Vicedo Lugo

PART II

COASTAL PROCESSES

Vicedo Lugo

CHAPTER 84

NON-UNIFORM SUSPENDED SEDIMENTS UNDER WAVES

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Abstract

An original mathematical formulation for suspended sediments in a two-dimensional wave boundary layer is presented. The model accounts for non-immediate adaptation of sediments to the hydrodinamic conditions, and allows to include the effect of sorting of the different diameters considered.
The mathematical model is numerically solved through a finite difference scheme.
It is suitable that results compare favourably with experimental data by Staub et alii.

1) Introduction

While in the last decade a number of mathematical formulations regarding the velocity distribution in a wave boundary layer have been proposed (including numerical and analytical solutions [Kesteren and Bakker,1984] [Kajiura,1968]), less effort has been devoted to the problem of predicting the sediment distribution [Justesen and Fredsoe, 1985] [Bakker, 1974]. In particular for aspects concerning bed-boundary conditions and non-uniformity of material.

In the present paper an original formulation of the bed-boundary condition in a two-dimensional wave boundary layer, accounting for non uniform material, is proposed. The problem has been solved through a mathematical model, in which the fluid flow and the sediment concentration equations are solved numerically.

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2) Equations of motion

The fluid flow equations are usually solved aside from the sediment presence by means of momentum equation.
For a two-dimensional turbulent boundary layer the general equation appears:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\partial}{\partial x}\overline{u'u'} - \frac{\partial}{\partial y}\overline{v'u'} \qquad 1)$$

For a wave boundary layer advection terms, as well as the horizontal diffusion term are neglected; moreover the pressure term is substituted by the inertial term external to the boundary layer; as a consequence, eq.1) reads:

$$\frac{\partial(u-U_o)}{\partial t} = \frac{\partial}{\partial y}\left(\epsilon\frac{\partial u}{\partial y}\right) \qquad 2)$$

where the turbulent diffusion coefficient:

$$-\epsilon = \frac{\overline{u'v'}}{\partial u/\partial y} \qquad 3)$$

has been introduced.
Upon introduction of the defect velocity $u_d = u - U_0$, eq.2) becomes:

$$\frac{\partial u_d}{\partial t} = \frac{\partial}{\partial y}\left(\epsilon\frac{\partial u_d}{\partial y}\right) \qquad 4)$$

To solve eq. 4), a turbulent closure hypothesis and adequate bed boundary conditions must be assigned.
The following expressions are assumed for the turbulent diffusion coefficient:

$$\epsilon = l^2\frac{\partial u}{\partial y} \qquad 5)$$

where l is a turbulent mixing length, function of the boundary layer thickness δ, for which the following Escudier formulation [Launder and Spalding,1972] is assumed:

$$l = 0.41\, y \qquad \text{for} \qquad y \leq 0.22\,\delta \qquad 6a)$$

$$l = 0.09\,\delta \qquad \text{for} \qquad y > 0.22\,\delta \qquad 6b)$$

The bed boundary conditions are imposed according to the universal law of the wall:

$$\frac{u}{u_*} = \frac{1}{K} \ln(y^+ E) \qquad \text{for} \quad y^+ = \frac{u_* y}{\nu} \geq 11.6 \qquad 7a)$$

$$\frac{u}{u_*} = y^+ \qquad \text{for} \quad y^+ < 11.6 \qquad 7b)$$

the latter being referred to the presence of a viscous sublayer next to the wall, and E being a roughness parameter (E=9 for hydraulically smooth walls).

Eq. 4), 5), and 6) are solved numerically through a Preissman finite difference-scheme [Lyn and Goodwin, 1987]. The first points of the integration mesh were chosen according to either eq. 7a) or 7b). Eq.7b), being a non-linear equation, solving for u* requires an iterative procedure based on Newton-Raphson scheme.

The mathematical model warrants sufficient generality to be applied both under simple waves and under superimposed waves and current conditions; however, only the first case is dealt with in this paper.

