BREAKER TYPE AND PHASE SHIFTS ON NATURAL BEACHES

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ABSTRACT

The kinematics of incipient breakers were measured on various complex natural beaches. Water velocities were found to consistently lag behind the wave crest. This phase shift appears to be partly inherent in shoaling waves and partly a result of partial wave reflection. Phase shifts vary from 5 degrees through to quadrature. This variation probably depends to a large extent upon beach reflectivity and distance of the break point from the shoreline. Phase shifting also varies vertically beneath the wave crests from uniform distributions to those in which maximum shifts occur near the bed. There is a strong relationship between the nature of phase shifting, the morphodynamic condition and the breaker type. Surging waves were associated with large, and vertically uniform phase shifts. Waves towards the plunging and spilling end of the breaker type continuum occurred where phase shifts were significantly larger at lower depths than near the surface.

INTRODUCTION

The importance of breaker type in differentiating surf-zone morphodynamic regimes is implicit in the delineation of beaches into reflective and dissipative categories (e.g., see Short,1978 and Wright et.al.,1979). The six morphodynamic states of Wright et.al.(1979) range from the highly dissipative type through intermediate complex and rhythmic forms to the highly reflective extreme. Breaker characteristics have received little direct attention in recent theoretical developments of the surf-zone dynamics associated with these beach states. However, it is obvious that spatial patterns of energy dissipation related to breaker type must be as important to morphodynamic response as incident angle and wave height distribution.

Peregrine and Cokelet (1980) use numerical solutions of water motions in deep water which provide apparently good simulation of the initial breaking process. Unfortunately, there exists at present no theory capable of successfully describing the shallow-water breaking process even under simple circumstances. Accordingly, in order to investigate the process in the context of breaker types occurring with each of the morphodynamic states, the kinematic structure of waves near breaking was investigated in association with each of these beach types. The investigation involved measurement of the vertical distribution of horizontal velocities.

The purpose of this paper is to present some results concerning one of the least considered properties of waves at breaking; the phase relationship between the wave crest and the horizontal velocity components below it. Two aspects of this problem will be considered: (1) the variation in this phase relationship for waves breaking in association with the different morphodynamic beach types; and (2) the influence of this phase relationship on breaker type through the effect on the vertical distribution of horizontal velocity components for waves on the respective beach types.

Both Adeyemo (1970) and Thornton et.al.(1976) report without comment as to cause or implications the occurrence of phase shifting between water surface and horizontal velocities in laboratory and field experiments respectively. Adeyemo employed photographic measurement of hydrogen bubble displacement in waves on both a steep beach (slope 1:9) on which plunging breakers occurred, and on a moderately flat beach (slope 1:18) with spilling breakers. In both cases Adeyemo reports wave crests leading velocity maxima by more than 30 degrees. Further, this phase shift was found to exist throughout the shoaling region seaward of the break point (see Table 1).

Thorntom et.al. used electromagnetic flow metres and capacitance wave staffs to measure waves and currents on field beaches. They report a 15 degree lag of water velocities behind the water surface at peak frequencies for surging breakers on a steep natural beach (average slope, 1:8). They found phase shifts varying from 5 to 30 degrees in the same direction in measurements made on a flat beach (average slope, 1:20) where spilling breakers prevailed (see Table 1).

TABLE 1

Typical Phase Shifts in Lower Half of Water Column

Example	Experiment	r	θ (degrees)	θ_{max} (degrees)	$\hat{\theta}$ (degrees)	$r_{\hat{\theta}}$	$r_{\hat{\theta}max}$	h/L
1	Adeyemo (1970) Curve a	.01	-0.1	0.5	32	n.p.	.29	.0823
	(tan β = 1:18) curve e	.01	0.5	0.5	32	.29	.29	.0907
	curve h	.01	0.0	0.5	32	.91	.29	.1130
2	Adeyemo (1970) curve a	.03	1.2	3.2	36	.64	.33	.0800
	(tan β = 1:9) curve e	.03	-3.2	3.2	36	n.p.	.33	.0938
	curve h	.03	0.3	3.2	36	.86	.33	.1135
3	Del Monte Beach; tan β = 1:20 (Thornton et.al., 1976)	.06	-1.7	7.0	27	n.p.	.24	.0111
4	Carmel River Beach; tan β = 1:8 (Thornton et.al., 1976)	.14	2.3	16.0	15	.6	.13	.0333
5	Offshore Bar, Umina Beach (1) (26-8-78, Run 1)	.00	0.0	0.2	30	.74	.27	.0211
6	Offshore Bar, Scotts Head (3-12-78, Run 2)	.00	-0.2	0.3	23	n.p.	.20	.0314
7	Transverse Bar, Palm Beach (7-4-79, Run 1)	.01	0.0	1.0	14	.71	.12	.0231
8	Rip-head bar, Palm Beach (7-4-79, Run 4)	.01	-0.5	0.5	22	n.p.	.32	.0241
9	Beach face, rip-bay, Palm Beach (7-4-79, Run 5)	.21	21.3	23.7	88	.97	.97	.0149
10	Cusp Horn, Pearl Beach (1) (3-8-78, Run 1)	.32	32.3	35.7	16	.16	.14	.0330
11	Cusp Horn, Pearl Beach (2) (7-8-78, Run 6)	.12	1.0	14.2	56	.96	.53	.0255
12	Cusp bay, Pearl Beach (2) (7-8-78, Run 5)	.11	1.5	12.2	30	.82	.27	.0245

