CHAPTER 14

MEASUREMENT AND COMPUTATION OF WAVE INDUCED VELOCITIES ON A SMOOTH SLOPE

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Abstract

Two main items are treated in this paper. The first item is measurement of wave induced velocities in a large scale model and the second one is a verification of the 1-D numerical model, developed by Kobayashi and Wurjanto (1989).

Wave induced velocities on a smooth slope were measured in Delft Hydraulics large Delta flume. The run-up and run-down velocities were compared with theoretically derived upper bounds and formulas for these velocities were derived on the basis of these upper bounds.

Computations were performed with the numerical model of Kobayashi and the results (velocities, pressures and run-up and run-down levels) were compared with measurements, partly from the investigation mentioned above. The results for run-down velocities and run-up levels were acceptable, the results for run-up velocities were a little worse and the results for pressures and run-down levels were bad.

Introduction

Knowledge of wave induced velocities on a slope is an important step towards a better understanding of the behaviour of coastal structures under wave attack. These velocities are important to calculate wave forces on rubble mound structures and on placed block revetments and to calculate run-up or overtopping.

Therefore, large scale physical model tests were performed in the Delta flume of Delft Hydraulics with regular waves on a smooth slope. Velocities were measured at various locations and for various wave conditions. Furthermore, the numerical model of Kobayashi and Wurjanto (1989) was used to calculate wave induced velocities and these results were compared with the measurements.

The numerical model is a 1-D model and is based on solving the non-linear equations for long waves on a slope. The model was a ver-

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sion, specially developed for CERC, USA and Prof. N. Kobayashi of the University of Delaware and Mr. J.P. Ahrens of CERC are gratefully acknowledged for the permission of using this model.

Tests in the Delta flume

Large scale model tests have been performed which were aimed on measuring the water velocities just above (about 3-5 cm) the slope surface. The dimensions of the Delta flume are 230 m long, 5 m wide and 7 m deep. All tests were performed with regular waves and on a smooth slope of 1:3. The slope consisted of a placed block revetment with block dimensions 0.5 m x 0.5 m and thick 0.15 m. Some blocks had circular holes in the center of the block, but these were filled with shingle during the tests.

The velocities were measured with four-quadrant electro-magnetic velocity meters (EMS) mounted on a horizontal rod just above the slope. The EMS was developed and constructed by the Instrumentation Section of Delft Hydraulics. The disc of the EMS has a diameter of 0.035 m and the range is +/- 5 m/s. The accuracy is 2% of the recorded value or 0.02 m/s for small velocities. The velocities were measured at seven locations around the still water level. The water depth was 5 m during all tests.

The test program is shown in Table 1. Four wave periods were generated and for each wave period 3-4 wave heights. The Table gives the wave periods, T, wave heights, H, wave steepnesses, s_{op}, defined as $s_{op} = 2\pi H/gT^2$ and the surf similarity parameter, ξ_o, defined as $\xi_o = \tan\alpha/\sqrt{s}$, where α is the slope angle of the structure.

T (s)	H (m)	s_{op}	ξ_o	T (s)	H (m)	s_{op}	ξ_o
2.25	0.20	0.025	2.09	4.0	0.65	0.026	2.07
2.25	0.29	0.037	1.74	4.0	0.84	0.034	1.82
2.25	0.38	0.048	1.51	4.0	1.11	0.045	1.58
3.0	0.37	0.026	2.05	5.0	0.30	0.008	3.78
3.0	0.50	0.036	1.77	5.0	0.51	0.013	2.92
3.0	0.62	0.044	1.58	5.0	0.70	0.018	2.48

Table 1 Tests for measurement of velocities on a smooth 1:3 slope

The time signals of the velocity meters were analysed. The maximum run-up and run-down velocity for each wave in a record of 30 s was established. The maximum run-up and run-down velocities in this paper are defined as the average of the highest three recorded values.

Theoretical considerations on velocities

A simple theory was developed to support the analysis of the measurements. The water velocity on a slope due to breaking waves depends largely on the process of wave run-up, run-down and wave impact. With respect to the run-down velocity, v_d, the theory is based on the fall velocity of a particle, falling without friction.

$$E_{kin} = 0.5\ mv_d^2 = mg(R_u - z) = E_{pot} \qquad (1)$$

where:
E_{kin} = kinetic energy
E_{pot} = potential energy
m = mass of water particle
g = gravity acceleration
R_u = level of maximum run-up relative to swl
z = location on slope, measured vertically relative to swl

Elaboration of Eq. 1 gives:

$$v_d = \sqrt{2g(R_u - z)} \qquad (2)$$

This means that the run-down velocity is independent on the wave height and period, but only on the run-up level and the location on the slope. In order to use a dimensionless velocity both parts can be divided by $\sqrt{gH}$. But again, strictly speaking, this is not required. The final formula for the run-down velocity becomes then:

$$v_d/\sqrt{gH} = \sqrt{2R_u/H}\ \sqrt{(1-z/R_u)} \qquad (3)$$

Eq. 3 shows that the dimensionless run-down velocity is a function of the dimensionless run-up level and the dimensionless location on the slope. This equation is an upper bound for the run-down velocity, since the friction influence is neglected.

