CHAPTER 10

A Numerical Solution to Transient Wave Induced Harbor Oscillations Using Boundary Element Technique

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Abstract

A new numerical technique for solution to transient linear long wave induced oscillations is introduced. It is based on the boundary element method, which is well recognized by its numerical efficiency and convenience. Several harbor models are investigated. Advantages of the proposed method as compared to the existing techniques are discussed.

1. INTRODUCTION

Studies concerning harbor oscillations can be classified into two major groups, periodic wave excitations and transient wave excitations. The assumption of periodic incoming waves reduces the problem from a solution of wave equation to one of Helmholtz equation. There are a number of well-established theories available in the literature for the time harmonic case. However, only a few studies have been reported on the transient problem. The traditional approach to the transient problem is to utilize the knowledge of frequency response of the harbor and derive the solutions by means of Fourier synthesis (Carrier and Shaw, 1969). An alternative approach using finite element method (FEM) was introduced by Lepelletier (1980) to solve the problem by a time marching scheme with a time dependent ocean boundary condition.

The present study introduces a boundary element method (BEM) solution for the transient oscillations of arbitrary-shaped, constant depth harbors. Unlike Fourier synthesis approach, this method provides the required information in a natural and direct way. Moreover, this procedure requires neither internal cells nor their associated domain integrals, making the method especially attractive from the computational point of view.

2. THEORETICAL BACKGROUND

Most of the existing numerical harbor models emerge on the validity of the linear wave theory. Indeed, good agreement between the linear theory predictions and experimental results has been reported by various authors, Lee (1969) and Lepelletier (1980), for example. Hence, the problem is formulated within the framework of linear wave theory. The theory assumes irrotational flow of an inviscid, incompressible fluid, and the wave amplitude to be

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infinitesimally small. In addition, the water depth is assumed to be constant and the wave length is long enough to meet criterion for long wave approximation.

Under these conditions, the governing equation in two horizontal dimensions x and y is given by

$$\frac{\partial^2 \phi}{\partial t^2} = c^2 \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) \quad . \tag{1}$$

Here ϕ is the unknown velocity potential and c is the speed of propagation. Following the long wave approximation, c is known as $c=(gd)^{1/2}$ with g and d being earth's gravitational acceleration and the water depth, respectively.

The solution of ϕ within the domain sketched in Fig-1 is sought in the present study. The configuration under consideration consists of an interior domain which will be identified as domain II and an exterior domain, domain I. Domain II is an arbitrary-shaped, constant depth harbor with vertical boundaries. It is connected to domain I with a partially or fully open entrance. Domain I represents the open ocean with semi-infinite boundaries. One part of the boundary coincides with the straight coastline extending from $-\infty$ to $+\infty$. The remaining part is a fictitious semi-circle of radius $R=\infty$, connecting both ends of the coastline.

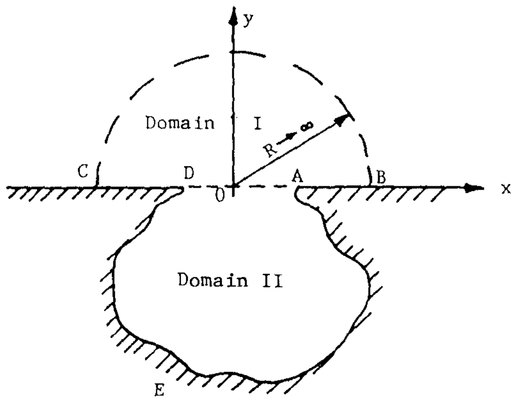

Fig-1. Geometric configuration of an arbitrary-shaped harbor.

Parallel to the time harmonic model of Lee (1969), the problem is formulated in the two regions separately. In domain I, decomposition of the velocity potential ϕ_I into components is conventional in linearized water wave problems:

$$\phi_I = \phi_i + \phi_r + \phi_s \tag{2}$$

where ϕ_i is the incident wave potential, ϕ_r is just its reflection as if the harbor did not exist, and ϕ_s is the scattered potential representing the disturbance introduced by the harbor. Since the incident wave potential is normally known, ϕ_i and ϕ_r are immediately determined. Therefore, only formulation and solution of ϕ_s will be sought in domain I.