Notation:
- r = Miche's $(2/\varepsilon i)$ reflection coefficient
- θ = phase predicted from r and kx
- θmax = phase predicted from r, $kx = \pi/4$
- $\hat{\theta}$ = measured phase
- $r_{\hat{\theta}}$ = reflection coefficient required for $\theta = \hat{\theta}$ at kx
- $r_{\hat{\theta}max}$ = reflection coefficient required for $\theta = \hat{\theta}$ at $kx = \pi/4$
- n.p. = not possible

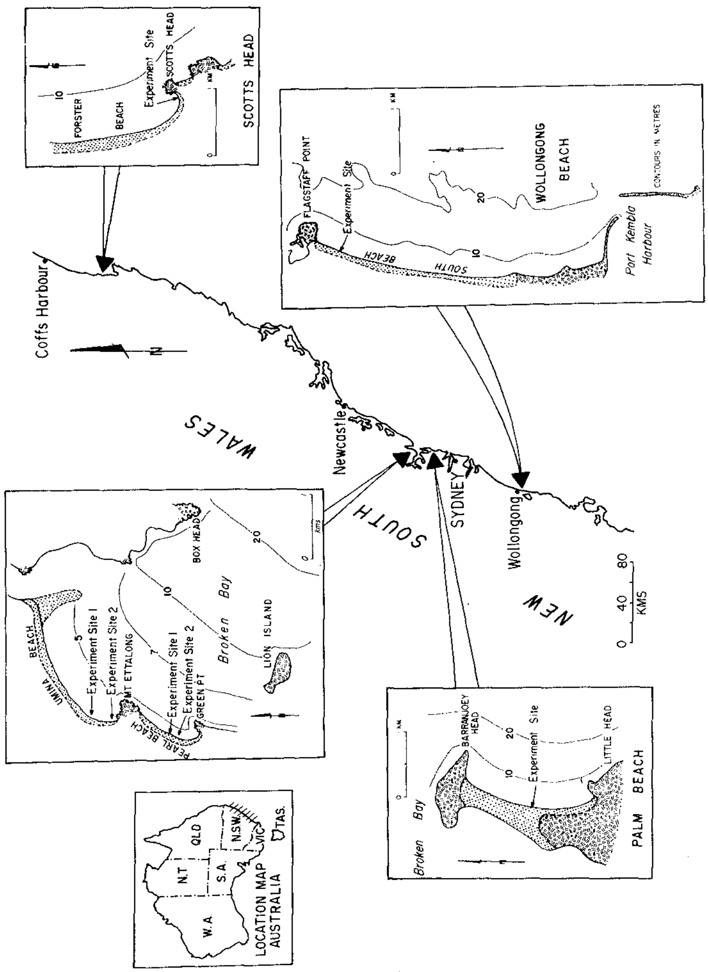


FIGURE 1 Field Site Locations

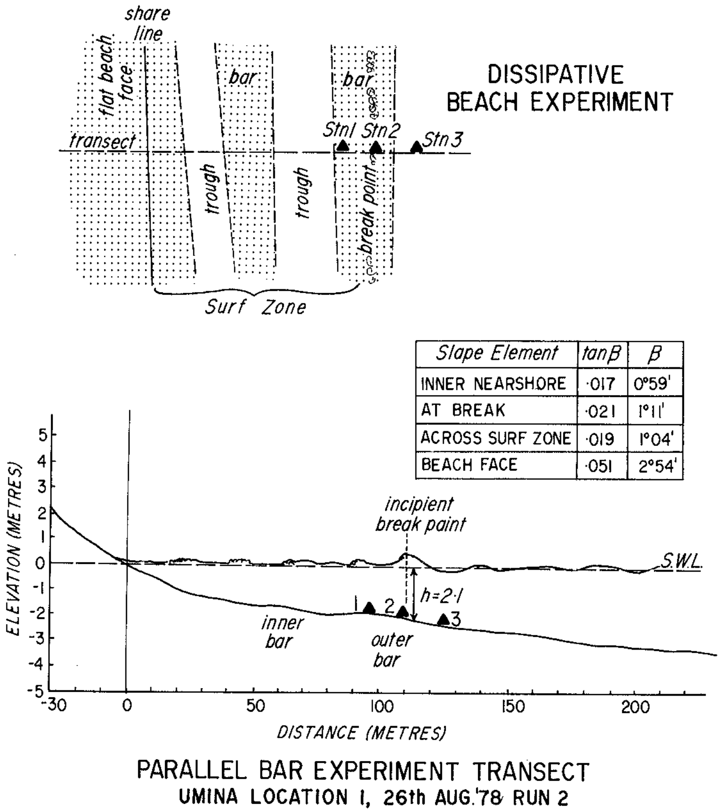
EXPERIMENTS AND METHODS

Field measurements of vertical distributions of horizontal velocities were made at various sites on the microtidal, New South Wales coast of Australia, shown in Figure 1. These beaches were chosen in each case as being characteristic of a particular morphodynamic type. Of these, experiments from three beaches (Umina, Pearl and Palm) will be used to illustrate the general results concerning phase shifting.

Umina beach provides an example of the dissipative beach type with low bed slopes, parallel bars and generally spilling breakers (Fig. 2). Beach sediments are largely fine sands (2.75 phi). The break point during experiments was located on the outer bar. Pearl Beach (Fig. 3) provided the reflective extreme with waves breaking against the beach face. The beach was heavily cusped during experiments (Fig. 6a). Steep beach slopes are enhanced by the coarse to very coarse sand (1 to -.5 phi). Palm Beach will be used to illustrate wave breaking on an intermediate-rhythmic beach type (Fig.4). Median diameter of sediments is slightly less than 1.5 phi. During experiments type 4 beach morphology of Wright et.al.