The results of the mathematical model have been compared with some experimental data obtained by Jonsson and Carlsen [1976] in an oscillating tunnel.
Comparison between measured and predicted velocities for different phase angles are reported in Fig. 1

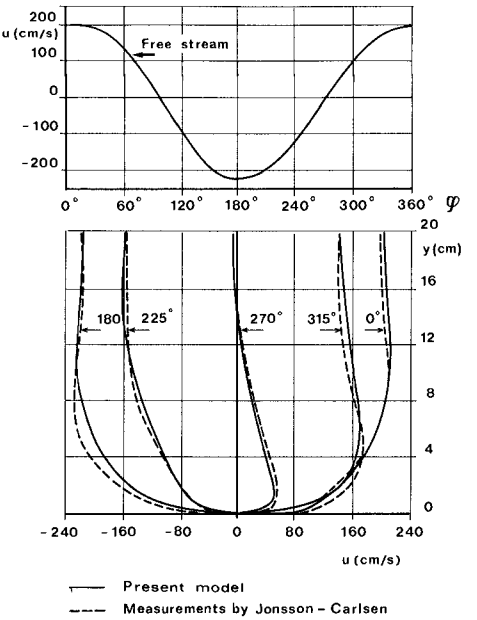


Fig.1. Velocities in the free stream and comparison between measured and predicted ones as a function of height for different phase angles.

3) Equations of sediment transport

3.1) Uniform grain-size hypothesis

The general suspended sediment transport equation in a two-dimensional turbulent boundary layer is written as:

$$\frac{\partial c}{\partial t} + \frac{\partial (u\ c)}{\partial x} + \frac{\partial (v-w)\ c}{\partial y} = - \frac{\partial \overline{u'c'}}{\partial x} - \frac{\partial \overline{v'c'}}{\partial y} \qquad 8)$$

in which w is the fall velocity and c the volume concentration of sediments.
The vertical component of velocity, the horizontal diffusion term and the advection term have been neglected, as usual.
The turbulent diffusion coefficient ε_s for the solid phase was introduced in the form:

$$- \varepsilon_s = \frac{\overline{v'c'}}{\partial c / \partial y} \qquad 9)$$

and was assumed equal to that of the momentum (eq. 3 - 6); therefore eq 8) becomes:

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial y} \left(w\ c + \varepsilon \frac{\partial c}{\partial y} \right) \qquad 10)$$

The boundary conditions generally considered to define the proper boundary value problem are: zero concentration far from the bed, and bottom concentration (at a reference level y=a), as a function of the grain properties and of the flow conditions (i.e. θ Shields parameter); The latter appears:

$$c(a) = c_b = f(\theta) \qquad \text{being} \qquad \theta = \frac{u_*^2}{(S-1) g\ D} \qquad 11)$$

A more appropriate bed-boundary condition, accounting for non-instantaneous adaptation of sediment concentration to the flow conditions, has already been proposed by the authors [Armanini and Ruol, 1987]. The net flux, at the reference level y=a is, in fact:

$$D = -\left(w\ c + \varepsilon \frac{\partial c}{\partial y} \right)_a \qquad 12)$$

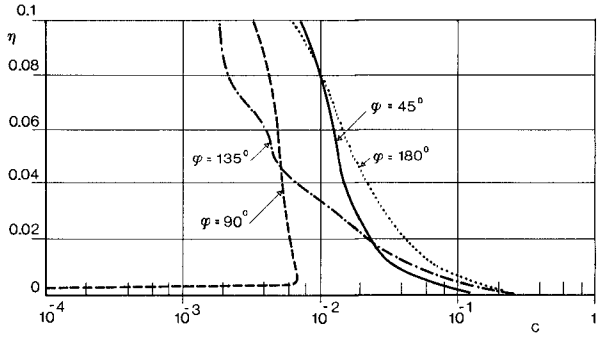
The first term represents the downward flux and the second one the upward flux (pick-up rate). It is reasonable to assume [Di Silvio and Armanini 1982, 1985] that the first term (deposition) depends on the upstream boundary condition, while the second term (erosion) depends on the flow conditions and is the same as for local equilibrium condition.
Under this assumption the erosion flux (pick-up rate) is

just equal to (-w c_b) where c_b is the bottom concentration in a channel flow with the same Shields parameter. In this way the bottom boundary condition becomes:

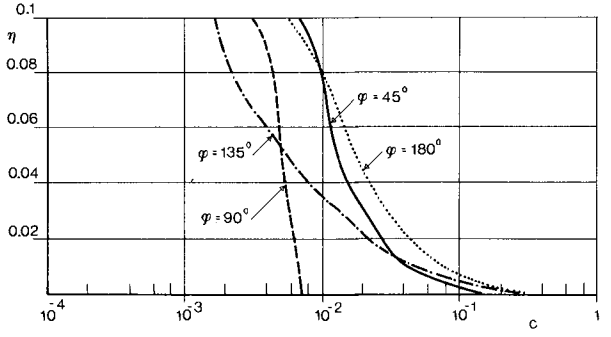
$$D = - w [c(a) - c_b] \qquad 13)$$

According to the methodology considered for the solution of the motion equations, eq. 10) is solved numerically through the Preissman finite difference scheme [Lyn and Goodwin, 1987].

In fig.2) the vertical distribution of the concentration under the two bottom boundary conditions is shown. It does appear that the hypothesis of immediate adaptation of the bottom concentration to the equilibrium condition (eq.11), produces a positive gradient of the concentration next to the bed, which is somewhat devoid of physical content, whenever the friction velocity falls under the critical Shields threshold value; this phenomenon does not appear with the suggested boundary condition (eq.13).



a) boundary condition according to eq.11



b) boundary condition according to eq.13

Fig.2 Comparison between the two different bed-boundary conditions in determining the concentration c as a function of dimensionless height η = (y-z)/(H-z) for mean grain size D_{50} = 0.19mm.

In Fig.3 the variation of dimensionless concentration during a wave period is compared with the experimental results of Staub et alii [1984]. In the same figure the simulation carried out by Justesen and Fredsoe [1985] is also reported: the present model seems to yield a better agreement with the experimental data.

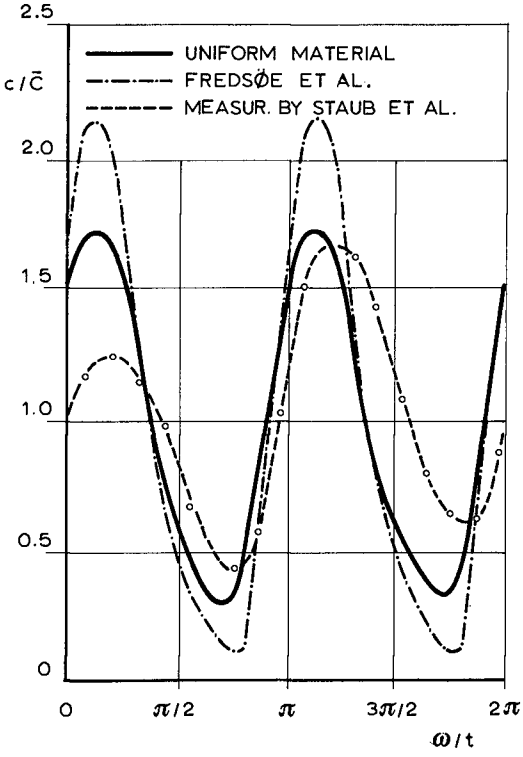


Fig.3 Comparison between measured and predicted variation of dimensionless concentration during one wave period at 1.8cm above the bottom.
(D_{50} = 0.19mm; u_{max} = 1.28m/s; T = 9.1s)

3.2) Non uniform grain size hypothesis

In many cases the mean diameter alone is not able to represent correctly the concentration distribution and the sediment transport. In fig. 4) the mean concentration integrated over the boundary layer thickness and along the period is reported.

From the figure it does appear that, at least in a certain range of diameters (0.12-0.17mm), at a small variation of particle diameter a strong variation of mean concentration is observed.

