CHAPTER 79

SURF BEAT GENERATION ON A MILD-SLOPE BEACH

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ABSTRACT: Two dimensional generation of surf beats by incident wave groups is examined theoretically. An inhomogeneous wave equation describes the amplitude of the surf beat wave. The forcing function is the modulation of the radiation stress. The short waves are amplitude modulated both outside and inside the surf zone causing the long wave generation to continue right to the shore line. Resonant generation as shallow water is approached is included. The analytical solution is evaluated numerically and shows a highly complicated amplitude variation of the surf beat depending on the parameters of the problem.

1. INTRODUCTION

Field observations show that on mild slope beaches a significant amount of wave energy occurs at frequencies far below the peak frequency of the incoming sea waves. The existence of such long waves were first reported by Munk (1949) who also speculated that the components with period around 2 minutes were caused by variation in height of the surf and he coined the name "surf beats." Also Tucker (1950) found long waves of 1-5 minute period with a height that increase linearly with the height of the short period sea, and a time lag corresponding approximately to the time it would take for a wave group to reach the breaker zone or beach and for a long wave generated there to be reflected back to the observation point. Longuet-Higgins and Stewart (1962,64) suggested that while the short waves are destroyed by breaking, the set-down wave generated by and following the wave groups outside the surf zone as a forced wave is reflected at the beach and propagates seaward as a free wave.

Since the first recording, numerous observations have shown that the energy of surf beat can actually be very substantial and in some cases even exceed that of the high frequency wind waves (Wright et al., 1982) and the amplitudes at the shoreline can be comparable to that of wind waves (Guza & Thornton, 1982,85).

Although no final proof has been established, it seems widely accepted today that the surf beats are generated by mechanisms in the nearshore region and that they are associated with modulations of the amplitude of the short-period incoming waves. However, several possible ways in which energy can be transferred from the high

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frequency wind waves to the surf beats have been considered with more or less decisive results. It is likely that several possible mechanisms can be active either separately or at the same time. Closely related to the question of generation is the nature of the long wave motion in the nearshore region. In particular, it has been discussed extensively whether the nearshore long wave motion is dominated by forced or free waves; by progressive or standing waves, and whether it consists of crosshore directed (essentially two-dimensional) waves or trapped edge waves, which are three dimensional reflection-refraction wave patterns.

The present work assumes that the surf beat can exist as a two-dimensional motion. It is inspired by the work of Symonds et al. (1982) who investigated a mechanism for generating two dimensional surf beat that had not been considered previously. That mechanism is based on the fact that waves initially of different height will break at different distances from the shore line and (more important) have a different height at breaking. The surf beats are generated by the variation which this breaking pattern causes in the total radiation stress in the region between the extreme seaward and shoreward positions of the breakpoint. The varying breakpoint generates a shoreward moving long wave with the frequency of the wave groups, and since the forcing takes place only in the breaker region the long wave is a free wave through the rest of the surf zone. At the shoreline this wave is fully reflected so that a standing long wave is formed in the surf. Outside the surf zone the reflected wave continues seaward as a progressive wave.

In the model of Symonds et al., the groupiness of the waves is totally destroyed at the breaking point which is why there is no surf beat generation in the actual surf zone. A saturation model with a constant wave height to water depth ratio is used for the waves in the surf zone.

In the work presented in the following, we allow the wave groups to be maintained all the way to the shoreline. Measurements as well as experiments with the surf zone model developed by Svendsen (1984) indicates that if there is a variation in wave height at the breaker point the waves will remain different through the surf zone. For simplicity, we model this by introducing a breaker height variation at a fixed breaking point. This will represent the other extreme relative to the situation studied by Symonds et al. Observations show that the true picture probably represents a combination of the two: a varying breakpoint with some groupiness left in the surf zone. A somewhat similar problem was studied by Foda & Mei (1981) using a multiple scale expansion and different assumptions about the relative magnitude of the wave components involved. Our result contains a resonant interaction which was also included in the analysis by Freilich & Guza (1984) although under different assumptions about the magnitude of the bottom slope.

2. DESCRIPTION OF MODEL CONDITIONS

Since the phenomena associated with wave groups are quite complicated it may be useful to give a more detailed description of the situation which is created mathematically in the following.

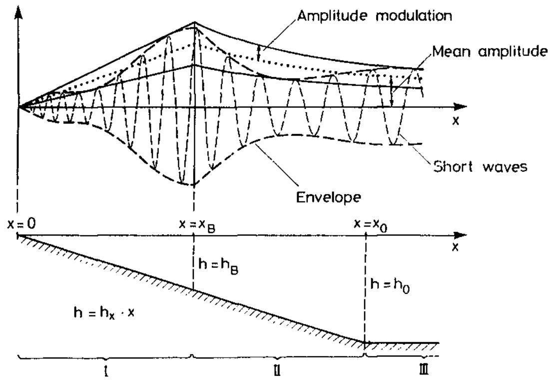


Fig. 1: Sketch of bottom topography and short wave variation considered.