(1979) prevailed. Three break points with distinctly different breaker type characteristics were distinguished on this beach: On the rip head bars waves were heavily plunging; at the beach face in rip embayments waves were of the surging type after reforming across the rip channel; and on the seaward margins of transverse bars between rip channels breaker types were weakly plunging to spilling.

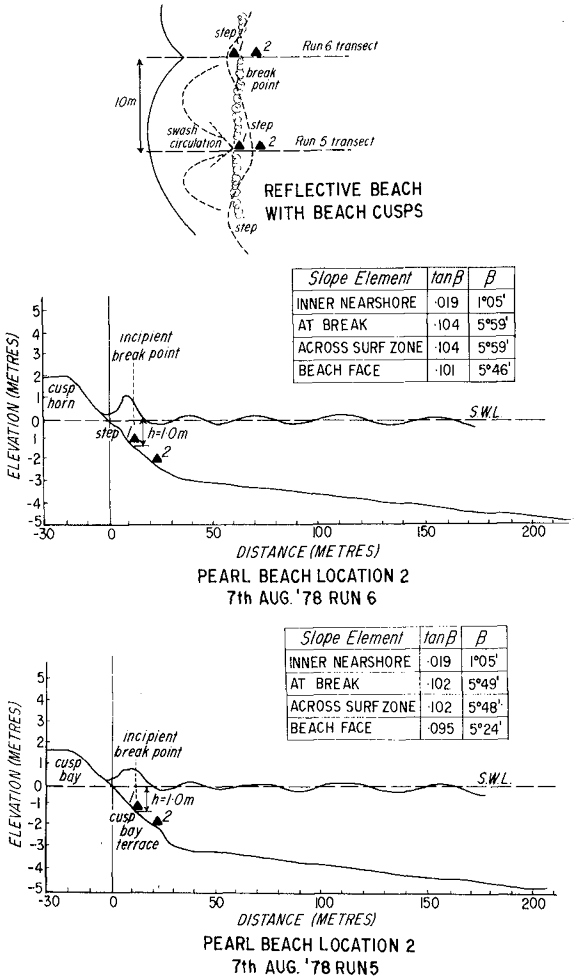
Wave and current sensors were recorded both digitally on magnetic tape and continuously on strip chart. Photographic records were obtained using a movie camera with single frame exposures synchronised with the digital scanning of wave and current sensors. Pressure and water surface measurements were made using bottom mounted pressure transducers together with the photographic record of surface piercing, visually calibrated staffs attached to each instrument station used. From one to three stations were used in experiments. They are depicted in Figures 2, 3 and 4. At the incipient break point the instrument station also supported a vertical array of up to 7 miniture, bidirectional, ducted impellor-type flow metres. The axes of these flow metres were orientated shore normal in the horizontatal plane. The photographic record was also used to determine breaker type geometries of each passing wave. A description of the data acquisition system can be found in Bradshaw et.al. (1978).