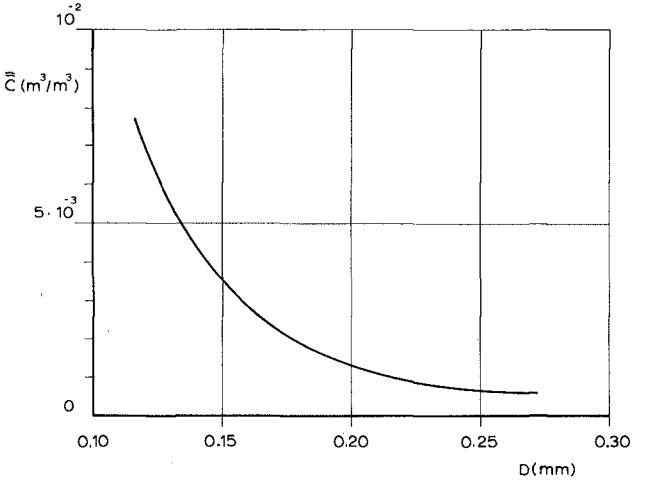


Fig.4 Average concentration (over depth and period) versus grain diameter in the case of uniform material model. (u_{max} = 1.2m/s; T = 9.1s)

In order to overcome these limitations it is then convenient to divide the grain size distribution curve into a discrete number of classes j [Ruol, 1988].
The continuity equation of suspended sediments for each class is then the same as for uniform material (eq.10):

$$\frac{\partial c_j}{\partial t} - w_j \frac{\partial c_j}{\partial y} = \frac{\partial}{\partial y}\left(\varepsilon \frac{\partial c_j}{\partial y}\right) \qquad 14)$$

On the other hand in assigning the bed- boundary conditions it is necessary to account for the mutual interactions amongthe various classes.
The upward flux for each class can be assumed, as for uniform material,the same as in equilibrium condition, but multiplied by the actual concentration for the j-th class on the bed (β_j):

$$\beta_j w_j c_b(\theta_j) \qquad 15)$$

the downward flux, instead, depends on the effective concentration near the bed. As a consequence, the net flux at the reference level a, becomes:

$$D_j = - w_j [c(a)_j - \beta_j c_b (\theta_j)] \qquad 16)$$

Eq. 16), which represents the new bed-boundary condition, contains the new variable β_j; one more equation is then necessary to solve the problem. This equation is given by the volume sediment balance in the active layer at the

bottom surface:

$$\frac{\partial}{\partial t}(\beta_j \Delta) = - D_j + \beta^*_j \sum_k D_k \qquad 17)$$

in which Δ is the thickness of the bed layer instantaneously involved in the mixing process during the erosion-deposition process, that is, in the exchange process between the sediments of the bottom and of the stream.
It is easy to show that β^*_j equals β_j whenever sedimentation prevails over erosion, and the actual bed concentration above the exchange layer during erosion processes.

Finally it must be:

$$\sum_k \beta_k = 1 \qquad 18)$$

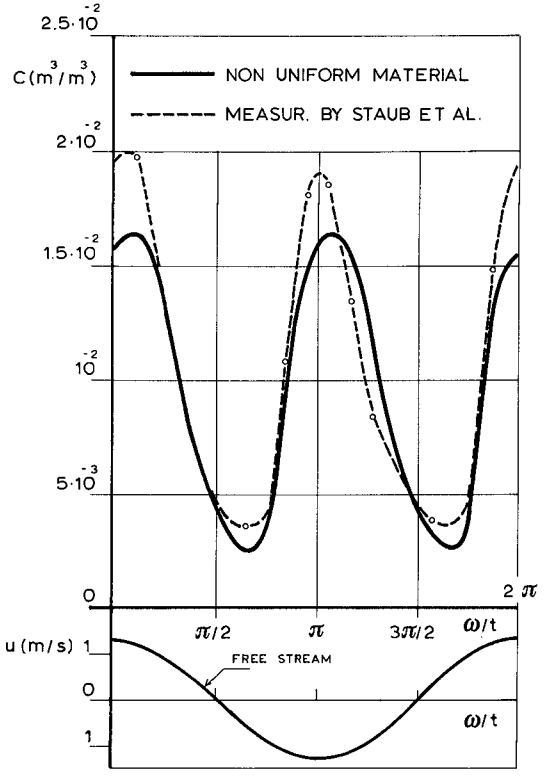


Fig.5 Velocities in the free stream (u_{max} = 1.2m/s; T = 9.1s) and comparison between measured and predicted variation of total concentration during one wave period at y=1.2cm above the bottom. (D_1=0.13mm {20%}; D_2=0.20mm {80%})

In fig.5) the concentration obtained from computations accounting for two grain-size fraction are compared with experimental data by Staub et Al. [1983].

