CHAPTER 63

DELAY DISTANCE IN SUSPENDED SEDIMENT TRANSPORT

by

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Abstract

In the previous paper (Hattori, 1969), a theory for the distribution of suspended sediment concentration due to standing waves was obtained on the basis of the concept of "delay distance" which accounted for the fluid-particle interaction and the inertia effect of sediment particles.

The purpose of this paper is to make clear the physical dependence of delay distance to wave and sediment characteristics.

The evaluation of delay distance from experimental results of distribution profiles of suspended sediment concentration is made by the assumption that the horizontal diffusion coefficient of suspended sediment is nearly equal to the vertical one. An empirical expression of delay distance is obtained as follows;

$$|\delta_x|/\hat{\xi} = 1.35(w_0/\hat{u})^{3/2}.$$

The following theoretical expression for delay distance is obtained from the equation of motion for a discrete particle suspended in one-dimensional unsteady fluid flow.

$$\frac{|\delta_x|}{\hat{\xi}} = A\left(\frac{w_0}{\hat{u}}\right)^{3/2}\left(\frac{h}{w_0 T}\right)^{1/2}.$$

INTRODUCTION

It has been noticed from experimental evidences that the horizontal distribution of suspended sediment concentration due to standing waves became a periodic pattern. As seen in Fig. 1, an example of experimental results obtained by the writer, the sediment concentration at a level from the bottom attaines a maximum value at the antinode and a minimum one at the node of standing waves.

The vertical distribution of suspended sediment concentration represents a linear relationship with respect to the height from the bottom on semi-logarithmic scale.

Although the vertical distribution profile of sediment concentration suspended by wave actions has been investigated by many researchers(1), only few investigations have been made on the horizontal distribution profile.

As well known, the linear wave theory gives no convective transport of suspended sediment, because of the orbits of fluid particles caused by wave motions are closed. The diffusive transport by the turbulence induced by wave motions is of the gradient type and has a tendency to uniformalize the concentration of suspended sediment in the horizontal direction. Thus, the formation of periodic distribution profile of suspended sediment concentration can not be explained either by the convective transport or by the diffusive transport.

The writer(2) previously presented a theory by the analytical model based on the concept of a "delay distance" in order to give a physical explanation in the formation of periodic distribution profile of suspended sediment concentration. The delay distance introduced in the theory accounts for the fluid-particle interaction and the inertia effect of sediment particles.

In the previous theory, the velocity difference between the sediment and the fluid particles was assumed to be equal to the production of the delay distance and the local gradient of fluid particle velocity. Kennedy and Locher(3) also presented a theory for suspended sediment concentration due to progressive waves. In their theory, they introduced the concept of a "delay time", which based on the same idea as the writer's.

The theory by the writer was substantiated by comparisons with experiments performed in a two-dimensional wave flume, 30 m long, 0.80 m wide, and 0.70 m deep. However, the physical properties of delay distance, such as the dependence of wave characteristics, has not fully examined.

The purpose of this paper is to clarify the relation between the delay distance and wave characterictics together with sediment properties on the basis of experimental results and the theoretical consideration.

OUTLINE OF THE PREVIOUS THEORY

As mentioned in the introduction of this paper, the concept of delay distance was introduced into the theory for the horizontal and vertical distribution of suspended sediment concentration. The relative motion between the sediment and the fluid particles gives rise to the delay distance in suspended sediment transport. The assumption was made that the sediment particle moves at the same velocity as that of the fluid some distance behind it.

Assume that the delay distance and the diffusion coefficient of suspended sediment are independent of the space and time variables. The conservation equation for suspended sediment concentration in a two-dimensional case can be written as

$$\frac{\partial C}{\partial t}=-\frac{\partial}{\partial x}\left[C\left(u+\delta_x\left|\frac{\partial u}{\partial x}\right|\right)\right]-\frac{\partial}{\partial z}\left[C\left(w+\delta_z\left|\frac{\partial w}{\partial z}\right|\right)\right]+\varepsilon_{sx}\frac{\partial^2 C}{\partial x^2}+\varepsilon_{sz}\frac{\partial^2 C}{\partial z^2}+w_0\frac{\partial C}{\partial z}\ , \qquad (1)$$

in which C is the concentration of suspended sediment, w_0 is the settling velocity of sediment particles, δ is the delay distance, ε_s is the diffusion coefficient of suspended sediment, u and w are the horizontal and vertical velocities of fluid particles, x and z are the horizontal and vertical coordinates, and t is the time. Subscripts x and z represent the values in the x- and z- directions respectively. The coordinate system and notations are shown in Fig. 2.

