CHAPTER 37

Oscillating Water Column Modelling

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Abstract

The development of a mathematical model to calculate the response of a simple oscillating water column wave energy device (OWC) to a known wave climate is described. The power available from the device is calculated and this is compared with results from model tests in a wave flume.

Particle Image Velocimetry (PIV) has been used to map the flow of water within the chamber of the device and vorticity maps have been calculated. The estimation of energy loss in vortices due to viscous dissipation is discussed in detail and is shown to account for as much as 8% of the total energy of an incident wave.

Introduction

The oscillating water column wave energy converter (OWC) is now a well established means of extracting useful energy from water waves both simply and cost effectively. It consists of a chamber with two orifices. One is open to the sea below the low water level and the other is typically at the top of the chamber and connects to a turbine. A water column is formed in the chamber and oscillates due to the force of the incident waves. This in turn forces air through the turbine thus generating electricity. Several devices are now in operation worldwide but these are single prototype units working at only a fraction of their true potential either due to turbine restrictions or to large scale dissipation of energy within the chamber itself.

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This paper attempts to develop a mathematical model to predict the power available from a simple design of OWC and to use Particle Image Velocimetry (PIV) to study the characteristics of water flow in the chamber of a model of the same OWC. Analysis of the highly detailed velocity maps produced with PIV allows regions of energy dissipation to be identified and the magnitudes of these energy losses to be estimated.

The OWC under study here is a simplified $\frac{1}{36}$th scale model of the prototype designed by Whittaker[4] and now operational on the island of Islay off the west coast of Scotland. This working example utilises a self rectifying axisymmetric Wells turbine[3] for the conversion to electrical energy.

The OWC model

The scale model measures approximately 0.5m × 0.5m × 0.4m - the dimensions chosen to coincide with those of models built and tested at Queen's University, Belfast. The water depth shallows from a maximum of 0.75m 'offshore' to 0.12m at the mouth of the OWC, the variation occurring steadily along a beach of slope 1:12 (figure 1).

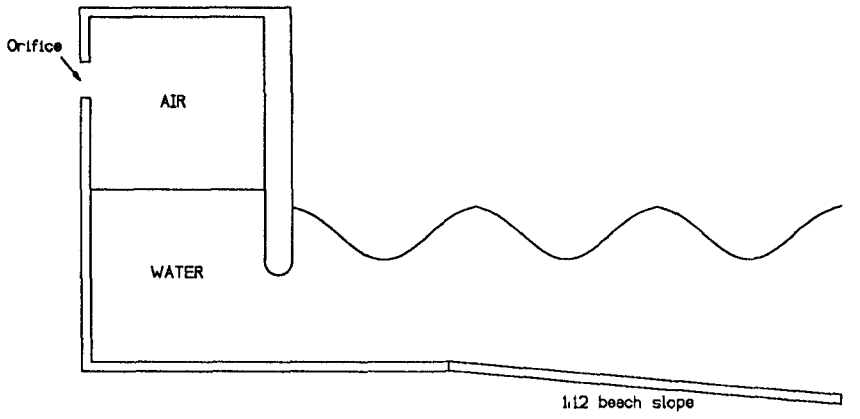

Figure 1: OWC model at top of 1:12 beach

Mathematical modelling

It is important that an OWC and it's turbine are capable of extracting the maximum amount of energy from the waves incident upon the chamber. In order to achieve this the natural frequency of the chamber must be chosen to be

close to that of the prevailing wave climate. Therefore it is useful to be able to predict theoretically the resonant frequency of an OWC chamber and the air pressure variations within it since it is these pressures which provide the force to drive the turbine. Linear wave theory is used as a first approximation here to model the typically low amplitude waves produced in the wave flume experiments.