Region III

The situation is depicted in Fig. 1. We consider a region of constant depth which could be a continental shelf, say, (region III in the figure). In the present study we simplify by letting the groupiness of the waves be caused by two regular incoming wave trains with slightly different wave number $k_0(1+\epsilon)$ and $k_0(1-\epsilon)$ the second with a much smaller amplitude than the first. This implies that the surf beat wave becomes a simple wave of constant period which allows us to seek analytical solutions to the problem. The basic equations used, however, can also be applied to the general case of a time varying wave height as in a train of irregular Stokes waves, but this would require a numerical solution.

In region III we assume a quasi-uniform state. The modulation of the waves causes a variation in radiation stress which generates a set down wave, and we assume that the length of the wave height modulation is much larger than the depth so that the set down wave is a long wave. This set down wave and its propagation into the shore region was neglected by Symonds et al. (1982).

Present in region III is also an outgoing long wave which represents the results of the transformations and reflections taking place in the nearshore region. The amplitude and phase of that wave is one of the unknowns of the problem.

Region III is primarily introduced to be able to establish well defined and reasonably simple seaward boundary conditions for the problem.

Region II

In region II the depth is decreasing sufficiently gently that we can assume local solutions for short and long wave components. Thus the short wave will be shoaling towards the breaking point x_B. During this process, however, the balance between the variation in radiation stress and set down wave is constantly changing which causes the forced set down wave to grow. As the short waves approach breaking their group velocity approaches that of the forced wave creating a state of almost resonance. We describe this dynamical process (in contrast to the equilibrium situation in region III) by a WKB approximation.

For most values of the governing parameters we find that in spite of the resonant interaction the energy transferred from the short wave to the forced wave is relatively small, and we therefore omit in the present paper to take the effect of this energy flux into account in the development of the short waves. There is no principle difficulty though in extending the formulation to include this effect as well.

At the transition point x_0 between region III and II the solutions for the two regions are matched by requiring continuity in mass and momentum flux for the long wave solution. These conditions cause a partial reflection at x_0 of the set down waves, whereas it is assumed that no such reflection occurs for the short waves (either because of sufficiently deep water or because the transition between horizontal and sloping bottom actually takes place sufficiently smoothly to allow the short waves to adjust).

Region I

At x_B the short waves are assumed to break. Since there is a temporal variation in the height of the waves reaching this point, we have a time-varying breaker height at x_B. The dissipation of energy in the surf zone is proportional to H^3 and therefore, although the wave height variation will decrease shorewards, those waves that are highest at the (fixed) breakpoint will always remain the highest, etc. Thus this mechanism implies that the groupiness of waves remains present all the way to the shoreline. It also implies that the generation process responsible for the modification of the forced long wave continues through the surf zone.

Again we have chosen to use the simplest possible description of the processes involved. Thus a saturation model based on $a=\gamma h$, where h is the water depth, is used to describe the wave height variation in the surf zone of the short waves. Essentially the specification of wave height as a fraction of depth replaces solution of the energy equation for the short waves. Since the short wave height varies as the groups propagate shorewards, this means that the parameter γ is $\gamma(x,t)$.

The matching at x_B again requires continuity in the mass and momentum flux for the long wave motion. The abrupt shift in the rate of change of the radiation stress represents a source of difference at the two sides of the matching point.

At the shoreline the long wave is fully reflected and radiated seaward as a free wave. The background for this assumption is discussed below and involves some aspects that, although known from other areas of wave dynamics, do not seem to have been applied before to analyze the behavior of surf beats near the shoreline.

3. THE GOVERNING EQUATIONS

The governing equations for the surf beat are derived from the depth integrated and time averaged equations for waves and currents (see e.g. Phillips, 1977; Mei, 1983). In those equations the set-up/set-down will then correspond to the long wave surf beat and the time-varying current will represent the particle motion in that long wave. Since the equations are derived under the assumption that the current velocity is uniform over depth, their use implies that we assume the surf beat is everywhere a long wave (as already indicated). On the other hand, the equations contain all relevant non-linear terms in the current discharge and set-up (including the momentum flux due to the mass flux Q_S in the waves). Thus the description of the long wave component of the wave motion is actually equivalent to that given by the nonlinear shallow water equations.

The total water particle velocity u is split into

$$u = u_w + U \tag{3.1}$$

where U is the velocity in the current, u_w that of the oscillatory motion so that $\overline{u_w} = 0$ below wave trough level ($\overline{}$ meaning average over a wave period).

The relevant equations are then the continuity equation

$$\frac{\partial b}{\partial t} + \frac{\partial \overline{Q}}{\partial x} = 0 \tag{3.2}$$

where b is the elevation of the mean water surface and

$$\overline{Q} = Q_c + Q_s \quad ; \quad Q_s = \overline{\int_{-h_0}^{\eta} u_w dx} \quad ; \quad Q_c = \int_{-h_0}^{c} U dz \tag{3.3}$$

η is the total surface elevation measured from a horizontal reference level. The equation of shore-normal (x) momentum is

$$\frac{\partial \overline{Q}}{\partial t} + \frac{\partial}{\partial x}\left(\frac{\overline{Q}^2}{h} + \frac{1}{\rho} S_{xx}\right) + gh\frac{\partial b}{\partial x} + \frac{\overline{\tau_b}}{\rho} = 0 \tag{3.4}$$

in which h is the local depth (including the long wave surface elevation), S_{xx} is the shore wave radiation stress and $\overline{\tau}_b$ the mean bottom shear stress (which we will neglect here).