The three following assumptions were used to obtain the analytical solution of Eq. (1); 1. The horizontal distribution profile for suspended sediment concentration is given by the same relation as obtained from the one-dimensional equation for sediment concentration by neglecting terms with respect to the vertical motion in Eq. (1). 2. The vertical distribution profile at various sections is geometrically similar. 3. The fluid motion due to standing waves is given by the long wave theory. Then, the horizontal and vertical velocity components of fluid particles are expressed as

$$u=H\sqrt{\frac{g}{h}}\cos kx \sin \omega t\ , \qquad (2)$$

and

$$w=H\omega\frac{z}{h}\sin kx \sin \omega t\ , \qquad (3)$$

in which H is the amplitude of standing waves, h is the water depth, k is the wave number, ω is the angular frequency, and g is the gravitational acceleration.

With the preceding assumptions stated, the time averaged concentration of suspended sediment due to standing waves was obtained as

$$\overline{C}(x,z)=\overline{C}(0,a)\exp\left[\alpha(1-\cos kx)-\frac{\beta}{h}(z-a)\right], \qquad (4)$$

in which

$$\alpha=\widehat{u}|\delta_x|/\varepsilon_{sx}\ , \qquad (5)$$

$$\beta=w_0 h/\varepsilon_{sz}\ , \qquad (6)$$

and a is the reference level with respect to the vertical distribution of suspended sediment concentration. The bar represents the time averaged value over one wave period. The mean horizontal velocity of fluid particles averaged over the horizontal distance between the nodal and antinodal sections of standing waves is obtained from Eq. (2) as

$$U=\frac{4}{L}\int_0^{L/4}u\,dx=\frac{2HL}{\pi h T}\sin \omega t\ . \qquad (7)$$

In Eq. (5), $\widehat{u}$ denotes the amplitude of mean horizontal velocity of fluid particles and is written by Eq. (7) as follows;

$$\widehat{u}=2HL/\pi hT\ , \tag{8}$$

where T is the wave period and L is the wave length.

EVALUATION OF THE DELAY DISTANCE FROM EXPERIMENTAL RESULTS

Since the delay distance can not directly be measued in model experiments, the evaluation of delay distance has to be made with experimental results of suspended sediment concentration profiles.

According to the theory, the distribution profile of the time averaged concentration of suspended sediment in the horizontal direction is determined by the parameter α which is given in terms of the delay distance, the diffusion coefficient and the wave characteristics.
As seen in Eq. (5), the diffusion coefficient of suspended sediment in the horizontal direction is also an unknown quantity to be determined from experimental results.

Figures 3 through 5 illustrate comparisons with the theory and the experiments for the horizontal and vertical distributions of suspended sediment concentration. In those figures, α and β are assumed so as to best fit to the experimental values.

It is noticed from Figs. 4 and 5 that there is a high concentration layer near the bottom. Since suspended sediment in this layer behaves as if it were a viscous fluid, the theory can not apply to the motion of suspended sediment near the bottom.

As seen in Figs. 4 and 5, the vertical distribution of suspended sediment concentration in the upper region, $z/h>0.2$, is expressed by a linear relationship on semi-logarithmic scale. This fact means that the vertical diffusion coefficient, ε_{sz} , is constant with respect to the height from the bottom. Therefore, the diffusion coefficients for each test run are calculated from Eq. (6) with aids of experimental results of the vertical distribution of sediment concentration at the nodal section ($x=0$).

It needs to estimate the horizontal diffusion coefficient in advance of the evaluation of delay distance. The wave motion is unsteady and has the two velocity components both in the horizontal and vertical directions.

As a first approximation, the evaluation of delay distance is made by the assumption that the horizontal diffusion coefficient is almost equal to the vertical one. Under this assumption the delay distance is computed from Eq. (5) by putting $\varepsilon_{sx} \fallingdotseq \varepsilon_{sz}$.

Two quantities related to the standing wave motion are defined to obtain a non-dimensional expression for the delay distance. The amplitude of mean horizontal velocity of fluid particles, $\hat{u}$ given by Eq. (8), is used as a velocity scale. As a length scale we define the following quantity being propotional to the amplitude of the mean horizontal displacement of fluid particles due to standing waves.

$$\hat{\xi}=H/(h/L)\ . \tag{9}$$

Figure 6 illustrates the relation between $|\delta_x|$ and $\hat{u}$. Although the evaluated values of delay distance are rather scattered, it may be considered that the delay distance of suspended sediment has a decreasing tendency with the increase in the mean horizontal velocity of fluid particles.