It is assumed that the air column compresses adiabatically and the variation of the mass of this column due to air movement through the orifice is considered. It is further assumed that the air travels through the orifice with a velocity proportional to the excess pressure in the chamber. This yields a differential equation giving the rate of change of air pressure within the OWC:

$$\frac{dp}{dt} = \frac{\gamma p}{d-y}\left[\frac{dy}{dt} - \frac{\kappa}{\rho_0 A}\left(\frac{p_0}{p}\right)^{\frac{1}{\gamma}}(p-p_0)\right] \tag{1}$$

where p is the air pressure in the chamber, y is the height of the water surface in the chamber above mean water level (MWL), γ is the ratio of the specific heat of air at constant pressure to that at constant volume, d is the length of the air column at MWL, p_0 is atmospheric pressure, κ is a constant relating to the size of the orifice, ρ_0 is the density of air at atmospheric pressure, and A is the cross sectional area of the air column.

Consideration of both the static and dynamic pressure under an incident wave and the upward force this exerts on the water column leads to the second differential equation :

$$\frac{d^2y}{dt^2} = \frac{1}{\rho_w(L+y-d)}\left[\rho_w g\left(asin\omega t - y - \frac{asinhk(h-b)sin\omega t}{k(b-h)coshkh} + \frac{ga^2 ksinh2k(h-b)}{8\omega^2(b-h)cosh^2kh}\right) + p_0 - p\right] \tag{2}$$

where L is the chamber height, ρ_w is the density of water, h is the mean water depth at the OWC, b is the distance below mean water level of the lip of the front wall, a is the amplitude, k is the wavenumber and ω is the angular frequency of the incident wave.

Equations (1) and (2) are insoluble analytically. They are solved numerically using a fourth order Runge-Kutta algorithm with a small time step. The program

which does this starts with deep water waves of known frequency and amplitude and calculates their amplitude and wavenumber in the shallower water at the mouth of the OWC before calculating the wave pressures at that point. This allows more reliable duplication of wave flume experiments where the amplitude and wavenumber of waves produced is only known accurately at their point of origin in deep water.

From calculated values of water elevation and chamber air pressure the instantaneous power is given approximately by :

$$P = A\frac{dy}{dt}(p_0 - p) \tag{3}$$

The PIV system

PIV is a full field velocity measurement technique consisting of two distinct stages. First the displacement of small, neutrally buoyant, seeding particles in the flow is recorded on a photographic negative using a high quality medium format camera. Then the negative is analysed using the Young's fringe method to produce a detailed map of regularly spaced flow velocity vectors with a typical maximum error of the order of 5% of the maximum velocity magnitude.

The seeding particle displacements are recorded by illuminating the flow several times on a single frame with intense laser light. In the experiments carried out in Edinburgh the beam from a 15W continuous wavelength argon ion laser is scanned at high speed into the centre of the tank via a spinning mirror and a parabolic mirror which produce a 'sheet' of light illuminating a region about 1m wide.

One of the drawbacks of a PIV system is that although particle spacings on the film can be accurately determined the sense of direction of flow cannot. There is a 180° ambiguity in the direction of flow vectors which was overcome by the development of a rotating mirror image shifting system. This consists of a high quality front silvered mirror placed in a vertical plane and rotating on a turntable about a vertical axis through it's centre. The turntable rotates at a constant known speed. The PIV pictures are then taken with the flow reflected in the mirror. The rotation effectively superimposes a known horizontal velocity on the flow. This is chosen to be large enough such that the sense of all the resulting velocity vectors can be inferred at the time of analysis. Finally the superimposed shift velocity is subtracted to give the true flow vectors.

The technique of PIV is discussed in greater detail by Gray & Greated[1] and

by Greated et al[2].

Wave flume experiments

All wave flume experiments were carried out in a flume purpose built for PIV applications. The flume is 9.75m long, 0.4m wide and has a depth of 0.75m at mean water level. The walls and base of the tank are constructed from glass to allow unobstructed optical access from the base and sides of the tank. Waves are produced by a computer controlled hinged paddle with the capability to simultaneously produce waves and absorb reflections to minimise any seiche effects in the flume.

