CHAPTER 152

Equilibrium Beach Profiles with Random Seas

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Abstract

Three quantitative equilibrium beach profiles models are presented for random wave fields. Two models follow the work of Larson where improvements to the spilling breaker assumption are made with spectral estimations of wave breaking. The third model is established by imposing a no-net sediment transport condition on Bailard's (1982) cross-shore sediment transport formulation. Modelling the flow field as in Roelvink and Stive (1989), an equilibrium solution is achieved which includes a longshore bar.

Introduction

It has been observed that the beach profile shape is a function of the characteristics of the incident wave field. Dean (1977) examined a number of U.S. East and Gulf Coast beach profiles and found that the majority of the profiles could be described by the following simple equation:

$$h = Ax^m \tag{1}$$

where $h(x)$ is the water depth and x is the offshore distance perpendicular to the shoreline. The value of m which best fit all of the data was 0.667, though m ranged between 0.1 and 1.4. Dean developed a theoretical justification for a value of m of 2/3, based on the concept that the rate of energy dissipation per unit volume of fluid, $\mathcal{D}_*$, would be constant across the equilibrium beach profile.

$$\frac{d\mathcal{F}}{dx} = -\epsilon_b = -h\mathcal{D}_* \tag{2}$$

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where $\mathcal{F}$ is the energy flux, defined as the energy per unit area times the group velocity of the waves, $C_g(=\sqrt{gh}$ in shallow water), and ϵ_b is the dissipation per unit surface area:

$$\mathcal{F} = \frac{1}{8}\rho g H^2 \sqrt{gh} \tag{3}$$

Assuming a spilling breaker ($H = \kappa h$, where $\kappa \sim O(1)$), and substituting into Eqn. (2) results in an equation for h:

$$h(x) = Ax^{2/3} \tag{4}$$

This model, with its concave upwards shape, has proved to be useful in a number of coastal applications (Dean, 1991).

In this paper, several models for equilibrium profiles will be developed using different representations of the wave field. The first, developed by Larson (1988) for monochromatic waves, is extended to a random wave field. The second and third models are based on random breaking models by Thornton and Guza (1983), which were developed from field data taken from California beaches.

Larson's (1988) Model–Larson (1988) and Larson and Kraus (1989) replaced Dean's spilling breaker assumption with the breaking model of Dally *et al.* (1985):

$$\frac{d\mathcal{F}}{dx} = -\frac{K}{h}(\mathcal{F} - \mathcal{F}_s) \tag{5}$$

where $\mathcal{F} = EC_g$. K is an empirical constant equal to 0.17, and $\mathcal{F}_s$ is the stable energy flux the wave field is trying to obtain. $\mathcal{F}$ is based on a stable breaking wave height, $H_s = \gamma h$, where γ is a constant empirically determined to be 0.4. Equating this energy flux relationship to Dean's constant energy dissipation per unit volume yields the following equation for H:

$$H = \sqrt{\frac{8h^2 D_*}{\rho g \sqrt{gh} K} + \gamma^2 h^2} \tag{6}$$

which provides the wave height across the surf zone as a function of the profile depth, h. Substituting this into the energy flux equation, (2), leads to

$$2\frac{h}{K} + \frac{5}{24}\rho g^{3/2}\left(\frac{\gamma^2 h^{3/2}}{D_*}\right) = x \tag{7}$$

This equation, which relates the equilibrium water depth to the distance offshore, is best solved for the distance x in terms of h. Note the inclusion of a linear term which removes the infinite slope at the shoreline that occurs in Dean's model.

Application of Random Wave Breaking–To apply random waves to Larson's model, we first determine the breaker line location via the breaking depth, h_*.

At this depth, we assume that the breaking index is the correct predictor of breaking wave height.

$$H_b = \kappa h_* = \sqrt{\frac{8h_*^2 D_*}{\rho g \sqrt{g h_*} K} + \gamma^2 h_*^2} \tag{8}$$

from Eqn. 6. Solving for h_*, we have

$$h_* = \left(\frac{8D_*}{\rho g^{3/2} K(\kappa^2 - \gamma^2)}\right)^2 = \left(\frac{D_*}{\beta}\right)^2, \tag{9}$$

which defines the parameter, β for later use. Rearranging,

$$D_* = \frac{K}{8}\rho g \sqrt{g h_*}(\kappa^2 - \gamma^2) \tag{10}$$

which indicates that the magnitude of the constant energy dissipation/unit volume is dependent on the incident wave height (through h_*).

From these expressions we can determine the width of an equilibrium surf zone. Denoting W_* as this width, we have, from Eqn. 7,

$$W_* = 2\frac{h_*}{K} + \frac{5}{24}\rho g^{3/2}\left(\frac{\gamma^2 h_*^{3/2}}{D_*}\right) = 2\frac{h_*}{K}\left[1 + \frac{5}{6}\left(\frac{\gamma^2}{\kappa^2 - \gamma^2}\right)\right] \tag{11}$$

By putting in the values of κ and γ, we have $W_* = 15.0\ h_* = 19.3 H_b$ for the equilibrium profile width.