The boundary and initial conditions are specified as:

Domain I	Domain II	
Boundary conditions:		
$q_s=0$ on the straight coastline	$q_{II}=0$ on DEA	(3)
$\phi_i+\phi_r+\phi_s=\phi_{II}$	on DA	(4-a)
$q_s=-q_{II}$	on DA	(4-b)
Initial conditions, at t=0:		
$\phi_s = \dfrac{\partial\phi_s}{\partial t} = 0$	$\phi_{II} = \dfrac{\partial\phi_{II}}{\partial t} = 0$	(5)

Here, q is the normal derivative of ϕ, $q=\vec{n}\cdot\nabla\phi$, and $\vec{n}$ is a unit outward normal vector along the boundary.

Condition (3) assures that boundaries are impermeable by forcing the particle velocities to vanish. Conditions (4-a) and (4-b) are basically necessary to have a continuous surface elevation and fluid velocity across the boundary interface of the two domains. Notice that condition (4-b) is obtained by differentiating (4-a) with $\partial(\phi_i+\phi_r)/\partial n=0$ being understood. The negative sign in (4-b) indicates that the outward normal vectors are directed in opposite directions in I and II on DA. The last constraints are due to the hyperbolic character of the governing equation where one needs two initial conditions. They are chosen to be zero for convenience. It implies that the analysis will be started from a moment when the body of water in the harbor is at rest. However, this is not on approximation but rather a mathematical convenience; as will be seen later, a BEM formulation may include any initial state. It is worth mentioning that the radiation condition of the time harmonic problem is replaced by the causality principle to ensure the mathematical uniqueness. This constraint will be imposed in the determination of the Green's function of the problem. For a complete derivation of the Green's function and a detailed formulation of the problem, one is referred to Demirel (1986).

3. NUMERICAL ANALYSIS

No analytical solution of the problem described in the previous section exists. On the other hand, a numerical solution is always possible by utilizing one of the solution techniques depicted in Fig. 2.

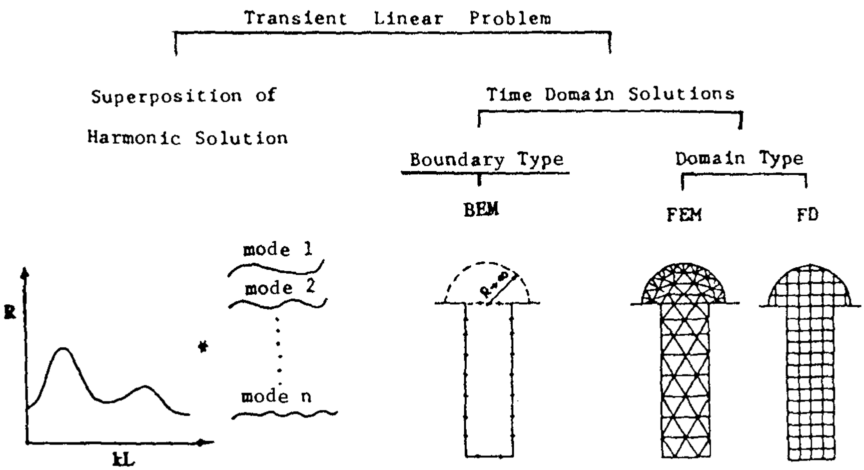

Fig-2. Classification of available solution techniques.

The first method is the traditional Fourier synthesis technique. This is a two-step procedure in which the transfer function of the harbor must be determined first. Afterwards, the transfer function and the Fourier transform of the incident wave system are convolved in the frequency domain followed by an inverse Fourier transform of the product. The second group contains time domain solution techniques. They can be sub-grouped into two categories, domain type solutions and boundary type solutions. Two major differences stand out between these categories as shown in Fig. 2. First of all, in the domain type technique, determination of the potential at any point requires the simultaneous solution of the entire domain. In contrast, in the boundary type solution technique, information of the boundary potential is sufficient to determine the interior potential at any point. Secondly, the open sea region can be extended to infinity in the boundary type method without introducing any approximation. In the domain type technique, however, the exterior domain has to be terminated at a finite distance. Because of these two reasons, BEM is used in this study. An outline of the method is summarized in the following.