Since we are particularly interested in the surface elevation b of the long waves we first eliminate $\overline{Q}$ from the linear terms of (3.2) and (3.4) by cross differentiation. This yields

$$\frac{\partial^2 b}{\partial t^2} - \frac{\partial}{\partial x}\left[gh\frac{\partial b}{\partial x}\right] - \frac{\partial}{\partial x}\left(\frac{\overline{Q^2}}{h} + \frac{1}{\rho}S_{xx}\right) = 0 \tag{3.5}$$

It is assumed that the total motion of any point consists of a quasisteady regular wave on which we superimpose a small perturbation (the amplitude modulation) that varies in time. Thus all wave averaged quantities have the form

$$f(x,t) = f_0(x) + f_1(x,t) \tag{3.6}$$

In particular the depth h is

$$h(x,t) = h_0(x) + b_0(x) + b_1(x,t) \tag{3.7}$$

By assuming that there is a steady state basic solution of wave height variation it can be inferred that for this solution there is no net mass flux. Hence by continuity $Q_0 = 0$ and

$$\overline{Q}(x,t) = Q_1(x,t) \tag{3.8}$$

When these assumptions are substituted into (3.5) we get the following equation for b_1

$$\frac{\partial^2 b_1}{\partial t^2} - \frac{\partial}{\partial x}\left[g(h_0+b_0)\frac{\partial b_1}{\partial x} + gb_1\frac{\partial b_0}{\partial x}\right] - \frac{\partial^2 S_{xx,1}}{\partial x^2} = 0 \tag{3.9}$$

where $S_{xx,1}$ represents the variation in S_{xx} due to the wave height modulation for the short waves (vide (3.6)).

Finally, realizing that at most points $b_0 << h_0$ we neglect terms proportional to $b_0 b_1$ so that the governing equation for the surf beat amplitude b_1 becomes

$$\frac{\partial^2 b_1}{\partial t^2} - \frac{\partial}{\partial x}\left(gh_0 \frac{\partial b_1}{\partial x}\right) - \frac{1}{\rho}\frac{\partial^2 S_{xx,1}}{\partial x^2} \tag{3.10}$$

This is a wave equation with a forcing term which represents the effect the wave height modulation has on the slowly varying surf beat. It is the same equation that was used by Symonds et al. (1982) but only in the region of the breaker point variation. Chu & Mei (1970) and Mei & Benmoussa (1984) derive the same equation for 3D by a multiple scale expansion of slowly varying Stokes waves.

Following Mei & Benmoussa (1984) we let the short wave motion be composed of two waves with almost the same wave numbers

$$\begin{aligned} \eta_1 &= \frac{1}{2}\, a_1 \exp\left[i\int k_1 dx + \omega_1 t\right] + * \\ \eta_2 &= \frac{1}{2}\, \delta a_1 \exp\left[i\int k_2 dx + \omega_2 t\right] + * \end{aligned} \tag{3.11}$$

where * means complex conjugate.

In region III we have k_1 and k_2 constant and we define k_0 and ϵ so that

$$k_1 = k_0(1+\epsilon) \quad ; \quad k_2 = k_0(1-\epsilon) \qquad \text{for } x > x_0 \tag{3.12}$$

The equivalent change in wave frequencies are given by the dispersion relation. We have

$$\omega_g = \frac{1}{2}(\omega_1 - \omega_2) \simeq \frac{\partial\omega}{\partial k} \cdot \epsilon k_0 = c_{g0}\, \epsilon k_0 = \frac{c_{g0}}{c_0}\, \epsilon\omega_0 \qquad \text{for } x > x_0 \tag{3.13}$$

where c_{g0} is the group velocity in region III for the wave with wave number k_0 and frequency ω_0 and $c_0 = \omega_0/k_0$. Equation (3.13) also defines ω_0 as approximately $(\omega_1+\omega_2)/2$.