The Larson model shows that the wave height distribution across the surf zone and the beach profile depends on $\mathcal{D}_*$, the volumetric dissipation, which in Eqn. 10 is shown to be related to the incident wave height (through the breaking depth). This development was for a single wave train; now we extend it to the case for an random wave field.

Given that the wave height probability distribution is a Rayleigh distribution and that the variables, H, h_* and D_* are related, the probability density function for H can be related to those for h_* and D_* through a transformation, e.g.,

$$P(h_*) = P(H_*)|_{H_*=\kappa h_*}\frac{dH_*}{dh_*} = \frac{2\kappa^2 h_*}{H_{\text{rms}}^2}\, e^{-\left(\frac{\kappa^2 h_*^2}{H_{\text{rms}}^2}\right)} \tag{12}$$

Using (9), the probability distribution for D_* is found by

$$P(D_*) = P(h_*)|_{h_*=D_*^2/\beta^2}\frac{dh_*}{dD_*} = \frac{4\kappa^2 D_*^3}{\beta^4 H_{\text{rms}}^2}\, e^{-\left(\frac{\kappa^2 D_*^4}{\beta^4 H_{\text{rms}}^2}\right)} \tag{13}$$

with β defined by (9). To find the expected value of D_*, we integrate:

$$\bar{D}_* = \int_0^\infty D_*\, P(D_*)\, dD_* = \sqrt{\frac{\beta^2 H_{\text{rms}}}{\kappa}}\,\Gamma(\frac{5}{4}) = \frac{\kappa}{8}\rho g\,(\kappa^2 - \gamma^2)\sqrt{g\bar{h}_*} \tag{14}$$

The last expression is similar to (10), except that *expected* values for h_* are used to find $\bar{D}_*$

Random Wave Dissipation Models–A number of random wave breaking models have been developed. For example, Battjes and Janssen (1978) utilized the conservation of energy flux within the surf zone with dissipation determined from a random model of the breaking wave field (based on truncating the Rayleigh probability density function at the local breaking wave height) and a turbulent bore energy dissipation term to predict root-mean-square (or H_{rms}) wave heights in the surf zone. This model was modified by Thornton and Guza (1983), who developed two empirical distributions of breaking wave heights within the surf zone: one with a simplified energy dissipation and another with a more accurate dissipation term. Substituting their breaking wave distribution into a bore dissipation model, they determined the expected dissipation of energy within the surf zone. Wave heights predicted from their models agreed with both field and laboratory data very well.

The energy flux wave model for normal wave incidence is given by Eqn. (2). The forms for dissipation term, ϵ_b, from Thornton and Guza (1983) are

$$\underline{\text{Simple}}: \ \epsilon_b = \frac{3\sqrt{\pi}}{16}\rho g \frac{B^3 \bar{f}}{\gamma^4 h^5} H_{\text{rms}}^7 \tag{15}$$

$$\underline{\text{Complete}}: \ \epsilon_b = \frac{3\sqrt{\pi}}{16}\rho g \frac{B^3 \bar{f}}{\gamma^2 h^3} H_{\text{rms}}^5 \left[1 - \frac{1}{(1 + (H_{\text{rms}}/\gamma h)^2)^{5/2}}\right] \tag{16}$$

where B is an empirical constant smaller than unity, $\bar{f}$ = frequency of the incident wave train, and $\gamma = 0.42$, the breaking index based on the assumption that $H = 0.42\ h$ for the inner surf zone.

Simple EBP Model– The first step in the simple random EBP model is to use righthand side of the energy equation (2) with the dissipation term (Eqn. 15): $\epsilon_b = h\mathcal{D}_*$. This yields a representation for H_{rms} after some rearranging:

$$H_{\text{rms}} = \left(\frac{\gamma^2 \mathcal{D}_*}{A}\right)^{1/7} h^{6/7} \text{ where } A = \frac{3\sqrt{\pi}\rho g B^3 \bar{f}}{16\gamma^2} \tag{17}$$

The equation for H_{rms} indicates that for equilibrium beaches under random waves the statistical wave height should vary nearly linearly with the water depth.

Substituting this relationship for H_{rms} into the left hand side of the energy flux relationship (Eqn. 2) allows the calculation of the water depth across the profile, after using the shallow water limit of $C_g = \sqrt{gh}$:

$$\frac{\rho g^{3/2}}{8}\left(\frac{\gamma^2 \mathcal{D}_*}{A}\right)^{2/7} \frac{d\left(h^{12/7}\sqrt{h}\right)}{dx} = h\mathcal{D}_* \tag{18}$$

Simplifying and integrating with respect to the distance offshore, x, with the initial condition, $h(x) = 0$, at $x = 0$,

$$h(x) = \beta\, x^{14/17} \text{ where } \beta = \left(\frac{136\mathcal{D}_*}{31\rho g^{3/2}} \left\{ \frac{A}{\gamma^2 \mathcal{D}_*} \right\}^{2/7} \right)^{14/17} \tag{19}$$

Thus we have arrived at a simple model for the description of the equilibrium beach profile and the associated wave height distribution in closed form for a random sea state.