The first step is to transfer the governing differential equation to an integral form. Afterwards, the solution of this integral equation is sought. The resulting integral equation for the wave equation in two dimensions is in the following form (Mansur and Brebbia, 1982):

$$4\pi\lambda\phi(r,t) = \int_o^t \int_\Gamma \frac{\partial R}{\partial n} (\phi B + \frac{v}{c} \phi^*) \, d\Gamma \, d\tau + \int_o^t \int_\Gamma q\phi^* d\Gamma \, d\tau$$

$$+ \frac{1}{c}\int_\Omega (\frac{v_o}{c} \phi_o^* - \phi_o B_o + \frac{\partial \phi_o}{\partial R} \phi_o^* + \phi_o \frac{\phi_o^*}{R}) \, d\Omega \qquad (6)$$

where v denotes the time derivative of ϕ, $v = \partial\phi/\partial t$, q is the normal derivative of ϕ and R is the distance between the observation point r and the source point ξ, $R=|r-\xi|$. The parameter λ is equal to 1, 0 or 1/2 for r to be inside, outside or on a smooth part of the Γ boundary, respectively. Definition of the terms ϕ^* and B are given as,

$$\phi^* = \phi^* (r,t/\xi,\tau) = \frac{2c}{[c^2(t-\tau)^2-R^2]^{1/2}} H[c(t-\tau)-R] \qquad (7)$$

$$B = B(r,t/\xi,\tau) = \frac{2c[c(t-\tau)-R]}{[c^2(t-\tau)^2 - R^2]^{3/2}} H[c(t-\tau)-R] \qquad (8)$$

with $H[c(t-\tau)-R] = \begin{cases} 0 & c(t-\tau)<R \\ 1 & c(t-\tau)>R \end{cases}$, a unit step function .

The term ϕ^* is the Green's function for the two-dimensional wave operator, which may be considered as the effect of a source applied impulsively at $t=\tau$ located at $r=\xi$. A subscript o, as appeared in (6), indicates initial time, t = 0. Integrals over Γ are boundary integrals, while over Ω are domain integrals. All integrals are Cauchy principal-value integrals.

In order to evaluate the line integrals in (6), the boundary is discretized into a series of elements and the integrals are computed on each element piecewise. The boundary of the harbor ADE is discretized into N straight segments. In addition, the time dimension is divided into F time steps. Furthermore, functions ϕ and q in equation (6) are assumed to vary within each element and time step according to the space and time interpolation functions such that;

$$\phi(r_j,t_f) = \psi_j^T \gamma^f \phi_j^f \qquad (9)$$

$$q(r_j,t_f) = \mu_j^T \theta^f q_j^f \tag{10}$$

Here ψ_j and μ_j are space interpolation functions, γ^f and θ^f are time interpolation functions, whereas ϕ_j^f and q_j^f are column vectors containing the nodal values of ϕ and q, respectively, within the jth segment. Substituting (9) and (10) into equation (6) and applying a time stepping scheme to initiate the time integration always from the initial time yields the following set of algebraic equations:

$$\sum_{f=1}^{F} [H^{Ff}] \{\phi^f\} = \sum_{f=1}^{F} [G^{Ff}] \{q^f\} . \tag{11}$$

The elements of the matrices $[H^{Ff}]$ and $[G^{Ff}]$ are given by

$$h_{ij}^{Ff} = 4\pi\lambda\delta_{ij}\delta_{fF}\phi_i^F - \int_{\Gamma_j} \frac{\partial R_{ij}}{\partial n} \psi_j^T \int_{t_{f-1}}^{t_f} \left(\gamma^f B + \frac{\partial\gamma^f}{\partial\tau}\frac{\phi^*}{c}\right) d\tau d\Gamma$$

$$g_{ij}^{Ff} = \int_{\Gamma_j} \mu_j^T \int_{t_{f-1}}^{t_f} \theta^f \phi^* d\tau d\Gamma . \tag{13}$$

Note that all the domain integrals in (6) are dropped out since ϕ and $\partial\phi/\partial t$ are zero in accordance with the initial conditions (5) and the time integrals are evaluated always from the initial time t=0.