An OWC model was built entirely from perspex to allow access for the laser sheet from below and for photography from the side. The model was fitted with a high precision differential air pressure transducer. A resistance type wave gauge was fitted inside the chamber of the model and three further gauges were placed at different water depths on the sloping beach to measure the amplitudes of incoming and reflected waves. All five channels of data collection were sampled at a rate of 80Hz.

Building a scaled down Wells turbine for the model would not only be impractical but would also be unlikely to replicate the linear damping effect of a real turbine on the oscillation of the water column. Instead the square orifice of the model was closely packed with many small diameter tubes to reduce the Reynolds number of the airflow so that the flow would be laminar and thus proportional to the difference between chamber pressure and atmospheric pressure.

Wave gauge and air pressure records were taken for various amplitudes and frequencies of sinewaves and for nine different sizes of orifice ranging from a closed orifice to a maximum opening of 39mm $\times$ 39mm.

Results

PIV was used to produce velocity vector maps of the flow within the OWC for various frequencies and amplitudes of forcing waves and several different orifice sizes. Each map contains about 2500 vectors with a resolution of about 5mm.

At frequencies below about 0.8Hz a strong vortex forms just within the lip of the front vertical wall of the model when water rushes in to the chamber. Figure 2 shows the flow of water into the OWC chamber just after the crest of a 0.6Hz, 15mm amplitude wave is incident. A vortex then forms as the water level rises in the chamber as shown in figure 3 and this is shed as the level drops again.

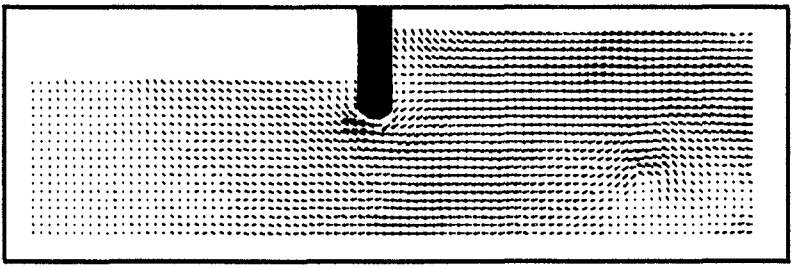

Figure 2: PIV vector map, f=0.6Hz, wave amplitude=15mm

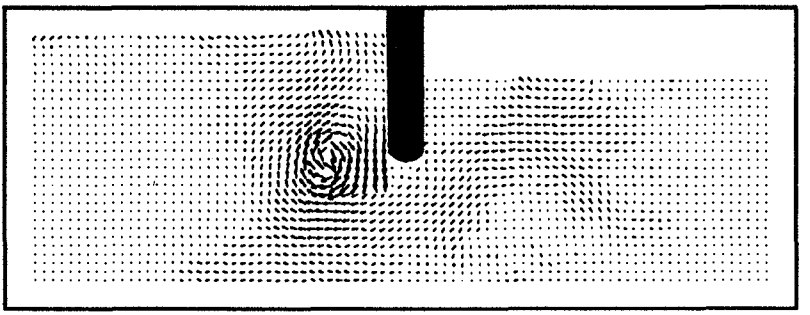

Figure 3: PIV vector map, f=0.6Hz, wave amplitude=15mm

Figure 4 shows the comparison of typical pressure and chamber water elevation results from theoretical and wave flume experiments. It can be seen that the theoretical results correctly predict the amplitude of oscillation of water in the column but underestimate the pressure variations. This may be due to the difficulty of determining the effective mass of water involved in the oscillations of the water column. However a more likely explanation is that the small diameter tubes placed in the orifice of the OWC model failed to lower the Reynolds number sufficiently to ensure laminar flow through them.