The total short wave motion $\eta = \eta_1 + \eta_2$ can then be written

$$\eta = \frac{1}{2}\, A \exp\left\{i\left(\int k_0 dx + \omega_0 t\right)\right\} + * \tag{3.14}$$

where A is a complex amplitude. With

$$\Omega_0 = \frac{c_{g0}}{c_0}\,\omega_0 \quad , \quad K_0 = k_0 \cdot \frac{c_{g0}}{c_g} \tag{3.15}$$

we get

$$A = a_1\left[\exp\left(i\epsilon\left(\int K_0 dx + \Omega_0 t\right)\right) + \delta \exp\left(-i\epsilon\left(\int K_0 dx + \Omega_0 t\right)\right)\right] \tag{3.16}$$

or

$$\left.\begin{aligned} A &= a_1\left(e^{i\theta} + \delta e^{-i\theta}\right) \\ \text{where}\quad \theta &= \epsilon\left(\int K_0 dx + \Omega_0 t\right) \end{aligned}\right\} \tag{3.17}$$

The radiation stress S_{xx} for the short waves is then given by

$$\frac{1}{\rho}\ S_{xx} = \frac{g|A|^2}{2}\left[\frac{2c_g}{c} - \frac{1}{2}\right] \tag{3.18}$$

If we therefore write S_{xx} according to (3.6) we see that

$$\frac{1}{\rho}\ S_{xx,0} = \frac{g}{2}\ a_1^2\ (1+\delta^2)\left[\frac{2c_g}{c} - \frac{1}{2}\right] \tag{3.19}$$

$$\frac{1}{\rho}\ S_{xx,1} = ga_1^2\delta\ \left[\frac{2c_g}{c_0} - \frac{1}{2}\right]\ \exp(2i\theta) \tag{3.20}$$

Inside the surf zone the more general expression applies:

$$\frac{1}{\rho}\ S_{xx} = g|A|^2 P \tag{3.21}$$

In the present context, however, we will, for simplicity, allow P to be approximated by $(2c_g/c_0 - 1/2)$ since this only changes the results quantitatively not qualitatively.

The variation of b_1 and $S_{xx,1}$ is described by

$$b_1 = b_a\ \exp(2i\epsilon\Omega_0 t) \tag{3.22}$$

$$S_{xx,1} = S_a\ \exp(2i\epsilon\Omega_0 t) \tag{3.23}$$

so that

$$S_a = a_1^2\delta\ \rho g\left(\frac{2c_g}{c_0} - \frac{1}{2}\right)\ \exp\left(2i\epsilon\int K_0 dx\right) \tag{3.24}$$

which substituted into (3.10) yields

$$\frac{\partial}{\partial x}\ \left(h_0\ \frac{\partial b_a}{\partial x}\right) + 4\epsilon^2\ \frac{\Omega_0^2}{g}\ b_a = -\ \frac{1}{\rho g}\ \frac{\partial^2 S_a}{\partial x^2} \tag{3.25}$$

This is the equation we solve.

Variation of Carrier Wave Amplitude

The two wave components a_1 and $a_1\delta$ which together form the carrier wave will show a variation that in region II corresponds to a simple shoaling under conservation of energy flux. Thus in region II we have

$$a_1 = \left(\frac{c_{g\infty}}{c_g}\right)^{\frac{1}{2}} a_\infty \quad ; \quad a_\infty = a_1(h{=}\infty) \tag{3.26}$$

In the surf zone (region I) the wave heights actually ought to be determined by one of the surf zone models developed in recent years as e.g. Svendsen (1984). This implies solving the energy and momentum equations using realistic descriptions for the relevant wave properties such as radiation stress, energy flux, and energy dissipation. If we assume that the wave height modulation in the wave groups is moderate, the breaker type will be virtually the same for all waves. Except perhaps for violently plunging waves (for which we know very little) this means that if two waves initially have different heights at the breaker point, the highest will remain so throughout the surf zone. Essentially this further implies that some groupiness is conserved also beyond the breaker point. How much of the original wave height modulation that actually is maintained will depend on how the variation in wave height influences the position of the breaker point, an effect we, as mentioned earlier, have neglected here by using a fixed breaking point.

The above mentioned groupiness would result also from the wave model by Svendsen (or other wave models based on solving the energy equation). For simplicity, however, we choose a simpler representation of this physical feature by using a modified saturation model. In the surf zone we let

$$|A| = \gamma h \tag{3.27}$$

where $\gamma^2 = \gamma_0^2 + \gamma_1^2 e^{2i\theta}$. Particularly for $|A|^2$ we get

$$\begin{aligned} |A|^2 &= |A_0|^2 + |A_1|^2 \\ &= \left(\gamma_0^2 + |\gamma_1|^2 \left(e^{2i\theta} + *\right)/2\right) h^2 \end{aligned} \tag{3.28}$$

The values of γ_0 and γ_1 are then according to the saturation hypothesis determined by the breaker heights of the waves. Since before breaking we have

$$|A|^2 = (a_\infty^2(1+\delta^2) + a_\infty^2\delta(e^{2i\theta} + *)) \frac{c_{g\infty}}{c_g} \tag{3.29}$$

we get

$$\gamma_0^2 = \frac{a_\infty^2(1+\delta^2)}{c_{g_B} h_B^2} c_{g\infty} \tag{3.30}$$

and

$$\gamma_1^2 = \frac{2a_\infty^2\delta}{c_{g_B} h_B^2} c_{g\infty} \tag{3.31}$$

where index ∞ refers to deep water, index $_B$ to the values at the breaking point.