Equation (11) is simultaneously applied to both domains. In each domain, the velocity potential ϕ is expressed in terms of the unknown normal derivatives of ϕ at the harbor entrance. These unknowns are determined by satisfying the matching conditions (4) on the entrance. Having obtained the derivatives of the potential along the entrance, the interior potential is calculated by letting the observation point approaching to any interior point desired.

4. NUMERICAL EXAMPLES

In the numerical computations, three model harbors were considered. A rectangular harbor and a circular harbor were taken first. Both harbors have an uniform water depth and are connected to an open-sea. The third is a model of the East and West basins of Long Beach Harbor, California. The interpolation functions ψ_j, μ_j and θ^f were taken as constant whereas γ^f was taken as linear in the computation.

The results given here were computed for two incident wave forms, an exponentially decaying cosine wave and a solitary wave:

$$\eta_i = A\ e^{-\alpha|t|} \cos(\omega_o t) \tag{14}$$

$$\eta_i = A\ \mathrm{sech}\ (mt) \tag{15}$$

where A, α, ω_o and m are the amplitude of the incident wave, a decay factor, the incident wave frequency and a dummy frequency parameter, respectively.

In the following figures, the results of the present BEM model were compared against the Fourier synthesis solutions. In each figure, the incident wave system is plotted in the upper part. The lower part is allocated for the comparison of two different approaches. The solid curve and the circles represent the results of the Fourier synthesis solution and the present theory, respectively. Coordinates of the figures were nondimensionalized by suitable parameters. For instance, values of the surface elevation were nondimensionalized by $(\eta_i)_{max}$ defined as the maximum elevation of the incident wave. Likewise, the elapsed time is nondimensionalized either by the period of the incident wave, as in the case of exponentially decaying cosine waves, or by a representative time scale for the case of a solitary wave. For a solitary wave defined by (15), the representative time scale was taken as $\bar{T}=\bar{\lambda}/(gd)^{1/2}$. Here $\bar{\lambda}$, the so-called effective wave length, is defined as twice the distance between the peak of the sech(mt) and a point at which the amplitude reduces to 0.1% of its maximum.

Case 1: A rectangular harbor

A fully open rectangular harbor with a width to length ratio b/L= 0.5 (b=2.5 feet, L=5.0 feet) and a uniform water depth d=0.5 feet was considered as the first case. The boundary of the harbor was discretized into N=32 constant elements with 6 of them placed on the entrance. The observation point was taken to be located at the back boundary of the harbor.

Fig-3 shows the time history of oscillations due to an exponentially decaying cosine wave having a frequency equal to the first natural frequency of the harbor, kL=1.1. The figure indicates that the maximum amplification for this transient wave is 3.8 and the oscillations last a little longer than 4 times the period of the incident wave system.

In the next figure, harbor response due to a solitary wave of $\bar{\lambda}/L$=5.7 was considered. As Fig-4 shows, the magnitude of the first, peak, which represents the major impact of the solitary wave, is in good agreement although there exist relatively minor discrepancies in the subsequent time steps.

Case 2: A circular harbor

The second harbor considered is a circular model previously used by Lee (1969) with a radius r=0.75 feet, a constant depth d=0.5 feet and a 60 degrees gap. For this harbor, the observation point was taken at the center of the harbor. The boundary of the harbor was discretized to N=30 straight elements with 6 of them located at the entrance.

Fig-5 shows the harbor response due to an exponentially decaying cosine wave of kr=0.5. It is seen that the peak amplitude is slightly lower but in general agrees well with the Fourier synthesis solution. Oscillations last approximately 8 cycles, reaching a maximum amplification of 2.9. The subsequent amplitudes are slightly different and decay faster in the BEM solutions.