As a result of the underestimated pressure variations the theoretical model also underestimates the mean instantaneous power available from the device. The theoretical results in figure 5 suggest that the natural frequency of the OWC is about 0.55Hz with an orifice of 25×25mm whereas the experimental results suggest a lower value of less than 0.5Hz. Unfortunately it was not possible to produce lower frequency sinusoidal waves in the flume. The experimental results in figure 6 show an optimum orifice size of about 24×24mm whereas the the-

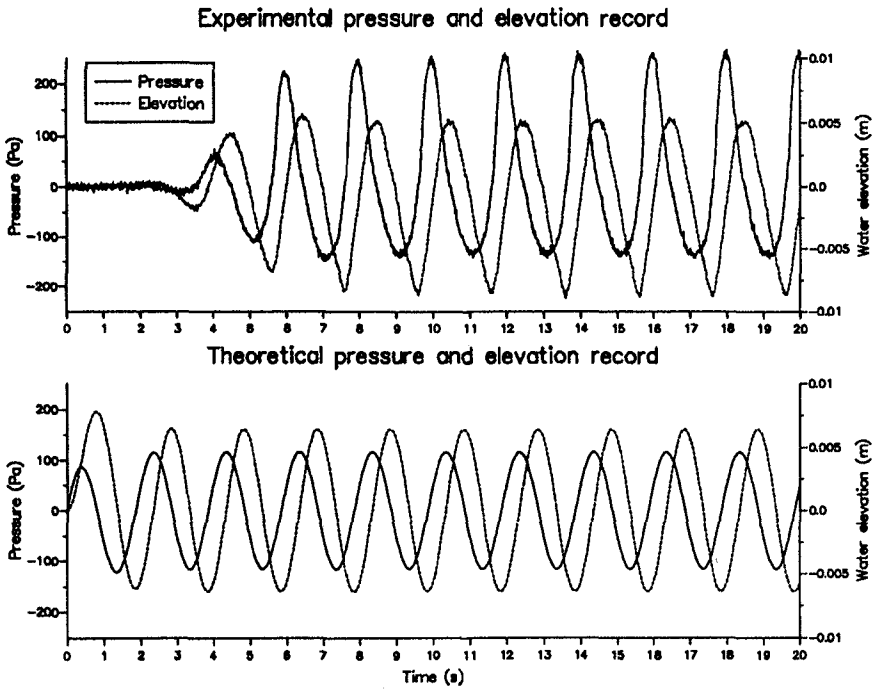

Figure 4: Air pressure and water elevation results, f=0.5Hz, wave amplitude=10mm, orifice=10x10mm

oretical results suggest an optimum size of 22×22mm at a frequency of 0.5Hz. These dimensions compare remarkably well despite the large discrepancy in peak air pressures between the two sets of results.

Vortex energies

Figure 3 shows a strong vortex within the chamber of the OWC. Much of the kinetic energy within the vortex is lost due to viscous dissipation. Therefore it is useful to attempt to calculate, from PIV vector maps, the kinetic energy contained in such a vortex.

For calculation of kinetic energies from PIV vector maps to be possible it is necessary to assume that the flow in the OWC is completely two dimensional i.e. in the plane of the PIV photographs. Although there is some movement through this plane it serves as good first approximation in this case.