With respect to the run-up velocity, v_{up} an upper bound can be derived from the wave celerity, c, defined for deep water by c = $\sqrt{gL/2\pi}$. If this wave celerity is assumed to be the run-up velocity at swl and if again $\sqrt{gH}$ is used to make the velocity dimensionless (again strictly speaking not required), the run-up velocity becomes:

$$v_{up}/\sqrt{gH} = \sqrt{1/2\pi s} \qquad (4)$$

where s = wave steepness, H/L. The run-up velocity will become zero at the maximum run-up level R_u. A similar term as used in Eq. 3 gives this effect. The final formula for the run-up velocity becomes then:

$$v_{up}/\sqrt{gH} = \sqrt{1/2\pi s}\ \sqrt{(1-z/R_u)} \qquad (5)$$

The dimensionless run-up velocity is a function of the (dimensionless) wave steepness and the dimensionless location on the slope. As quite a lot of energy is dissipated by the wave breaking process, Eq. 5 can be regarded as a high upper bound.

Analysis of measurements

Run-down velocity

The measured run-down velocities are plotted versus the location on the slope, z/R_u, in Fig. 1. The maximum run-up level, the still water level and the maximum run-down level are shown in the Figure. The maximum upper bounds given by Eq. 3, are shown for ξ_o = 1.5 and 2.5, giving more or less the measured range. As the run-up level R_u/H depends on ξ_o, the upper bound depends on this parameter too.

The relationship between R_u/H and ξ_o is shown in Fig. 5 and will be treated there.

Fig. 1 shows that all points are below the upper bound and that for $z/R_u > -0.4$ the same trend is found as for the theoretical upper bound. For smaller values of z/R_u (below the run-down point), the velocities decrease rapidly. In order to analyse the trend for the higher values of z/R_u, Fig. 2 was composed with on the horizontal axis the parameter $\sqrt{R_u/H}\ \sqrt{(1-z/R_u)}$, according to Eq. 3.