The solitary wave induced oscillations in the circular harbor of r=0.75 were plotted in Fig-6 along with the incident wave systems. The upper part of Fig-6 represents the incident wave system with $\bar{\lambda}/2r$=12.5. The computed surface elevation at the center of the

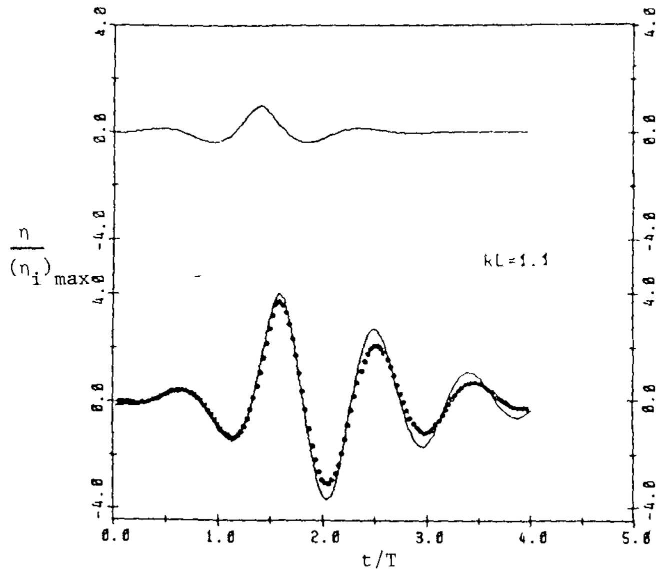

Fig-3. A comparison of time history of oscillations at the back boundary of a rectangular harbor, b/L=0.5, due to an expo nentially decaying cosine wave of kL=1.1 with α=0.3.

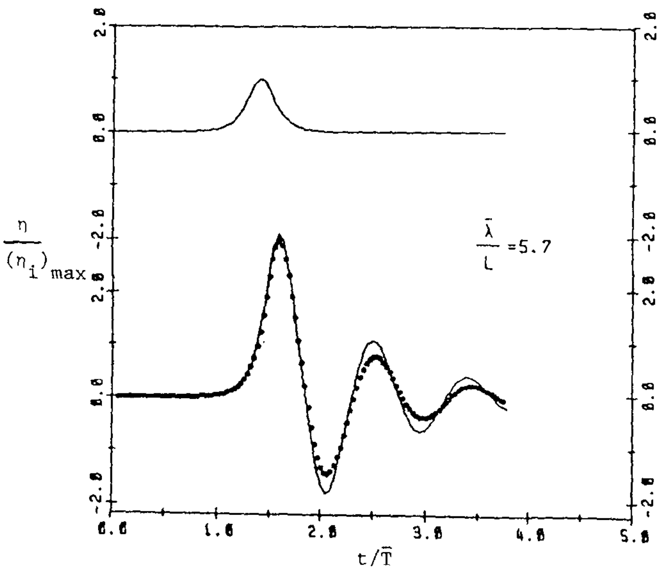

Fig-4. A comparison of time history of oscillations at the back boundary of a rectangular harbor, b/L=0.5, due to a solitary wave of $\bar{\lambda}$/L=5.7.

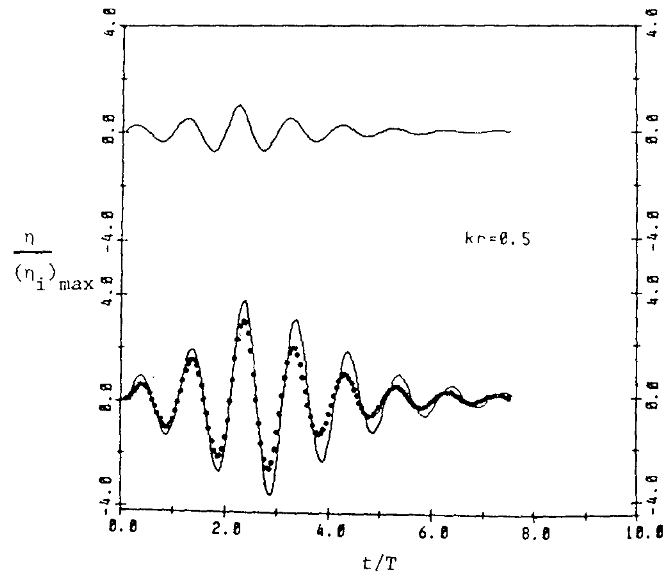

Fig-5. A comparison of time history of oscillations at the center of a circular harbor, r=0.75, due to an exponentially decaying cosine wave of kr=0.5 with a decay factor α=0.3.