The ratio of the delay distance to the mean horizontal displacement of fluid particles, $|\delta_x|/\hat{\xi}$, is plotted in Fig. 7 as a function of $w_0/\hat{u}$ which represents the inertia effect of suspended sediment particles. Within the limit of this study, the relation between $|\delta_x|/\hat{\xi}$ and $w_0/\hat{u}$ is shown by the full line in Fig. 7, from which the following expression is obtained.

$$|\delta_x|/\hat{\xi}=1.35(w_0/\hat{u})^{3/2}. \tag{10}$$

THEORETICAL CONSIDERATION ON THE DELAY DISTANCE

Let us consider a discrete sediment particle suspended in one-dimensional unsteady flows. The equation of motion for the particle is obtained by the equilibrium condition between the inertia and the drag forces of the particle and is written as follows;

$$\frac{du_s}{dt}=\frac{3}{4}\frac{KC_{D_1}w_f}{d\ w_s}|u_f-u_s|(u_f-u_s)\ , \tag{11}$$

in which u_f and u_s are the velocities of the fluid and the sediment particles, d is the diameter of sediment particle, w_f and w_s are the specific weights of the fluid and sediment, C_{D_1} is the drag coefficient, and K is a propotional constant.

The terminal settling velocity of the sediment particle, w_0 , is given by Eq. (12).

$$w_0^2=\frac{4}{3}\frac{gd}{C_{D_2}}\frac{w_s}{w_f}\ . \tag{12}$$

In Eq. (12), C_{D_2} is the drag coefficient. From Eqs. (11) and (12), we obtain Eq. (13) under the assumption of $C_{D_1}\fallingdotseq C_{D_2}\approx C_D$.

$$\frac{du_s}{dt}=\frac{Kg}{w_0^2}|u_f-u_s|(u_f-u_s)\ . \tag{13}$$

By the definition of delay distance, the velocity difference between the fluid and the sediment particles can be written as

$$u_f - u_s = |\delta_x| \frac{\partial u_f}{\partial x} . \tag{14}$$

Substituting Eq. (14) into Eq. (13), we obtain Eq. (15).

$$\frac{du_s}{dt} = K\delta_x^2 \frac{g}{w_0^2} \left| \frac{\partial u_f}{\partial x} \right| \frac{\partial u_f}{\partial x} . \tag{15}$$

It may be supposed from the experimental results that the rate of velocity change of sediment particles is almost equal to that of fluid particles. By putting $du_s/dt \approx du_f/dt$, Eq. (15) is rewritten by Eq. (16).

$$\frac{du_f}{dt} = K\delta_x^2 \frac{g}{w_0^2} \left| \frac{\partial u_f}{\partial x} \right| \frac{\partial u_f}{\partial x} . \tag{16}$$

In the previous theory, we assumed that the delay distance is constant with respect to the space and time variables. Therefore, with use of the relations, $u_f \approx \widehat{u}$, $x \approx L$, and $t \approx T$, Eq. (16) is replaced by Eq. (17).

$$\frac{\widehat{u}}{T} = A'\delta_x^2 \frac{g}{w_0^2} \left(\frac{\widehat{u}}{L} \right)^2 . \tag{17}$$

In Eq. (17) A' is a constant. From Eqs. (8) and (9), the relation of $\widehat{u}$ to $\widehat{\xi}$ is given as

$$\widehat{u} = 2\widehat{\xi}/\pi T . \tag{18}$$

Substituting Eq. (18) into Eq. (17), the relative delay distance of suspended sediment is expressed in the form

$$\frac{|\delta_x|}{\widehat{\xi}} = A \left(\frac{w_0}{\widehat{u}} \right)^{3/2} \left(\frac{h}{w_0 T} \right)^{1/2} , \tag{19}$$

where A is a constant.

The theoretical expression for delay distance by Eq. (19) confirms the experimental evidence that the relative delay distance is propotional to $(w_0/\widehat{u})^{3/2}$. However, the experimental results on Fig. 7 do not indicate the dependence of the term $(h/ w_0 T)^{1/2}$ in Eq. (19) to the relative delay distance. It may be considered that the difference between the theoretical and the empirical expressions results from the assumptions used in the evaluation of delay distance.

CONCLUSIONS

The dependence of delay distance to wave and sediment characteristics was discussed on the basis of experimental results and theoretical consideration.