4. MATCHING AND BOUNDARY CONDITIONS

To obtain a solution over the three regions described in Section 2, we need a boundary condition at the outer end of region III, matching conditions between II and III and between I and II, and a boundary condition at the shore line. Since (3.25) is a second order equation a total 2x3 conditions are required to establish the solutions in the three regions.

Seaward Radiation Condition

The seaward boundary condition is a radiation condition stating that there are no free waves propagating towards the shore, only the bounded (and known) set down wave.

Matching Conditions

At x_0 and x_B the propagation conditions change, which causes changes in the constants of the general solution. We ensure continuity at those points by requiring that

$$\Big[b_a\Big]_{x_0^-}^{x_0^+} = 0 \quad ; \quad \Big[b_a\Big]_{x_B^-}^{x_B^+} = 0 \tag{4.1a,b}$$

This corresponds to assuming continuity in mass flux across x_0 and x_B. Similarly, continuity in momentum flux can be obtained by applying (3.4) on the two sides of each matching point in combination with (4.1a,b). This yields

$$\left[\frac{\partial b_a}{\partial x}\right]_{x_0^-}^{x_0^+} = -\frac{1}{\rho g h(x_0)}\left[\frac{\partial S_a}{\partial x}\right]_{x_0^-}^{x_0^+} \tag{4.2a}$$

$$\left[\frac{\partial b_a}{\partial x}\right]_{x_B^-}^{x_B^+} = -\frac{1}{\rho g h_B}\left[\frac{\partial S_a}{\partial x}\right]_{x_B^-}^{x_B^+} \tag{4.2b}$$

These relations indicate that db_a/dx will show a discontinuity at the two matching points, of which the one at h_B is by far the most significant.

Boundary Condition at the Shore Line

The model assumes that the short waves are entirely destroyed by breaking. No similar mechanism, however, is available for the long forced wave which will approach the shore line with a finite amount of energy. The wave must therefore be fully reflected there.

It is worth noticing that near the shoreline we actually stress the assumptions underlying the solution, which are that the bottom slope is gentle (i.e., $\Lambda = h_x L/h$ remains small) and the amplitude to depth ratio γ_s is small. As $h \to 0$, however, wave length of the long wave will go to zero as $h^{1/2}$. Hence the slope parameter Λ will grow as $h^{-1/2}$ for a constant h_x. Similarly, the amplitude to depth ratio of the long wave does not remain small.

The growth of Λ and γ_s near the shoreline indicates that the motion is more appropriately described by the nonlinear shallow water equations. However, the basic equations (3.2) & (3.4), and therefore also (3.5), remain valid even under those conditions since no assumptions as to the magnitude of Λ or γ_s have been invoked at the derivation of those equations. As mentioned previously, (3.2) and (3.4) correspond to this approximation, so that, close to the shore those two equations represent the appropriate description. Further away from the shore the NLSW-equations can then be matched to the linear solution as e.g. the wave equation (see Carrier, 1966).

In the present case (3.10) is a linearized form of (3.5) since some approximations have been introduced to get from (3.5) to (3.10). Therefore (3.10) does not represent a proper approximation as $h \to 0$.

This also applies to the WKB approximation used to transform (3.10) to (3.25), since that approximation also requires $\Lambda << O(1)$ corresponding to small changes in depth over a wave length. It does of course also require $\gamma_s << 1$.

For the time being we have chosen to disregard the problem and simply assume that the linearized equation (3.22) applies to the shoreline. The requirement of full reflection then corresponds to requiring that close to the shoreline the wave motion is a purely standing wave with zero net energy flux. Since this approach is in agreement with the actual physical situation it only means that the description is not accurate near the shoreline. At some distance this solution should be the same as if a more correct matching with the NLSW approximation had actually been made.

5. SOLUTION FOR THE SURF BEAT AMPLITUDE

The complete solution to the homogeneous version of (3.25) can be expressed in terms of two linearly independent Hankel functions representing two waves propagating shoreward and seaward respectively. The complete solution to the inhomogeneous equation (3.25) can then be found by the method of variation of parameters. In region I and II we get (i = 1,2) the complex solution

$$b_{a,i} = \left[H_0^{(1)}(a\sqrt{x})\left[-C_i^{(1)}+\int^x H_0^{(2)}(a\sqrt{x})\frac{d^2Sa}{dx^2}dx\right]\right.$$

$$\left.-H_0^{(2)}(a\sqrt{x})\left[C_i^{(2)}+\int^x H_0^{(1)}(a\sqrt{x})\frac{d^2Sa}{dx^2}dx\right]\right] \frac{i\pi}{2\rho g h_x} \qquad (5.1)$$

where $a = 4\omega_g/\sqrt{gh_x}$. In region III the solution is

$$b_{a,3} = C_3\, e^{i2\omega_g x\sqrt{gh_0}} + b_{a,0} \qquad (5.2)$$

where $b_{a,0}$ is the incoming, forced set-down wave, for which we have

$$b_{a,0} = -\frac{1}{\rho}\frac{S_a}{gh_0 - c_{g0}^2} \qquad (5.3)$$

The five constants $C_{1,2}^{(1)}$, $C_{1,2}^{(2)}$ and C_3 are then determined by the four matching conditions and by the boundary condition at the shoreline.