Spectra and cross spectra were computed between adjacent



Slope Element	tanβ	β
INNER NEARSHORE	·017	0°59'
AT BREAK	·021	1°11'
ACROSS SURF ZONE	·019	1°04'
BEACH FACE	·051	2°54'

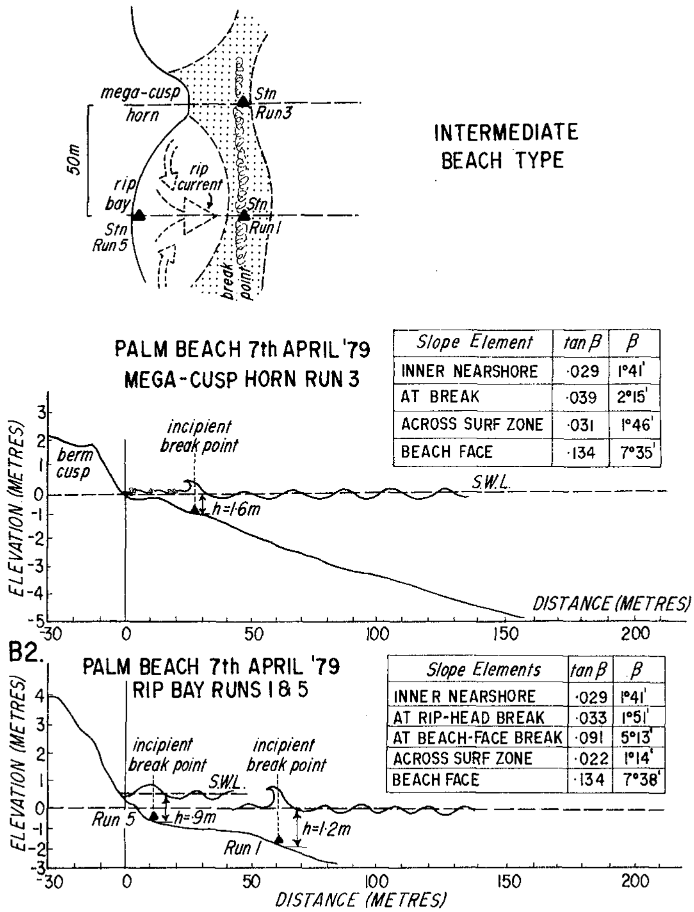
FIGURE 2 Dissipative Beach Experiment



Slope Element	tanβ	β
INNER NEARSHORE	·019	1°05'
AT BREAK	·104	5°59'
ACROSS SURF ZONE	·104	5°59'
BEACH FACE	·101	5°46'

Slope Element	tanβ	β
INNER NEARSHORE	·019	1°05'
AT BREAK	·102	5°49'
ACROSS SURF ZONE	·102	5°48'
BEACH FACE	·095	5°24'

FIGURE 3 Reflective Beach Experiment



Slope Element	tan β	β
INNER NEARSHORE	·029	1°41'
AT BREAK	·039	2°15'
ACROSS SURF ZONE	·031	1°46'
BEACH FACE	·134	7°35'

Slope Elements	tan β	β
INNER NEARSHORE	·029	1°41'
AT RIP-HEAD BREAK	·033	1°51'
AT BEACH-FACE BREAK	·091	5°13'
ACROSS SURF ZONE	·022	1°14'
BEACH FACE	·134	7°38'

FIGURE 4 Intermediate Beach Type Experiment

current meters in the vertical array, as well as between each current meter and the pressure transducer. These computations were made by Fourier transforming covariance functions calculated from the records and smoothed with a Tukey window. The covariance functions were computed using a maximum lag of 22% of the record length. The resulting spectral estimates contain 24 degrees of freedom.

Phase differences at peak spectral frequencies were used. These are shown in Figure 5 where the horizontal bars indicate 95% confidence intervals. These bars are located at the elevation (z) of each flow meter used. This elevation is shown on the ordinate normalized with respect to mean height of waves above mean water level (η) and the depth (h). Dashed confidence bars indicate phase estimates for sensors which occassionally emerged from the water, giving zero readings. The reliability of these estimates is therefore suspect although comparison of power spectra with those of non-emerging instruments suggests comparable frequency content. Solid phase curves are interpolated between flow meters. Dashed curves are less reliable due to either extrapolation or current records being less reliable due to reasons just given.

The contribution to phase shifting resulting from instrument response characteristics was assessed by comparison of analyses and the variously obtained time series. The phase curves in Figure 5 were therefore determined for both phase between currents and pressure, and between each adjacent current record. Phase shifts and confidence intervals between each successive pair of adjacent current records were summed over the vertical array. This produced good agreement with phase between pressure and each of the current records.