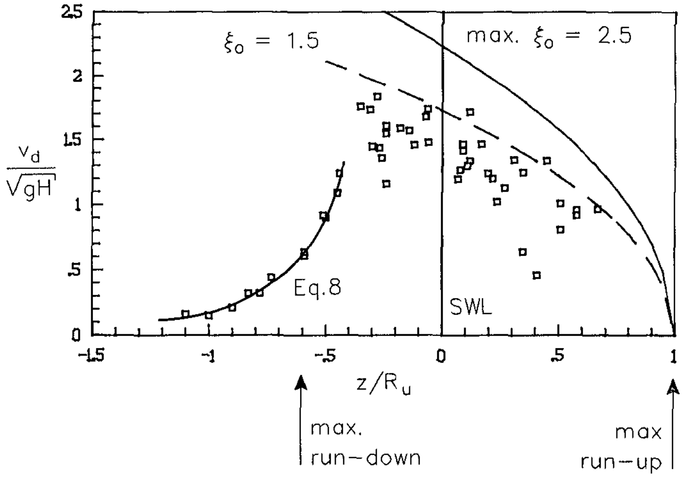

Figure 1 Run-down velocity as function of the location on the slope

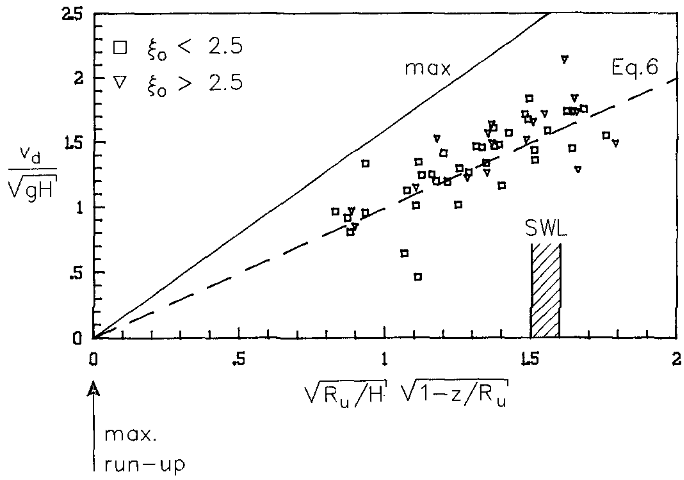

Figure 2 Run-down velocity for $z/R_u > -0.4$

This Fig. 2 gives the maximum upper bound for $\xi_o < 2.5$ and makes also a distinction between the plunging waves ($\xi_o < 2.5$, see Fig. 5) and surging waves ($\xi_o > 2.5$). Although there is some scatter, a linear expression between the parameters on respectively the horizontal and vertical-axis is acceptable, which means that the trend of Eq. 3 can be maintained. An expression for the run-down velocity on a smooth slope for $z/R_u > -0.4$ becomes then:

$$v_d/\sqrt{gH} = 1.1 \sqrt{R_u/H} \sqrt{(1-z/R_u)} \tag{6}$$

This equation is also shown in Fig. 2. As the run-down velocity is in fact not dependent on the wave height, a better expression is:

$$v_d/\sqrt{gR_u} = 1.1 \sqrt{(1-z/R_u)} \tag{7}$$

For the run-down velocity below the point $z/R_u = -0.4$, an expression can be found based on Fig. 1. The velocity decreases rapidly with decreasing z/R_u and the relationship can therefore be given by a power curve:

$$v_d/\sqrt{gH} = 0.18\ z/R_u^{-2.3} \qquad \text{for} \qquad z/R_u < -0.4 \tag{8}$$

Run-up velocity

The measured run-up velocities are shown in Fig. 3 as a function of z/R_u. Fig. 3 is similar to in Fig. 1. Both upper bounds for ξ_o = 1.5 and 2.5 are shown. The measured velocities are well below these upper bounds which means that quite some energy is dissipated by the wave breaking process. As the influence of the surf similarity parameter is large, see the difference between the two upper bounds, it can not be concluded on the basis of Fig. 3 that the velocities for $z/R_u > 0$ follow the same trend as the upper bounds. Therefore, another figure was composed based on Eq. 5.

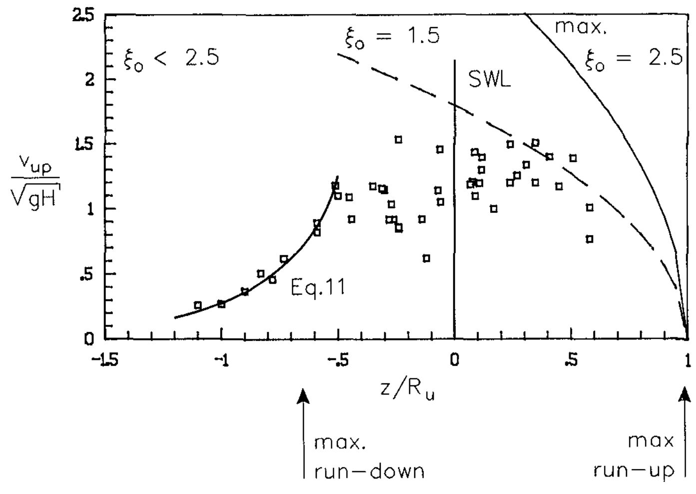

Figure 3 Run-up velocity as function of the location on the slope

Fig. 4 gives the run-up velocity as a function of $1/\sqrt{s}\ \sqrt{(1-z/R_u)}$, including the upper bound. Again a linear expression fits the data and the following equation for the run-up velocity for $z/R_u > 0$ can be derived:

$$v_{up}/\sqrt{gH} = 0.27\ 1/\sqrt{s}\ \sqrt{(1-z/R_u)} \tag{9}$$

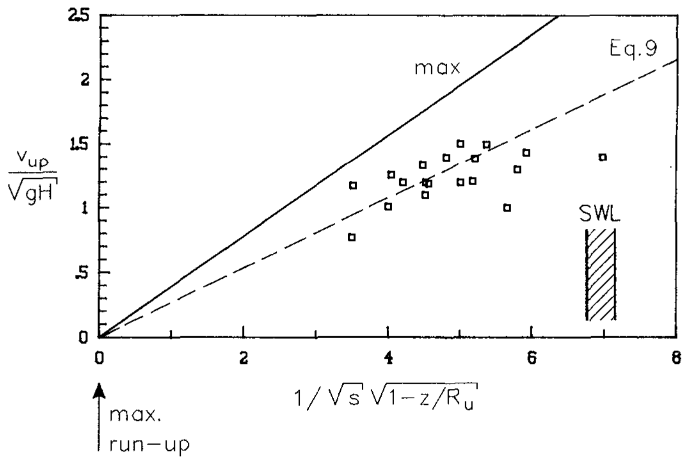

Figure 4 Run-up velocity for $z/R_u > 0$

This equation is shown in Fig. 4. Fig. 3 shows futhermore that the run-up velocity in the area $-0.4 < z/R_u < 0$ is not really dependent on the location. Analysis showed that it is only a function of the wave steepness:

$$v_{up}/\sqrt{gH} = 0.2/\sqrt{s} \qquad \text{for} \qquad -0.4 < z/R_u < 0 \tag{10}$$

Finally the run-up velocity decreases rapidly for decreasing z/R_u and for $z/R_u < -0.4$, see Fig. 3. A relationship similar to Eq. 8 which gives a good fit is:

$$v_{up}/\sqrt{gH} = 0.30\ z/R_u^{-2} \qquad \text{for} \qquad z/R_u < -0.4 \tag{11}$$

Eqs. 6 - 11 give empirical relationships for the run-down and run-up velocities on a smooth slope. The relationships are based on large scale experiments for a slope of 1:3. A limit for application of the formulas is therefore a slope close to 1:3.

The model IBREAK

Kobayashi and Wurjanto (1989) describe the model IBREAK that can simulate the wave motion on an arbitrary rough or smooth slope. The model IBREAK is a second and more user's friendly version than the original model and runs on a main frame computer. Broekens (1988) developed the same model, based on Kobayashi's papers and reports, but now for a personal computer and more user's friendly. This was done at the time that IBREAK was not yet available. As IBREAK is the original work of Kobayashi it was decided to verify this model and not the version of Broekens (although the results would be similar).

The model is based on the non-linear long wave equations and is a 1-dimensional model, i.e. only a depth averaged water velocity is assumed. This means that breaking (plunging) waves are simulated by a "bore-type" wave. The wave front can become almost vertically as a limit. Furthermore, pressures are assumed to be hydrostatic. A rough slope is simulated as a "smooth" slope with a large friction coefficient.

Applications of the model to stability of rock slopes were described by Kobayashi and Otta (1987) and applications to run-up and reflections on rough slopes by Kobayashi et al. (1987). The model of Broekens (1988) was used to describe the stability of rock slopes (De Graaf (1988)).

Most applications described by Kobayashi are for rough impermeable slopes. Only Kobayashi and Watson (1987) describe the application and verification of the model for a smooth slope. There conclusions were:
- Water velocities were not verified.
- Water pressures were verified with 3 model tests for one location on the slope. Large deviations were found for the maximum and minimum pressures.
- Run-up and run-down were verified with formulas described by Ahrens and Titus (1985). Run-up was a little smaller for the calculations and run-down was not accurate.
- The general conclusion was that the model is applicable for smooth slopes, although the friction coefficient should have a small value, greater than zero. A value of 0.05 was recommended.

The model IBREAK was used by permission of CERC and prof. Kobayashi. Calculations were performed in order to verify the model more in depth for a smooth slope. The large scale measurements in the Delta flume described in the first part of the paper, form the main basis for this verification. The following parameters were taken into account: the maximum run-up and run-down levels, the water velocities and the water pressures.

Verification of run-up and run-down

Fig. 5 shows the results of calculations and measurements on run-up and run-down levels on a 1:3 smooth slope with waves roughly between 0.2 and 1.1 m. On the basis of the measurements the following relationships for run-up and run-down levels were established:

$$R_u/H = \xi_o \quad \text{for } \xi_o < 2.6 \quad \text{(plunging waves)} \tag{12}$$

$$R_u/H = -1.5\xi_o + 6.5 \quad \text{for } 2.6 < \xi_o < 3.0 \quad \text{(collapsing waves)} \tag{13}$$

$$R_u/H = 2.0 \quad \text{for } \xi_o > 3.0 \quad \text{(surging waves)} \tag{14}$$

$$R_d/H = -0.1\xi_o^2 + \xi_o - 0.5 \tag{15}$$

Eqs. 12-15 are shown in Fig. 5. From this figure it can be concluded that:

- IBREAK gives a constantly increasing curve for the run-up and does not give a maximum for the transition from plunging to surging waves (ξ_o = 2.6 - 3.0). This is probably caused by the fact that the model simulates a bore-type wave. The run-up is a little too small for plunging waves and a little too large for surging waves.
- The average deviation between calculated and measured run-up is 12% which is reasonable. The general conclusion is that the run-up is predicted within acceptable limits.
- The calculated run-down is much smaller than the measured one, especially for the smallest ξ_o values. The deviation is very large there and not acceptable for practical use.