The differences between uniform and non uniform material models are much more pronounced as becomes more evident the non-uniformity of gradation curve. The results shown in the present paper are related to the experimental data by Staub et alii, where the gradation curve are almost uniform. Nevertheless the vertical distribution of concentration is more sensitive to the gradation of the material than the mean concentration.

In fig. 6) a comparison between the results obtained from the two-fraction and from the uniform material models are shown. The vertical distribution of the single diameter concentrations (D_1, D_2), of the total concentration (D_1+D_2), and the vertical distribution of the corresponding uniform material (D_{50}) concentrations are plotted together.

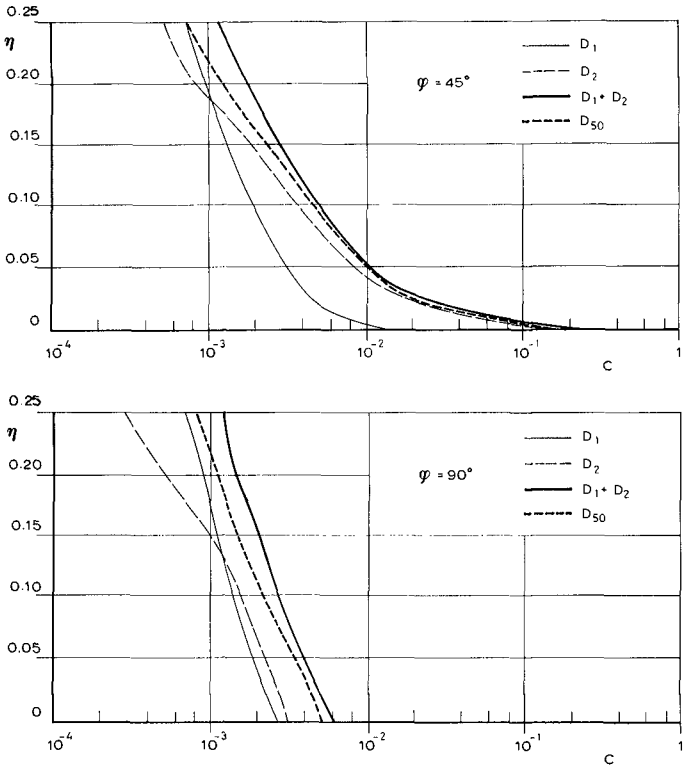


Fig.6a Vertical distribution of the concentration obtained from the two-fraction run (D_1=0.13mm {20%}; D_2=0.20mm {80%}) compared with the corresponding uniform material run (D_{50}=0.19mm) for different phase angles (φ =45°,90°).