Pressure records were also compared against current and photographic records with respect to frequency content. Close agreement is exhibited. The pressure record also appears to follow the photographically determined water surface time series fairly faithfully, which is important in terms of the coincidence of maximum pressure and the passage of the wave crest.

As explained by Thornton et.al.(1976), interpretation of such spectra requires caution due to nonhomogeneity and nonstationarity resulting from the break point wandering on flat beaches. For this reason cross-spectra phase was checked against wave by wave inspection of phase shifting between individual time series. The comparison revealed reasonable agreement. It also enabled use of reliable segments of records from those instruments which failed to produce a complete time series. These segments were used to estimate the interpolated and extrapolated parts of the curves in Figure 5.

DISCUSSION OF RESULTS

It is evident from Figure 5 that phase shifting is substantial under each set of morphodynamic conditions. Water velocities generally lag behind the wave crest, particularly in the lower half of the water column. This is consistent with the observations of both Adeyemo (1970) and Thornton et.al.(1976). For spilling breaker types on dissipative beaches (Fig. 2) velocities follow the water surface more closely towards the surface, with a sharp decrease in phase lags above mid depth. High waves heavily plunging on a steep beach (Fig. 3) exhibit a similar phase profile (Fig. 5c), as do waves of the outer break on the intermediate beach type (Fig. 5d). In the former case the phase shift in the cusp bay is smaller than at the break point on the cusp horn. This is probably attributable to the interaction of waves with the intense backwash drainage from the cusp bay illustrated in Figure 6a. Resulting turbulence causes strong vertical mixing, which may produce greater flow homogeneity throughout the water column, and thus reduced phase shifting in the lower half. Breaker type differences are related to this. Whereas heavily plunging breakers occur on the cusp horns, the general form in the bay is collapsing and even surging. In the latter case the wave is injected with turbulence before breaking. Hence the breaker form is described by the general water motion rather than by the water surface alone.

It is also apparent that the vertical distribution of phase is more uniform in situations where surging-collapsing breakers occurred (Figs. 5b and 5d, beach face). Collapsing breakers prevailed in association with the 20 degree phase shift at the break point (Fig. 5b). The standing wave apparent at the beach-face break of the rip embayment on the intermediate beach type occurred in association with surging waves. In this case the break point (and instrument station) was located one quarter wave length from the shore line antinode.

It is interesting to note that both the harmonic (Fig. 5a) and subharmonic (Fig. 5b) have phase shifts of roughly twice the primary wave. In the former case the development of a harmonic in the spectrum is expected on such a flat beach where significant cross-spectral transfer is likely (e.g., Guza and Thornton, 1980). It is expected that the harmonic is phase linked although this cannot be demonstrated from this analysis. On the steep beach it is consistent with expectation that spectra should exhibit a significant peak at subharmonic frequency rather than at harmonic frequencies (e.g., Guza and Bowen, 1975).