The evaluation of delay distance from experimental results of distribution of suspended sediment concentration was made by the assumption that the horizontal diffusion coefficient of suspended sediment is almost equal to the vertical one. The relation between $|\delta_x|/\hat{\xi}$ and $w_0/\hat{u}$ was shown in Fig. 7, and was obtained as follows;

$$|\delta_x|/\hat{\xi} = 1.35(w_0/\hat{u})^{3/2}.$$

On the other hand, the following expression for delay distance was obtained theoretically from the equation of motion for a discrete particle suspended in one-dimensional unsteady flow.

$$\frac{|\delta_x|}{\hat{\xi}} = A\left(\frac{w_0}{\hat{u}}\right)^{3/2}\left(\frac{h}{w_0 T}\right)^{1/2}.$$

It was considered that the difference between the theoretical and empirical expressions resulted from the assumptions underlying the evaluation of delay distance. Further detailed investigations are desirable to make more accurate evaluation of delay distance.

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3) Kennedy, J.F. and F.A. Locher: Sediment suspension by water waves, Waves on Beaches and Resulting Sediment Transport, Academic Press, 1972, pp.249-295.

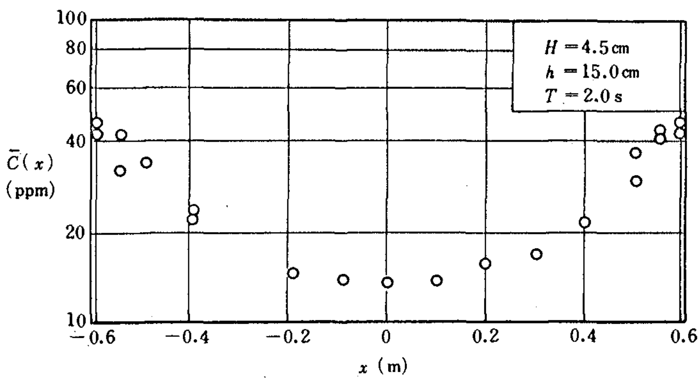

Fig. 1 Example of experimental result of the horizontal distribution profile of suspended sediment concentration.

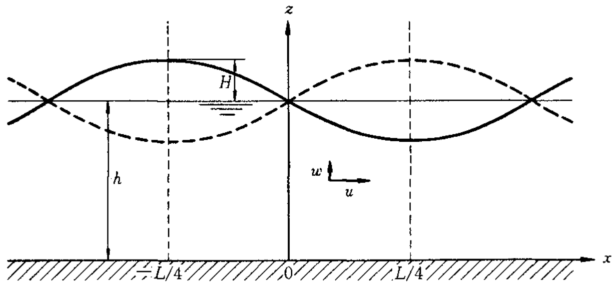

Fig. 2 Definition sketch.

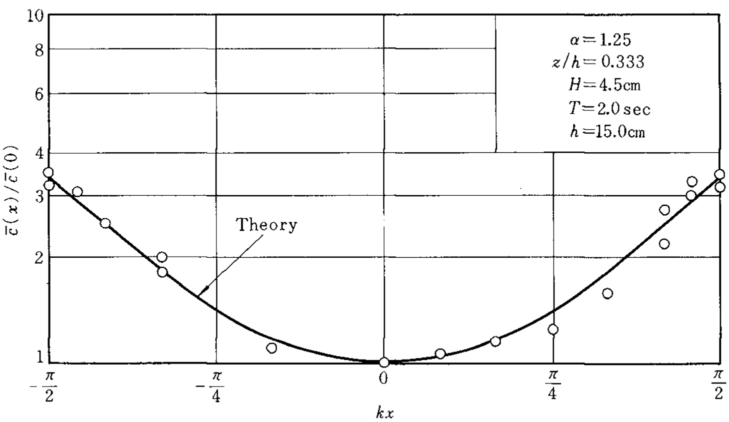

Fig. 3 The horizontal distribution of suspended sediment concentration.

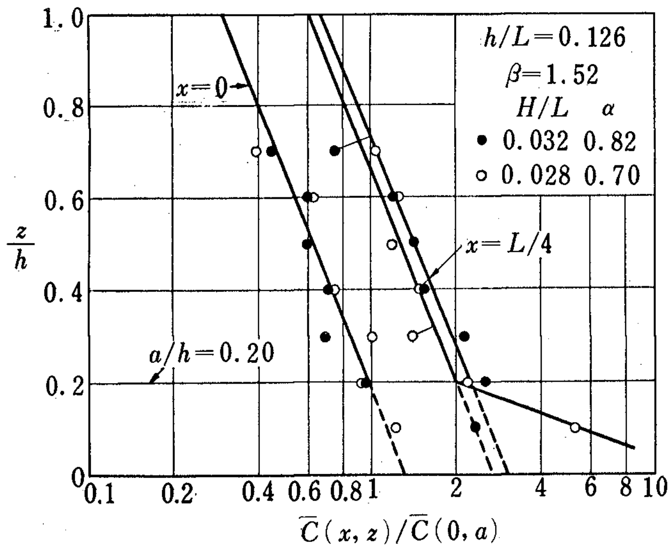

Fig. 4 The vertical distribution of suspended sediment concentration.