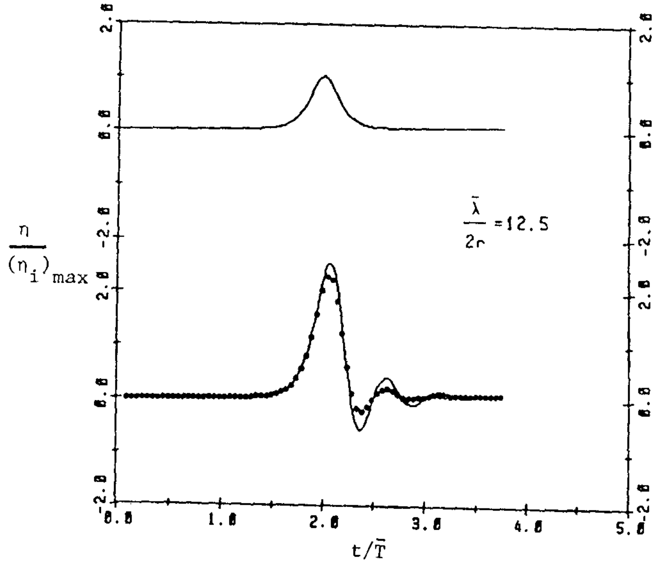

Fig-6. A comparison of time history of oscillations at the center of a circular harbor, r=0.75,due to a solitary wave of $\bar{\lambda}/2r=12.5$.

harbor is placed in the lower part of the same figure. As Fig-6 shows, the maximum amplification is 2.2 and there are few minor oscillations following this peak. This implies that the incident wave is so long that the presence of the harbor practically does not perturb the reflected wave pattern.

Case 3: Long Beach Harbor

A model of the East and West basins of Long Beach Harbor as shown in Fig-7 was considered as the last case. The same model was also used by Lee (1969). The entrance of the harbor is 0.2 feet, the water depth is d=1 feet and the characteristic length is L=1.44 feet. For this harbor, the observation point was chosen at the lower right corner of the harbor, Fig-7. The boundary of the harbor was discretized into N=75 straight elements with two elements located on the entrance.

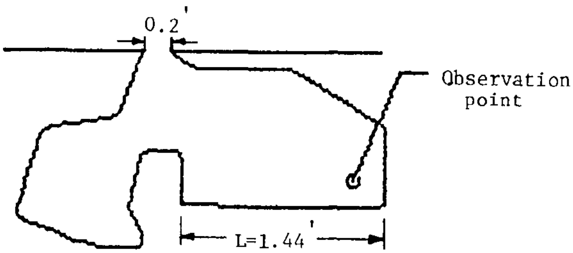

Fig-7. A model of East and West basins of Long Beach Harbor, California.

Fig-8 shows the results of a computation conducted for an exponentially decaying cosine wave input of kL=0.6. The figure indicates that the maximum amplification is 3.2 and the duration of oscillations is approximately 5 times the period of the incident wave system.

The transient response at the designated observation point of the Long Beach harbor due to a solitary wave of $\overline{\lambda}/L = 10.5$ was shown in Fig-9. It is seen that the present prediction matches well with the Fourier synthesis solution especially for the leading wave. According to the figure, the present theory estimates a maximum amplification of 2.5 and the duration of oscillations to be nearly 4 times the effective wave period $\overline{T}$.