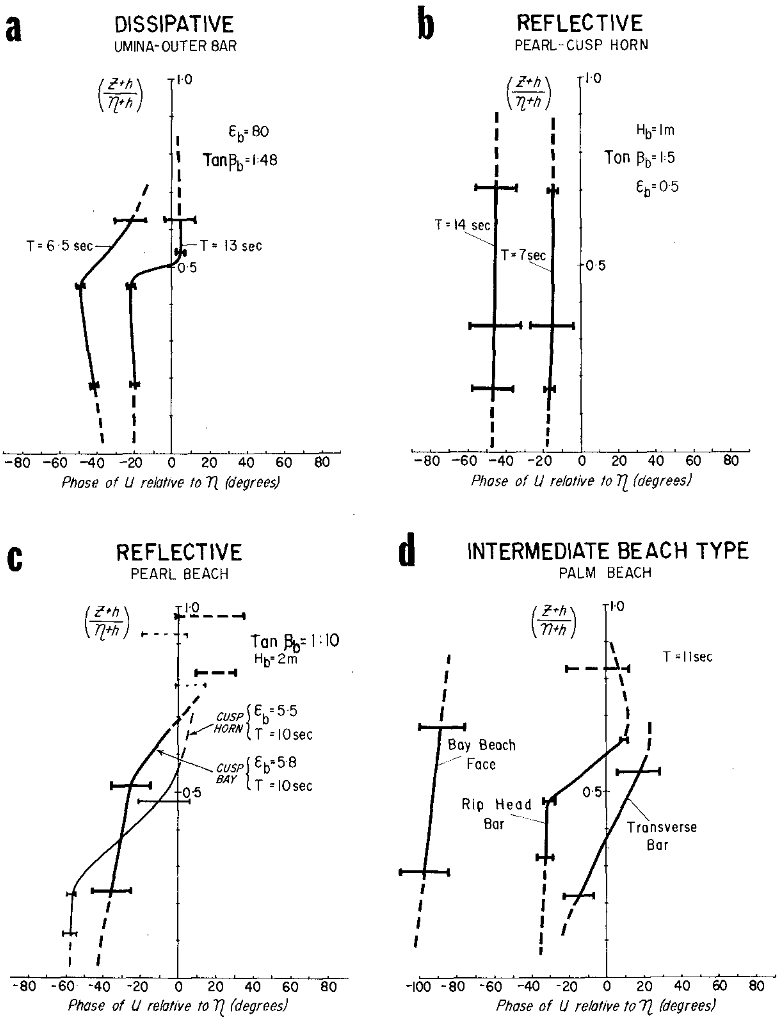


FIGURE 5 Cross-Spectra Phase

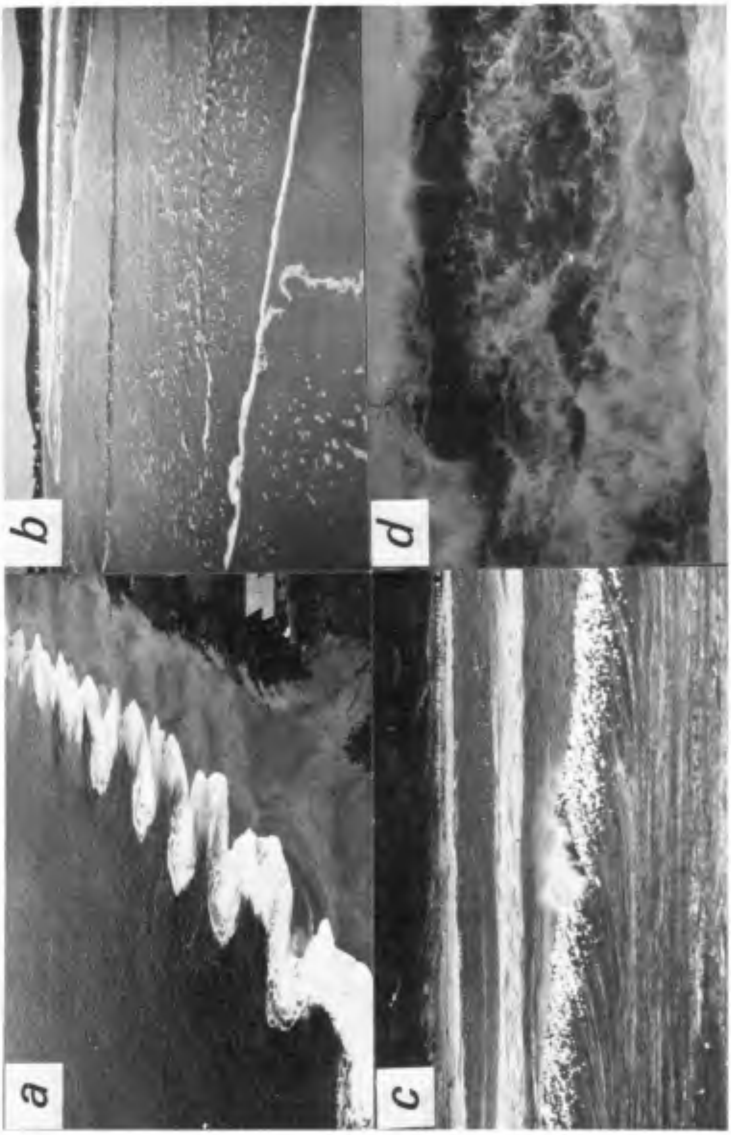


FIGURE 6. A. Beach-Cusp Drainage - Pearl Beach B. Swash Lense - Umina Beach

C Spilling Breaker D. Collapsing Breaker

However, the subharmonic might have been expected to be closer to quadrature phase than that actually displayed (Fig. 5b).

Before it becomes possible to account for the phase difference under the various morphodynamic conditions it will be necessary to explain the occurrence of phase shifting in general. Jonsson (1978) showed for oscillatory flows that the phase of water velocities within a laminar boundary layer leads those outside by $\pi/4$. In the fully turbulent case this phase shift is about $\pi/6$. This results from the inertia forces decreasing significantly near the bed whereas the accelerating pressure force the same in both the boundary layer and in the higher velocity free flow. Flow near the boundary therefore follows the phase of pressure more closely. However, the problem with the application of this argument is twofold: (1) phase shifting can only occur near the bed, whereas it has been measured by Adeyemo, Thornton et.al. and this study well away from the bed; and (2) the measured phase shifting occurs in the reverse sense to that predicted within the boundary layer.