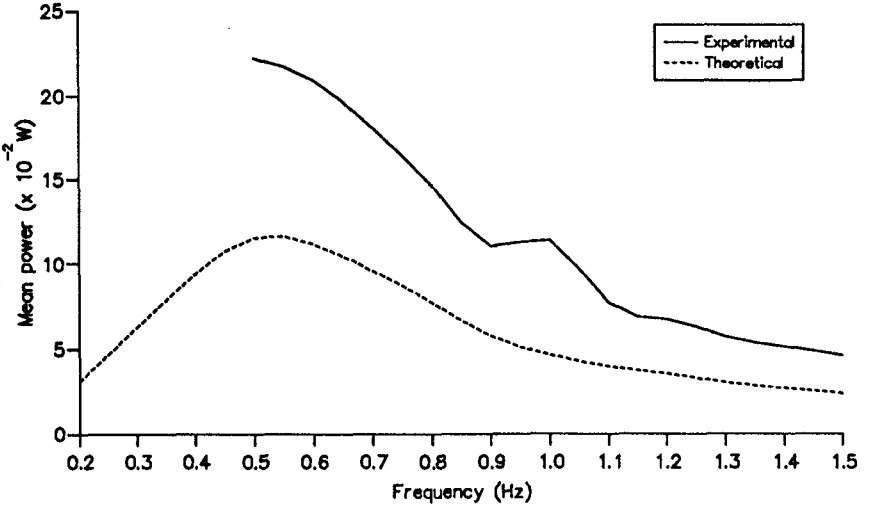

Figure 5: Calculated instantaneous power, orifice=25x25mm, wave amplitude=10mm

Each vector in the PIV velocity map is taken to lie at the centre of a cuboid of fluid with dimensions $d_x \times d_y \times w$ where d_x and d_y are, respectively, the horizontal and vertical spacings of vectors and w is the width of the OWC model. The cuboid is assumed to have a translational velocity equal to that of the vector at it's centre and an angular velocity equal to half the magnitude of the calculated vorticity at that point. This allows the translational and rotational energies to be calculated and combined yielding a total kinetic energy E given by :

$$E = \frac{1}{2}\rho_w d_x d_y w \left(v_x^2 + v_y^2\right) + \frac{1}{96}\rho_w w d_x d_y \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)^2 \tag{4}$$

where v_x and v_y are the horizontal and vertical velocities respectively and ρ_w is the water density. These values can be summed to give the total kinetic energy within the frame of the PIV picture or the total kinetic energy of a vortex subject to the difficulty of determining it's size. This can be compared with the calculated energy of the wave transmitted by the wavemaker in order to estimate the proportion of wave energy dissipated by these vortices.

In figure 3 the kinetic energy of the water in the OWC chamber is calculated to be 0.21J with about 0.16J being contained within the vortex. This corresponds to about 8 % of the original wave energy. However it clearly accounts for a much

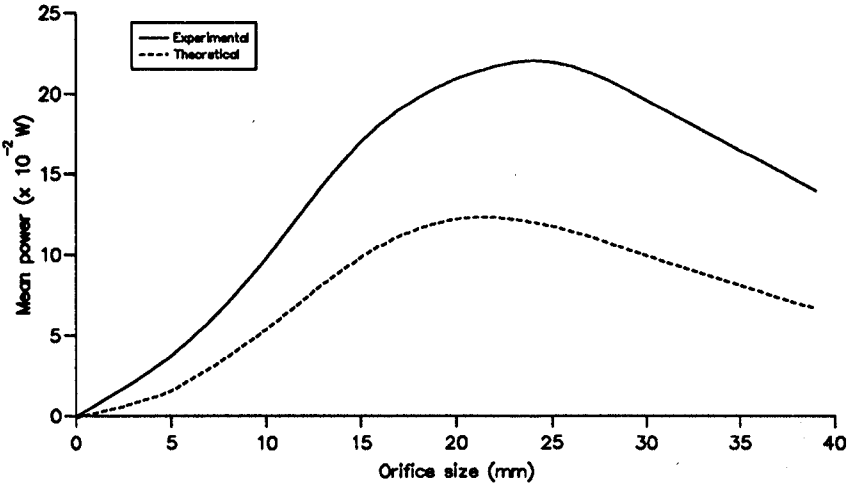

Figure 6: Calculated instantaneous power, f=0.5Hz, wave amplitude=10mm

higher proportion of the wave energy actually reaching the device since energy is dissipated as the wave travels from deep to shallow water. It should be noted that these energy calculations from PIV plots take no account of potential energy due to variations in water level.

Conclusions

An attempt has been made to develop a theoretical model of an oscillating water column wave energy converter. It has been shown that although the model does not satisfactorily predict the pressure variations in the chamber it is able to approximate the optimum orifice size and the optimum frequency of power extraction. PIV vector maps of the flow into the chamber of the OWC have been used to calculate the energy trapped within a large vortex. This has been shown to account for more than 8% of the wave energy incident on the device.

These PIV experiments should enable future OWC wave energy converters to be designed with a more quantitative approach to reducing hydrodynamic losses within the device.

References

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