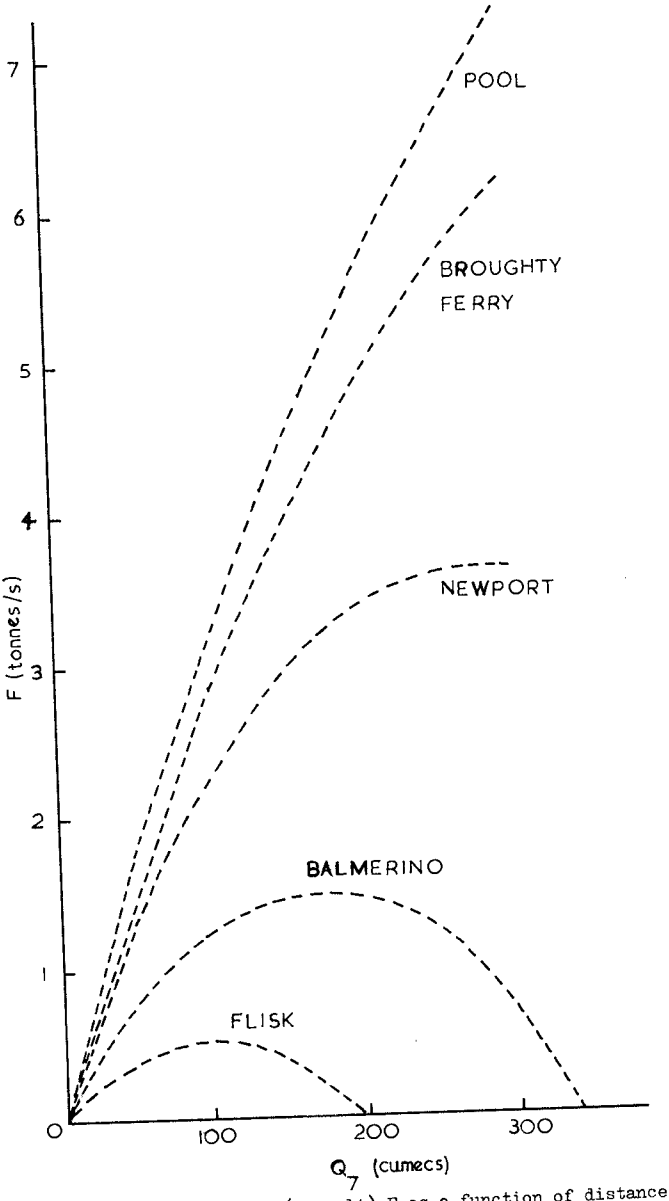


Fig. (8) Net mixing (of salt) F as a function of distance x from estuary mouth

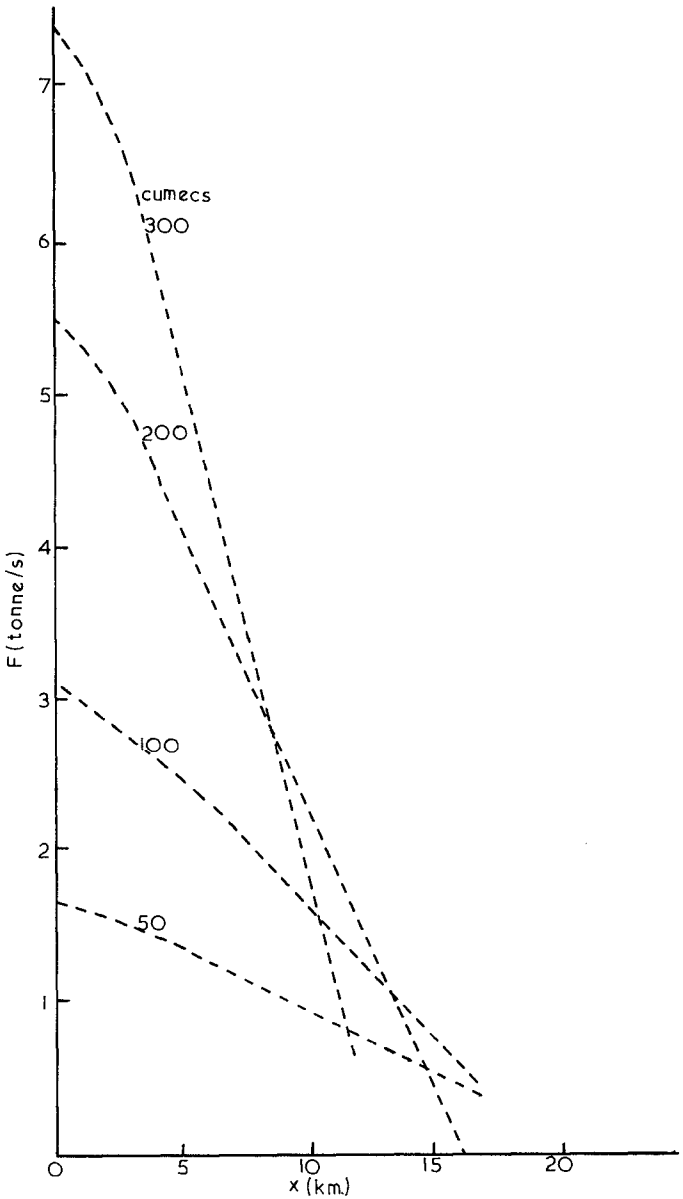


Fig. (9) Variation of net mixing (of salt) F with mean (7 day) river flow Q_7