The latter requires that the amplitude of the two wave components are the same and hence yields directly

$$C_1^{(1)} = C_1^{(2)} \qquad (5.4)$$

The four other constants are found numerically by solution of the flow complex equations resulting from applying the matching equations. The details are left out here.

6. NUMERICAL RESULTS

After determining the equations for the integration constants in (5.1) (5.2) and (5.3) numerical results have been calculated by evaluating the $\partial^2 S_a/\partial x^2$ term by numerical differentiation and similarly the integrals by numerical quadrature.

The problem under study has a substantial number of independent parameters. Clearly the bottom slope h_x is a parameter, but since the bottom steepness that the waves actually "feel" depends on the waterdepth to wavelength ratio the relevant measure of the bottom slope can be shown from the solution to be $\Lambda = h_x\, L/h$ where L is the local wave length defined as cT. Λ is assumed to be small in order to allow the waves to adjust to the local depth as assumed in the basic equations. In the solution Λ occurs in connection with the matching process at h_0 and h_B which leads to the two parameters

$$\Lambda_0 = \frac{h_x L_0}{h_0} \qquad \& \qquad \Lambda_B = \frac{h_x L_B}{h_B} \qquad (6.1)$$

The value of Λ_B is actually determined by the assumed breaker index γ.

Other (small) parameters are the ratio of group wave number to carrier wave number

$$\epsilon = (k_1 - k_2)/2k_0 \qquad (6.2)$$

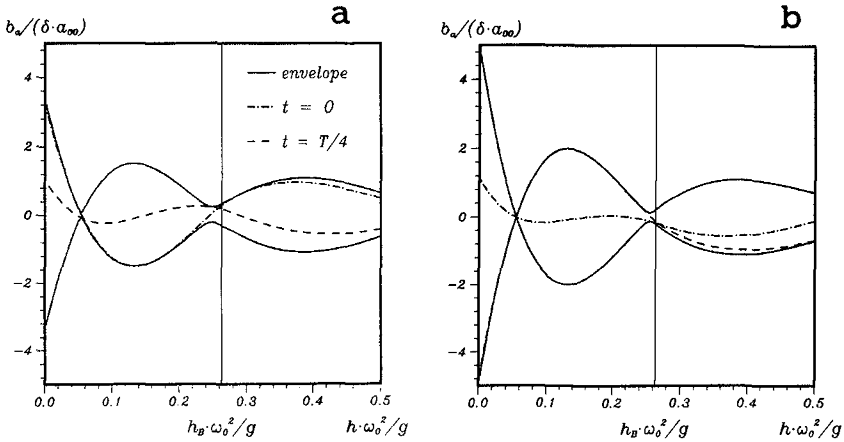
and the steepness of the carrier wave system

$$\epsilon_S = a_\infty \omega_0^2/g \tag{6.3}$$

The small parameter

$$\delta = a_2/a_1 \tag{6.4}$$

describes the weakness of the amplitude modulation causing the wave groups. Hence the problem is characterized by a total of five (small) parameters. In addition the dimensionless carrier wave frequency $\omega_0^2 h/g$ is of course important for the carrier wave description and particularly the value of $\omega_0^2 h_0/g$ connects the scale of the bottom geometry (h_0) to the length scale of the waves (g/ω_0^2).



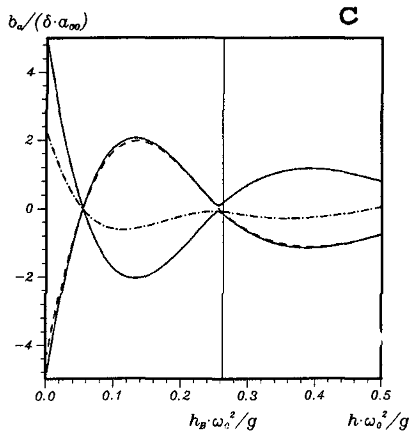
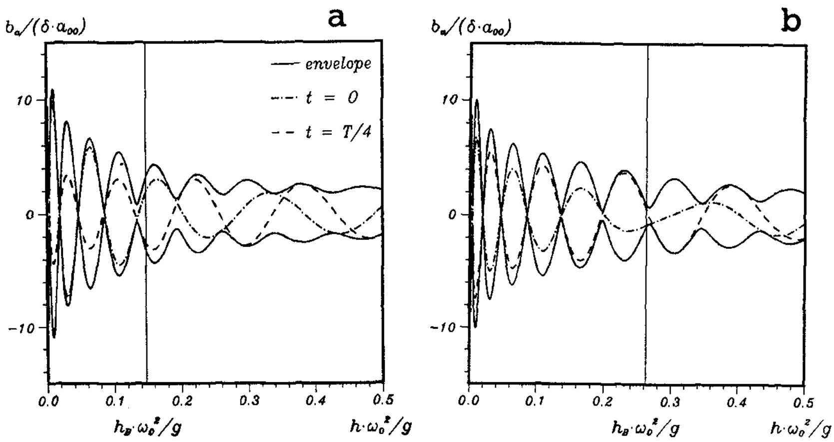


Fig. 2:
Envelope of surf beat motion for three different values of $H_0 = \omega_0^2 h_0/g$.
a: H_0=0.5, R=0.90
b: H_0=1.0, R=4.74
c: H_0=5.0, R=37.5.
In all cases ϵ_S=0.10 and $2h_x\omega_0/\omega_g$ = 0.20. The vertical line corresponds to $h_B\omega_0^2/g$. Instantaneous profiles at t=0, T/4 are also shown.