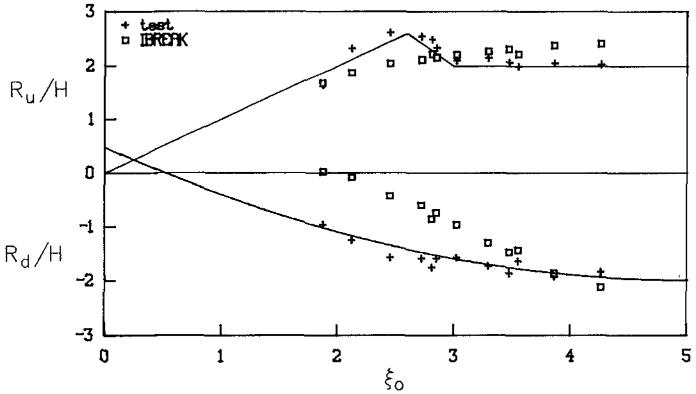

Figure 5 Measured and calculated run-up and run-down on a 1:3 smooth slope

Verification of velocities

A few tests from the series described in the first part of this paper were also calculated with the model IBREAK. Fig. 6 shows a part of the time signal of one of the tests with a surging wave of H = 0.3 m and T = 5 s. The location was -0.42 m which means about 1.5H below the still water level. The qualitative agreement between the measured and calculated signal is good. The quantitative results will be described below.

The maximum value (run-up velocity) and the minimum value (run-down velocity) of the time signal were used for comparison. First the influence of the friction factor f on the run-up an run-down velocities was studied. Friction factors of 0.0, 0.02 and 0.05 were used. Fig. 7 gives the results together with the measured values.

From Fig. 7 it can be concluded that a lower friction factor leads to higher velocities. A decrease of the friction factor from 0.05 to 0.02 leads to an increase in velocities of about 20%. A decrease of f from 0.05 to 0.0 leads to an increase of the velocities of about 40%. This means that even for a smooth slope the friction factor plays an important role with regard to velocities.

During run-down the energy dissipation is only due to friction, where for run-up the wave breaking process is important too. There

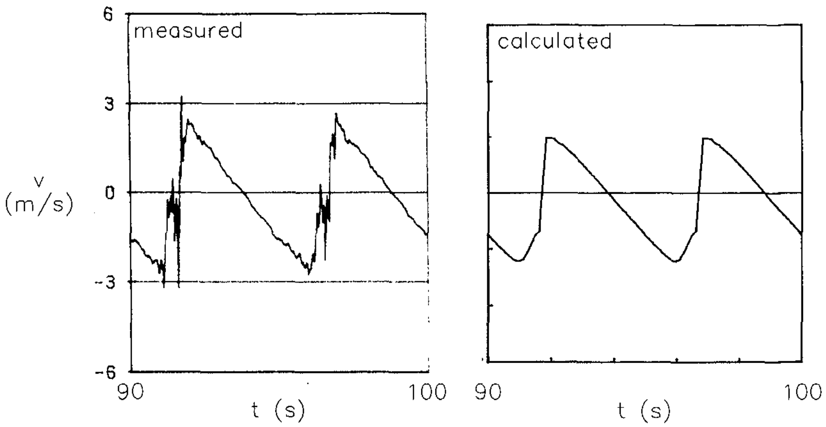

Figure 6 Time series for measured and calculated velocities on a 1:3 smooth slope

fore, the "calibration" of the friction factor for the model should be based on the run-down velocities. For the run-down values a friction factor of f = 0.02 gives the best results and this factor of 0.02 was used for the further calculations.