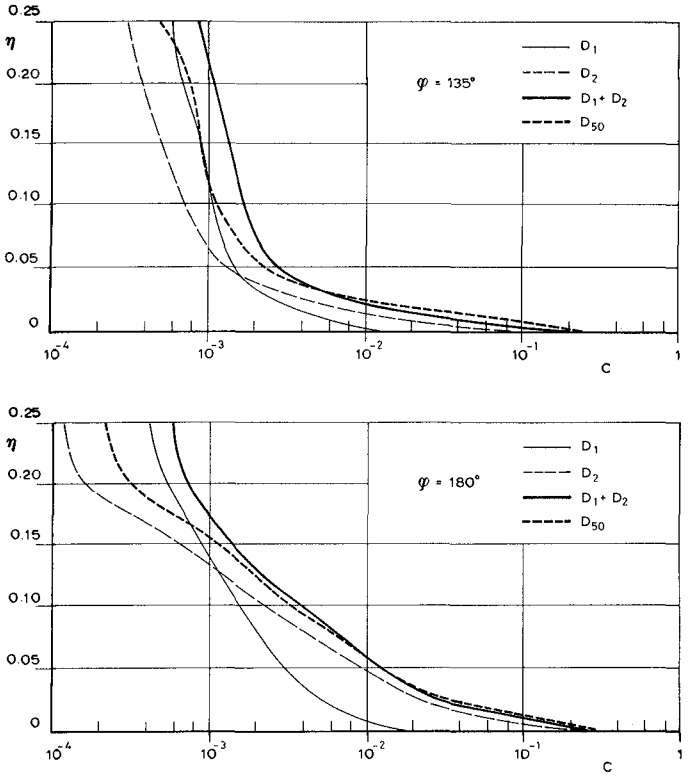


Fig.6b Vertical distribution of the concentration obtained from the two-fraction run (D_1=0.13mm {20%}; D_2=0.20mm {80%}) compared with the corresponding uniform material run (D_{50}=0.19mm) for different phase angles (φ=135°, 180°).

4) Conclusions

A mahtematical model of vertical distribution of concentration of suspended sediments in a two-dimensional wave boundary layer is presented. The model includes a bed boundary condition capable to account for non-immediate adaptation of concentration to hydrodinamic conditions. The model has been extended in order to describe the mutual interaction between different grain size classes in the case of non uniform grain size distribution.
It is opinion of the writers that the model will find a wider applicability if adapted to the sloping beach case by integration over the boundary layer thickness, (in a way already proposed by Armanini and Di Silvio [1982] [1988] for a channel flow). In this case in fact the model should be able to describe the sorting effect induced through the breaking zone.

5) References

[1] Armanini A.,Di Silvio G., 1982:"Sudden morphological modifications along a mountain river simulated by a mathematical model", APD-IAHR, Bandung.

[2] Armanini A.,Di Silvio G., 1985:"Transport of suspended sediments along channels with a trench" XXI Congr. I.A.H.R. vol.3 pp.527-532, Melbourne.

[3] Armanini A.,Di Silvio G., 1988:"A one-dimensional model for the transport of a sediment mixture in non-equilibrium condition" Journ. of Hydraulic Res.Vol.26, n.3 ,pp.275-291.

[4] Armanini A., Ruol P., 1987: "Distribution of suspended sediments in a wave boundary layer" Euromech 215, S.Margherita Ligure pp.80-83.

[5] Bakker W.T., 1974: "Sand concentration in an oscillatory flow". Proc. Coast. Eng. Conf. , ch.66.

[6] Di Silvio G., Armanini A., 1981: "Influence of the upstream boundary conditions on the erosion-deposition process in open channels" XIX Congr. IAHR n.22 New Delhi.

[7] Jonsson G., Carlsen N.A., 1976: "Experimental and theoretical investigation in an oscillatory turbulent boundary layer" Journ. Hydr. Res. 14, n.1 pp.45-59.

[8] Justesen P., Fredsoe J., 1985: " Distribution of turbulence and suspended sediment in the wave boundary layer" Rep. 62, pp.61-67, Inst. Hydrodyn. and Hydr. Eng., Tech. Univ. Denmark.

[9] Kajiura K., 1968: "A model of the bottom boundary layer in water waves". Bull. Earthquake Res. Inst., vol.46.

[10] Kesteren W.G.M., Bakker W.T., 1984: "Near bottom velocities in waves with a current; analytical and numerical computations". Proc. Coast. Eng. Conf. ch.79, pp. 1161-1177.

[11] Launder B., Spalding D.B. 1972: "Mathematical moels of turbulence". Accademic Press. London and New York. 169 p.

[12] Lyn D., Goodwin P., 1987: "Stability of a general Preissman scheme". Journ. of Hydr. Eng., vol.113 n.1.

[13] Ruol P. 1988: "How to represent bed materials in sediment transport models under waves" Proc. Conf. Trasp. sol. nei corsi d'acqua, Trento Italy, 9-10 jun.

[14] Staub C., Svendsen A., Jonsson I., 1983: "Measurements of the instantaneous sediment suspension in oscillatory flow". Prog. Rep. 58, Inst. Hydrodyn. Tech. Univ. Denmark.

[15] Staub C., Jonsson I., Svendsen A., 1984: "Variation of sediment suspension in oscillatory flow" Proc. Coast. Eng. Conf., ch.155 pp. 2310-2321.