Fig. 2 shows the variation of the solution for the envelope b_a for a selection of parameter values. The three parts a, b & c corresponds to three different values of the parameter $H_0 = \omega_0^2 h_0/g$ (0.5, 1.0 and 5.0). In all three cases all other parameters have been kept unchanged so that the three figures indicate the effect of increasing the depth h_0 in front of the slope (see Fig. 1) relative to the wave length of the carrier wave system.

Since it is difficult to compare the wave patterns as a whole we have chosen here to focus on the ratio between the amplitude of the wave propagating seawards in the constant region depth over the amplitude of the incoming set-down wave in the same region. This "reflection coefficient" R is a measure of how much the original set-down wave is amplified by energy transferred from the short wave system.

The variation of R we see in Fig. 2 is mainly due to the fact that the amplitude of the free (reflected) wave varies as $h_0^{-1/4}$ whereas the set-down wave decreases as h_0^{-1} as h_0 increases. Notice that for H_0 = 1.0 and 5.0 the figure does not show the whole slope region.



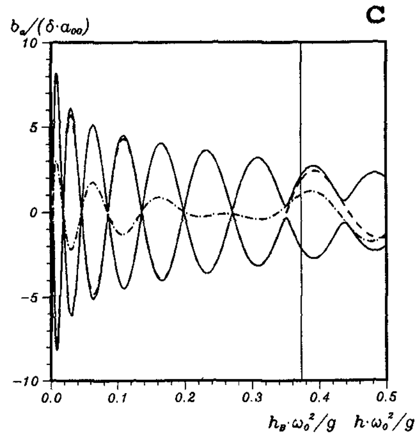


Fig. 3
Surf beat envelope for $2h_x\omega_0/\omega_g$ = 0.05, H_0 = 1.0 and three different short wave steepnesses. The data are a: ϵ_s = 0.05, R = 27.79
b: ϵ_s = 0.10, R = 12.77
c: ϵ_s = 0.15, R = 6.89.

In Fig. 3 the total slope length has been kept constant and the position of the breaker point varied. The reflection coefficients of up to 28 show a significant amplification of the set down wave. On the other hand, the value of R is biggest for the most narrow surf zone.

From those observations one might arrive at the conclusion that the surf beat generation primarily takes place on the slope seaward of the breaker point. This idea might be further supported by the fact that in the surf zone the minimum envelope amplitudes are very small as if the amplitude of the outgoing, reflected wave were nearly the same as the shoreward moving surf beat. (Clearly this becomes a better and better approximation the closer we get to the beach, as should be expected.)

To check this conjecture Fig. 4 shows numerical experiments with the same general data as in Fig. 3b (which is repeated in Fig. 4a for comparison). In Fig. 4b, however, we have artificially suppressed the surf beat generation inside the surf zone by letting the right hand side of (3.25) be zero in region I. Thus the long wave in the surf zone is now a standing free wave. We see there is a substantial reduction in the height of the surf beat generated as measured by the reflection coefficient, which drops from 12.77 to 8.58.