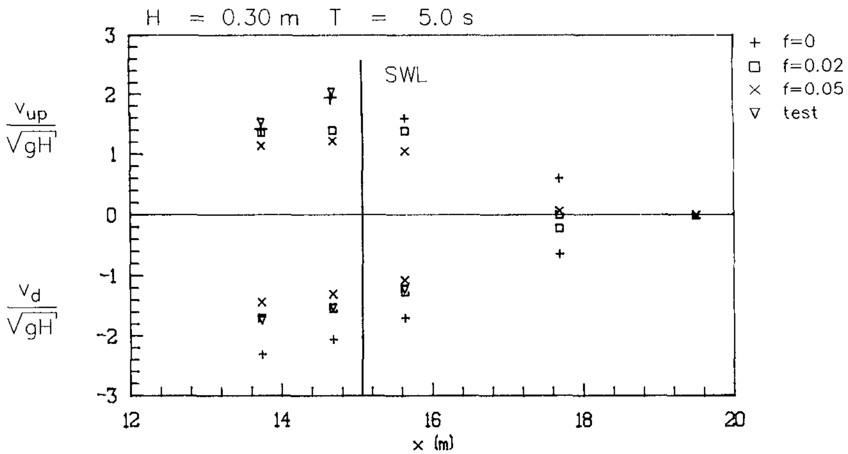

Figure 7 Influence of friction factor on run-up and run-down velocities

It should be noted again that the model gives a depth-averaged horizontal velocity and that the model tests give the velocity a few centimeters above and along the slope.

Fig. 8 gives the results on velocities for 3 tests, ranging from plunging to surging waves (ξ_o values of 1.51, 2.07 and 2.92 respectively). The average deviation of the calculated run-up velocities from the measured ones was 16%. This average deviation amounted to

10% for the run-down velocities. Values smaller than 40% of the maximum velocities on the slope were not taken into account for the calculation of the deviations. Fig. 8 shows furthermore that the calculated velocities were consequently smaller than the measured velocities. A smaller friction factor than 0.02 would therefore increase the agreement.

Fig. 8 shows no influence of the surf similarity parameter on the results. The deviations of the calculated velocities from the measured ones is similar for plunging, collapsing or surging waves.

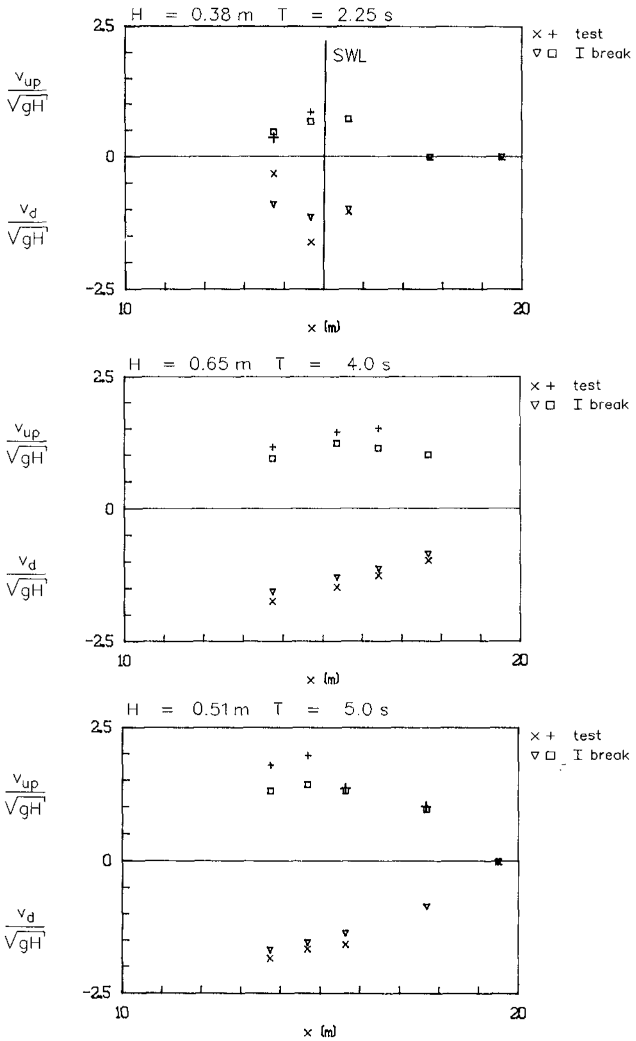

Figure 8 Measured and calculated maximum velocities on a 1:3 smooth slope

The general conclusion is that IBREAK gives a reasonable prediction of the velocities on a smooth slope.

Verification of wave pressures

A part of the time signal for the wave pressure on a location below the still water level is shown in Fig. 9 for one of the large scale tests on a slope of 1:3. The qualitative agreement is good. Fig. 10 shows again a part of a time signal for the wave pressure, but now for a small scale test on a slope of 1:2. Here the agreement is very poor. The only agreement is the irregularity of the signal.