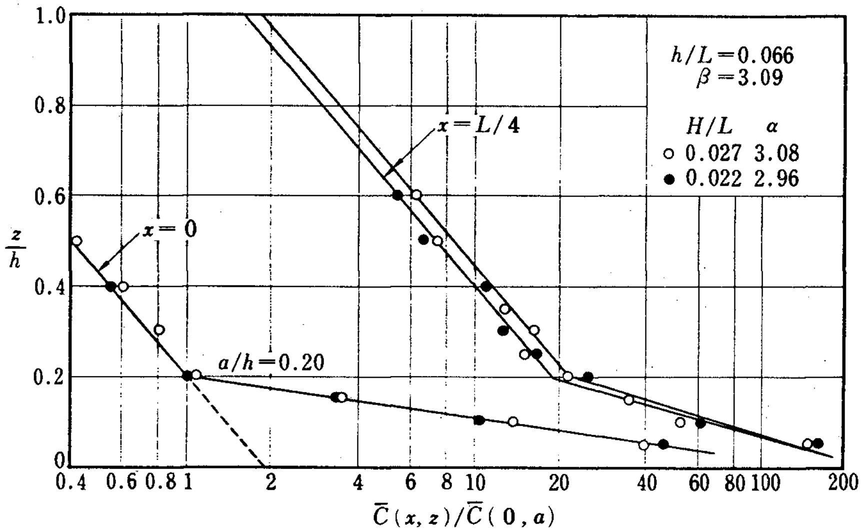

Fig. 5 The vertical distribution of suspended sediment concentration.

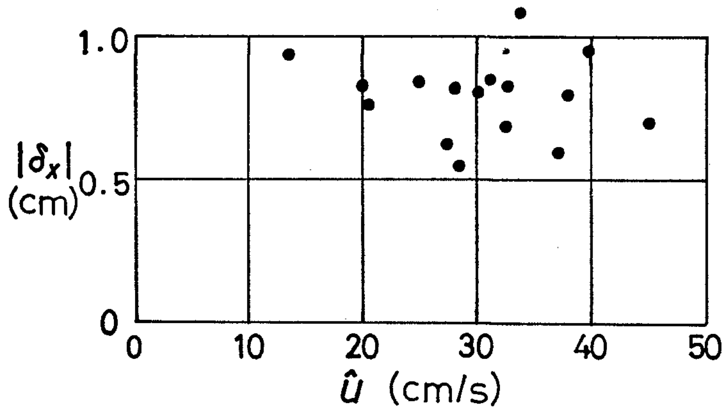

Fig. 6 Relation between the delay distance and the mean horizontal velocity of fluid particles.

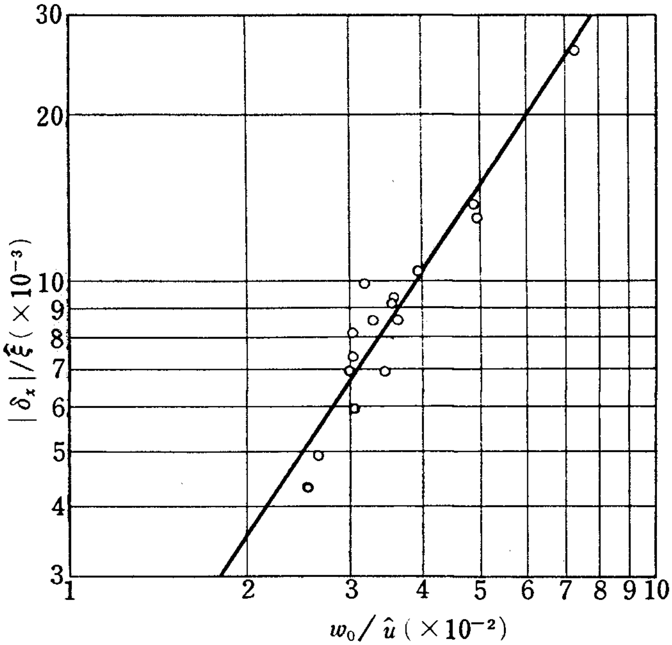

Fig. 7 Relation between $|\delta_x|/\hat{\xi}$ and $w_0/\hat{u}$.