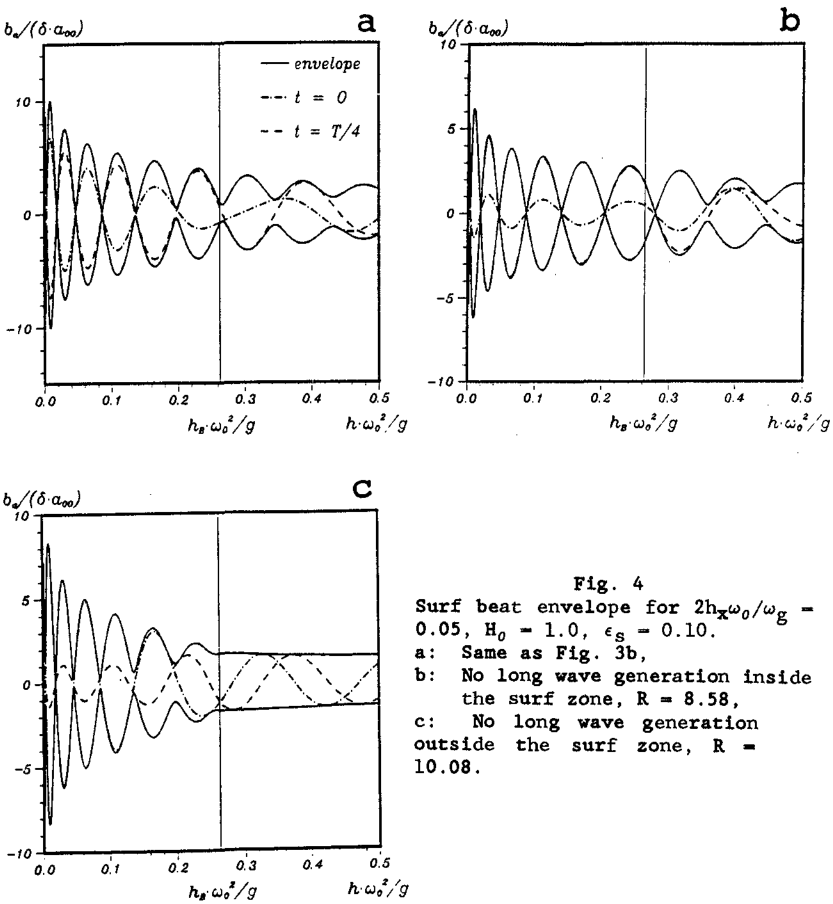
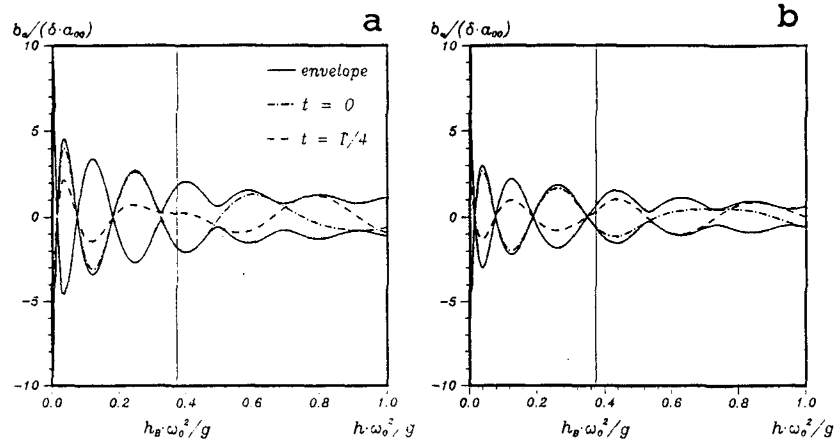


Fig. 4
Surf beat envelope for $2h_x\omega_0/\omega_g$ = 0.05, H_0 = 1.0, ϵ_s = 0.10.
a: Same as Fig. 3b,
b: No long wave generation inside the surf zone, R = 8.58,
c: No long wave generation outside the surf zone, R = 10.08.

Similarly Fig. 4c shows a numerical experiment in which there is no long wave generation at all until in the surf zone. There the long wave motion outside the breaker point is a purely progressive wave moving seaward after having been reflected from the shoreline. This seaward oriented wave is now 10.8 times the set-down wave we would have had in the constant-depth region III had the generation been normal. In Fig. 5a-c the same type of experiment has been repeated for the same wave period and slope width but with the breaker point moved seaward by a change in carrier wave steepness. The picture is seen generally to be the same.



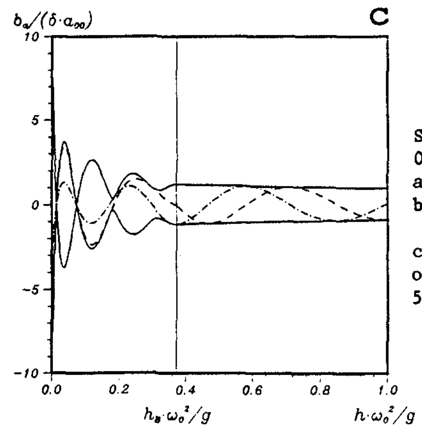


Fig. 5
Same as Fig. 4 except $2h_x\omega_0/\omega_g$ = 0.10, ϵ_S = 0.15, H_0 = 1.0.
a: R = 5.38,
b: no long wave generation inside the surf zone, R = 3.61,
c: no long wave generation outside the surf zone, R = 5.10.

As mentioned in section 2, as the waves move to shallower water the wave number of the wave groups approaches the wave number of the free wave solution to (3.25) and hence the generation process assumes the character of a resonant transfer of energy from the short waves to the shoreward moving long wave. This aspect of the process is equivalent to the resonant interaction first pointed out for water waves by Mei & Unluata (1972). Only here we meet a somewhat more complicated form where the wave number of the driving force instead of being constant is slowly drifting towards the resonant wave number as the waves move shoreward. Although we have not yet studied the details of this situation it is undoubtedly the resonant nature of the process which is responsible for the large amplification indicated by the enormous reflection coefficients. On the other hand, since the nearly resonant interaction only takes place over a finite distance the amplitude of the surf beat of course remains finite.

7. ACKNOWLEDGMENTS

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