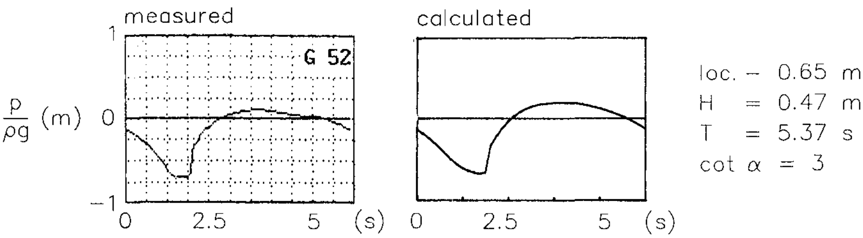

Figure 9 Measured and calculated wave pressures on a 1:3 smooth slope

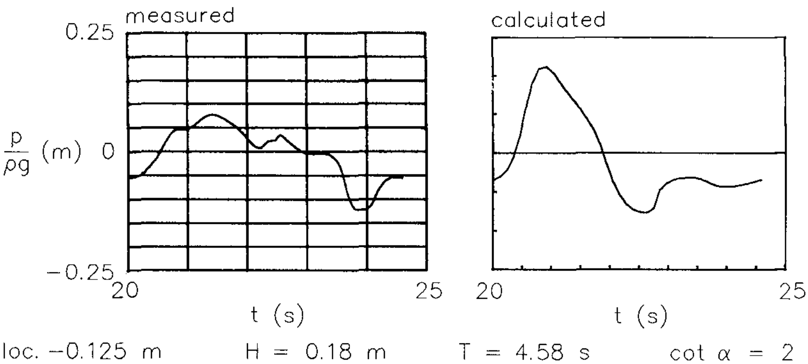

Figure 10 Measured and calculated wave pressures for a 1:2 smooth slope (small scale tests)

The influence of the friction factor on the calculated wave pressures was investigated for the same test as shown in Fig. 9. Fig. 11 gives the results for f = 0.02, 0.05 and 0.10. A value of 0.0 led to numerical instability. This figure shows that the influence of the friction factor on the wave pressures is small and not consistent. Sometimes the lowest value of the friction factor gives the highest wave pressures, sometimes the highest value, depending on the location of the slope.

Fig. 12 gives the results on wave pressures of three tests. The upper graph gives the results of a large scale test on a slope 1:3 with a wave height of 1.38 m and a wave period of 5.37 s. The other graphs give small scale tests on a slope 1:4. The middle graph that of a plunging wave and the lowest graph that of a surging wave.

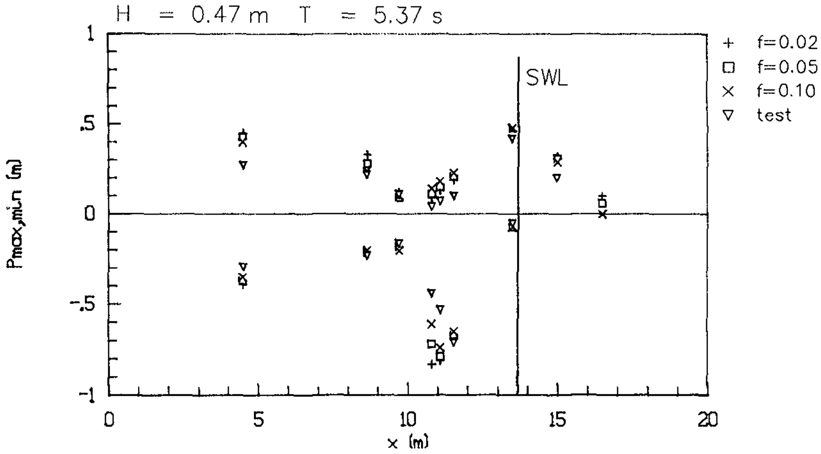

Figure 11 Influence of the friction factor on wave pressures

Calculations were performed for three large scale tests and for five small scale tests (two on a slope 1:2, one on a slope 1:3 and two on a slope 1:4). In all tests the pressures were measured on nine locations as shown in Fig. 12. From the comparison of measured and calculated pressures it could be concluded that the average deviation for the large scale tests was 37% for the minimum pressure and 44% for the maximum pressure. For the small scale tests this was respectively 55% and 113%. Values smaller than 40% of the maximum pressure on the slope were not taken into account for calculation of the deviation of the pressures.

The deviations between measurements and calculations are larger than for the velocities described in the previous section. Furthermore, the deviations for the small scale tests are larger than for the large scale tests. Although the deviations between measurements and calculations are large, the trend along the slope is similar. The location where the maximum pressure occurs on the slope is calculated fairly accurate. In most cases the calculated pressure is (much) higher than the measured one. A smaller friction coefficient than used, however, has almost no influence, see Fig. 11.