Another explanation for phase shifting is provided by partial wave reflection. Using linear theory for partial reflection, the amount by which the surface elevation phase (ψ) leads the velocity phase (ϕ) x is given by

$$= \psi - \phi$$

$$= \tan^{-1}\left(\frac{1+r}{1-r}\tan kx\right) - \tan^{-1}\left(\frac{1-r}{1+r}\tan kx\right) \qquad (1)$$

where kx is the distance from an antinode nondimensionalized with respect to the wave number (k), and r is reflection coefficient. The reflection coefficient can be approximated by

$$r = 2/\xi_i \qquad (2)$$

where,
$$\xi_i = \frac{a_\infty \sigma^2}{g} (2\pi)^{\frac{1}{2}} \tan^{-5/2}\beta \qquad (3)$$

a_∞ is the wave amplitude in deep water, σ is its radial frequency, g is the gravitational acceleration, and β is the bed slope (Guza and Bowen, 1976). Complete reflection occurs for $r \geqslant 1$. Equation 1 shows that phase shifting varies with reflectivity, the distance from the antinode and the wave length.

The results of Adeyemo (see Table 1) indicate that reflectivity does not provide a general explanation. It is evident from his results that phase shifting is relatively constant as h/L varies throughout the shoaling region (L being the local wave length). Also the relatively steep laboratory waves provide insufficient reflection to produce the observed phase shift at any location, even on the steeper beach. The very low r shown in Table 1 for the 1:18 beach is totally inadequate for explaining the measured phase shift ($\hat{\theta}$) even when kx = $\pi/4$, where the phase shift predicted from (1) reaches its maximum (θ_{max}). Nor is it conceivable that the reflection coefficient ($r_{\hat{\theta}}$) required for $\hat{\theta}$ at respective locations (kx) could be sufficiently large for such steep waves on such a flat beach. Similarly, reflection coefficients ($r_{\hat{\theta}max}$) at positions of phase shift maxima (kx = $\pi/4$) could not be expected to attain required values.

The same argument can be applied to the results from the dissipative Del Monte Beach of Thornton et.al. and Umina Beach of this study where substantial values of $\hat{\theta}$ were obtained (Table 1). The impossibility of reflection coefficients capable of explaining $\hat{\theta}$ values is demonstrated by the nature the runup at Umina Beach shown in Figure 6b.

Nevertheless, on steep beaches partial reflection theoretically and intuitively must be important in phase shifting. The linear predictions of θ in examples 4 and 9 to 12 (Table 1) for r approximated from (2) generally show fairly poor agreement with $\hat{\theta}$. However, θ is very sensitive to kx and so relies on accurate positioning of the antinode. Local kx at the break point has been used in this paper. It was determined from linear theory and phase measurements between instruments along transects. Better results may be possible using the average of local wave lengths between the break point and the shoreline. It is apparent in Table 1 that θ_{max} and $r_{\hat{\theta}max}$ are at least of the right order of magnitude for the steep beach examples (particularly examples 4 and 10).

It is therefore evident from waves on non-reflective, flat beaches that phase shifting is generally inherent in both shoaling and near-breaking waves. It is probable that phase shifts due to reflection will be superimposed upon these inherent shifts. In accounting for phase shifting due to reflection therefore, it will be first necessary to gain some understanding of the inherent phase relationship. Wave-wave interaction and associated cross-spectral transfer may be important in gaining this understanding (e.g., Thornton, 1979; Guza and Thornton, 1980).

IMPLICATIONS FOR BREAKER TYPE

Since phase shifting appears to be a significant factor in the kinematics of waves near breaking it should be an important determinant of the breaking process and resulting breaker type. Iwagaki et.al. (1974) found in laboratory waves that systematic differences exist in velocity profiles below the crests of breakers of different type. Vertical gradients in horizontal velocity components are shown in Figure 7 to be steepest toward the surging end of the breaker type continuum. This is particularly evident near the crest, though also general over the entire profile.