5. CONCLUSIONS

A numerical solution to transient linear wave induced harbor oscillations using BEM is introduced. The transient linear long waves are focused in two horizontal dimensions penetrating into a constant depth, arbitrary shaped harbor. Utilizing the fundamental

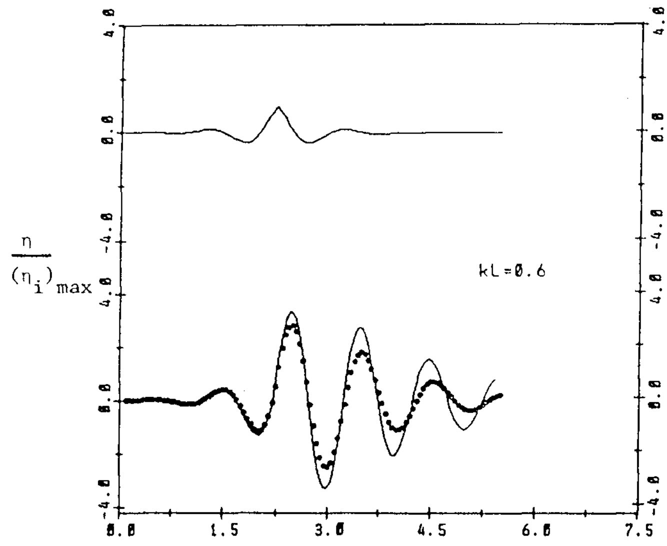

Fig-8. A comparison of time history of oscillations at the back boundary of the Long Beach Harbor model due to an exponentially decaying cosine wave of kL=0.6 with α=0.3.

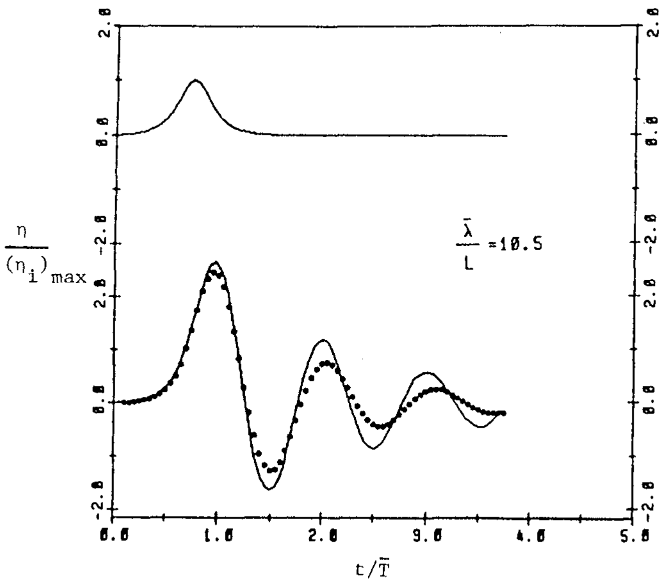

Fig-9. A comparison of time history of oscillations at the back boundary of the Long Beach Harbor model due to a solitary wave of $\bar{\lambda}$/L=10.5.

solution of Green's function for wave equation in two dimensions, only inputs of potential or surface elevation due to transient waves at the harbor entrance is required. The fluid domain is discretized into two regions but a matching technique is used at each time step to evaluate the condition at the entrance. The interior potential or elevation is obtained with only knowledge of the boundary potential.

This method has advantages over frequency domain solution in that it provides in a natural and direct way the time history of oscillations. Moreover, the computation time of the BEM solution is considerably less than that of the Fourier synthesis approach. This is because of the fact that a sufficiently large number of frequency responses are needed to carry out a proper Fourier synthesis solution.

Comparing with the theories based on the domain type techniques, the BEM approach carries forward some widely recognized priorities, such as less data preparation and reducing in the dimensionality of the problem. Also, solution in the exterior domain can be obtained easily without any approximation on the boundary conditions at the infinity. In contrast, in the FEM, a time dependent boundary condition must be specified along a large semi-circle in the open sea to avoid an artificial reflection. The accuracy of the results and the computation time are closely related to the choice of the radius of this circle.

6. ACKNOWLEDGEMENTS

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7. APPENDIX - REFERENCES

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