The general conclusion on pressures is that IBREAK can not accurately simulate wave pressures on a smooth slope. It can simulate the location where the maximum wave pressure occurs.

Conclusions

Eqs. 6 - 11 give empirical relationships for the run-down and run-up velocities on a smooth slope, based on theoretically derived upper bounds. The relationships are based on large scale experiments for a slope of 1:3. A limit for application of the formulas is therefore a slope close to 1:3.

Eqs. 12 - 15 give empirical relationships for maximum run-up and run-down levels on a smooth slope, based on large scale experiments on a slope of 1:3.

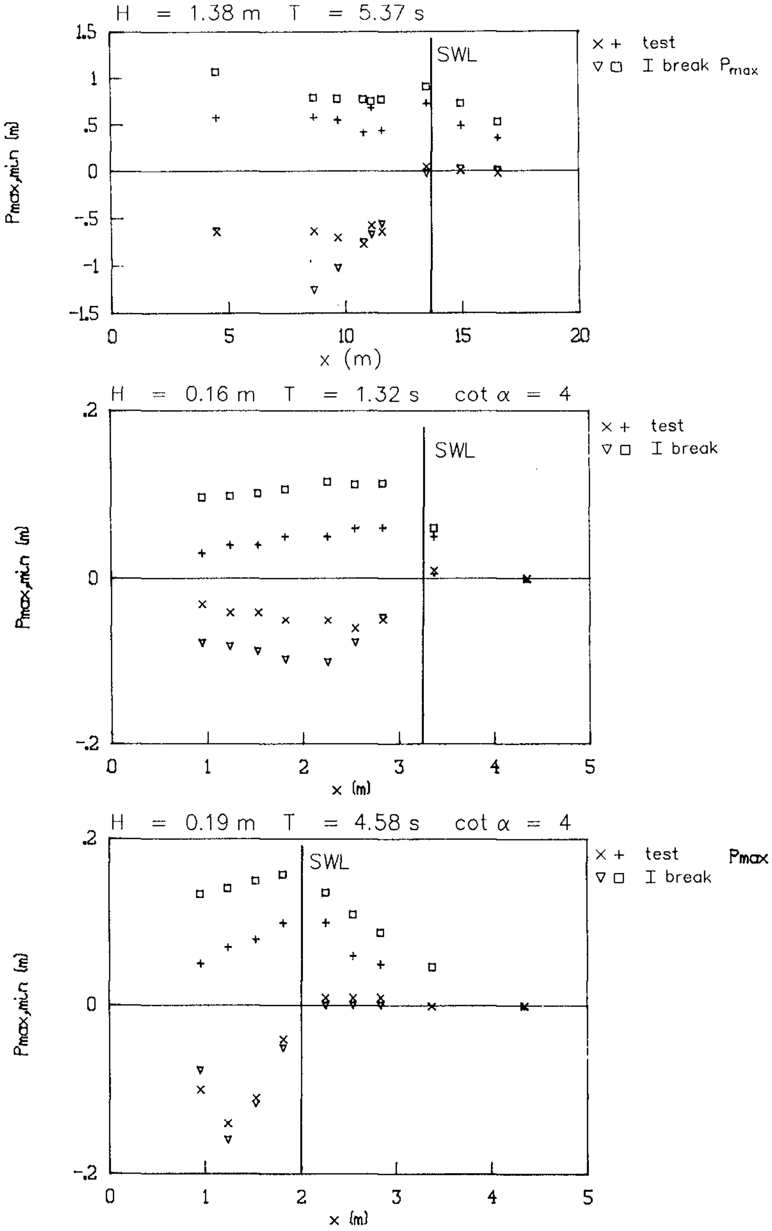

Figure 12 Measured and calculated maximum and minimum pressures

The model IBREAK was verified on maximum run-up and run-down levels, and velocities and water pressures along and on a smooth slope.

The average deviation between calculated and measured run-up is 12% which is reasonable. The general conclusion is that the run-up is predicted within acceptable limits. The calculated run-down is much smaller than the measured one, especially for the smallest ξ_o values. The deviation is very large there and not acceptable for practical use.

The friction factor plays an important role with regard to velocities, even for a smooth slope. The general conclusion is that IBREAK gives a reasonable prediction of the velocities on a smooth slope.

The friction factor has no influence on wave pressures when this factor is in the range of f = 0.02 - 0.10. The general conclusion on pressures is that IBREAK can not accurately simulate wave pressures on a smooth slope. It can simulate the location where the maximum wave pressure occurs.

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