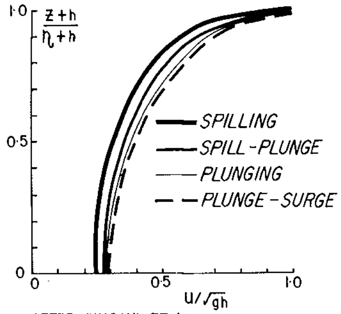
The vertical distribution of horizontal velocities together with rates of decrease in wave phase speed provide a useful concept of kinematic stability. This concept may provide a signature of general form $(\partial u/\partial z)\ (\partial C/\partial x)^{-1}$ which is the kinematic equivilent of previous breaker type indices (e.g., Galvin, 1968; Battjes, 1974), where u is the horizontal water velocity, z.=.-h, and C is the phase speed. Kinematic stability is related to the kinematic critereon for the initiation of wave breaking ($u_o \geqslant C$) extended through all z. Sharp crested waves breaking on gentle slopes (Fig. 6c) have relatively large vertical gradients in horizontal velocity. The significantly higher velocities at the crests of these waves are indicative of their strong non linearity. At the break point, the slow decrease in phase speed results in velocities exceeding the phase speed in the vicinity of the crest only and spilling breakers result. On steep slopes with surging-collapsing waves the vertical gradient in velocity components is smaller. Since the phase speed decreases abruptly velocities tend to exceed the phase speed through much of z almost simultaneously, as illustrated in Figure 6d.

If kinematic stability is significantly effected by phase shifting then the latter will be important in the determination of breaker type. Figure 8 shows for the different field results the distribution of horizontal velocities below the crest expressed as a percentage of horizontal velocity component maxima (u_{zmax}). These profiles are to be expected if the velocity at any point below the crest can be assumed to be a cosine function of u_{zmax} and θ_z.

For surging breakers it is apparent that the near quadrature phase at the break point results in very low horizontal velocities below the wave crest. This produces a stabilizing tendency such that horizontal velocities do not come to exceed the phase speed until the latter is very small. This stability is commonly recognized for surging waves as being the non-breaking, totally reflected case (e.g., $\xi_i \leqslant 2$ of Miche; see Guza and Bowen, 1976). Clearly however, since phase shifting can evidently occur by degrees this stability must also vary accordingly.

For the larger (relatively steeper) surging-collapsing waves in Figure 8, reduction of velocities below the crest due to phase shifting is not so great. However, the vertical gradient in velocities resulting from phase shifting is much smaller than in the more dissipative breaker types towards the plunging and spilling end of the continuum. In terms of the kinematic stability of these waves therefore, this phase shift produces no additional steepening of the velocity gradient below the crest.

For the more dissipative plunging and spilling waves in Figure 8 it is clear that vertical differences in phase shifting produce a considerable augmenting of the velocity gradient. This combined with lower gradients in phase speed should tend to confine initial breaking to the crest region.



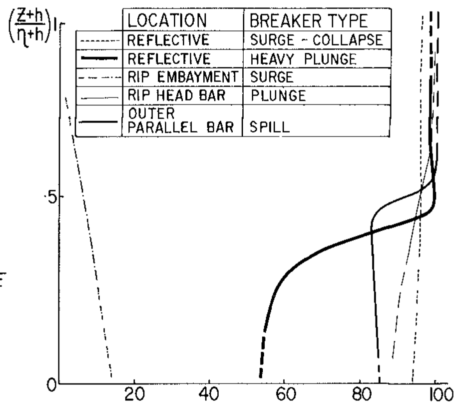


FIGURE 7. Horizontal velocities below crests of different breaker types

FIGURE 8. Velocity below crest as percentage of Umax

CONCLUSIONS

Phase of velocities lag behind the surface elevation under most conditions of waves near breaking. It appears that a certain amount of this phase shift is inherent in the shoaling process. However partial wave reflection must also produce some phase shifting. The contribution of each process to phase shifting has not been determined. It is obvious however that the greatest potential for phase shifting lies on reflective beaches where anything up to near phase quadrature may be obtained. It is also apparent that the vertical distribution of phase is more homogeneous on reflective beaches. On dissipative beaches substantial phase shifting appears to be restricted to depths midway beneath the crest.

Phase shifting must therefore be an important determinant of the kinematics of waves near breaking. It therefore must also be important in determining the breaker type. Large, vertically uniform distribution of phase shifting causes low velocities beneath the wave crest inducing stability with resulting surging waves. Smaller phase shifts uniformly distributed below the wave crest tend to develop with collapsing waves on steep slopes. Where phase shifting inreases markedly toward the bed velocities beneath the crest are reduced in that direction. The vertical velocity profile is thus significantly steepened. Breaking then tends to be confined to the crest region of the wave. This is consistent with the development of spilling waves where beach slope is low. It may also help explain the well known development of plunging waves where slope is steep.

Wave-current and wave-wave interactions occur extensively on complex natural topographies. It is likely that these also have significant effects on breaker kinematics both directly and indirectly through the influence on phase shifting. However, these effects are not yet obvious particularly with respect to phase shifting. Nevertheless it will be necessary to account for kinematic effects of phase shifting when considering breaker type response to surf-zone morphodynamics. Similarly, theoretical development of near breaking waves in shallow water will require inclusion of this factor.

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