Complete EBP Model–The more inclusive model involves the complete expression for the energy dissipation term within the surf zone, given in Eqn. 16. Equating the dissipation terms, we have

$$h\mathcal{D}_* = \frac{AH_{\text{rms}}^5}{h^3} \left[1 - \frac{1}{(1 + (H_{\text{rms}}/\gamma h)^2)^{5/2}} \right] \tag{20}$$

This equation can not be readily solved for H_{rms}; however, it is straightforward to use numerical techniques such as the Newton-Raphson method to obtain solutions. Note that the wave height will depend on parameters such as $\mathcal{D}_*$ and h, but not explicitly on the distance offshore.

To find the profile, the energy flux expression (Eqn. 2) is used. It can be rearranged into:

$$\frac{1}{8}\rho g \sqrt{gh} H_{\text{rms}} \left(2\frac{dH_{\text{rms}}}{dx} + \frac{H_{\text{rms}}}{2h}\frac{dh}{dx} \right) = h\mathcal{D}_* \tag{21}$$

Since H_{rms} does not depend explicitly on x, we use

$$\frac{dH_{\text{rms}}}{dx} = \frac{dH_{\text{rms}}}{dh}\frac{dh}{dx}$$

where the derivative of H_{rms} with respect to h is found from Eqn. 20. We therefore can rewrite the last equation as

$$\frac{dh}{dx} = \frac{8h\mathcal{D}_*}{\rho g \sqrt{gh} H \left(2\frac{dH_{\text{rms}}}{dh} + \frac{H_{\text{rms}}}{2h} \right)} \tag{22}$$

This equation is integrated numerically to determine the variation of the depth across the equilibrium profile and, given the depth, the wave height is found from Eqn. 20.

Results–Figure 1 shows a comparison of the simple and complete equilibrium profile models to data obtained under the National Sediment Transport Study at Torrey Pines Beach, CA (reference). The values of B and D_* are adjusted to

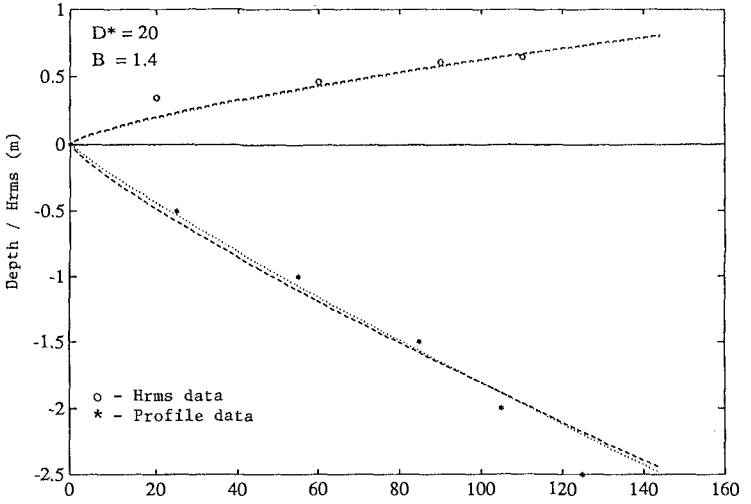

Figure 1: Equilibrium beach profile solutions for *simple* (dashed) and *complete* (dotted) dissipation models compared to wave and profile data. (Torrey Pines CA., November 18, 1978)

achieve the best fit to the wave height and profile data, respectively. As can be seen, both profile models adequately represent the data. The complete model is not a significant improvement over the simple model, suggesting that the analytical model is perhaps sufficient for describing a monotonic equilibrium beach profile under a random wave field.

Generalized Equilibrium Beach Profile Model

The generalized equilibrium beach profile model (GEBP) is founded on the work of Bowen (1980) which is based on the concept of local time-averaged no-net sediment transport across the nearshore region. More specifically, the GEBP follows the work of Bailard (1981), who, like Bowen, used Stokes' second order wave theory and Longuet-Higgins' (1953) streaming velocities to model the wave-induced nearshore flow and the mean longshore current, respectively. However, both theories have shortcomings in the nearshore area. Stokes' wave theory is only valid for relatively deep water, and thus inapplicable for most nearshore wave conditions. Longuet-Higgins' streaming velocity solution assumes mean flows in the direction of wave propagation only, which is clearly not the case in the surf zone with undertow present. Thus it is the objective of this work to determine the effects of more accurate representations of the nearshore wave

and current climate to the equilibrium solution of Bailard's sediment transport expression. To do this, the method of Roelvink and Stive (1989), denoted as R&S, is adopted by modelling the asymmetric velocities with a Stream Function representation of the unbroken wave field and the mean current with an undertow model that establishes an offshore directed mean flow inside the surf zone. Here, for simplicity, the turbulent flow induced by wave breaking, which is modelled by R&S to improve the location of the maximum intensity of the undertow, is neglected. Further, only short wave contributions to the total odd moments will be considered and thus group-induced long-wave flows will be excluded.

The GEBP is developed assuming wave energy, mean water surface level and bottom depth are coupled through the energy, momentum and bottom slope equations, respectively. Variations in wave energy across the nearshore region for a random wave field are modelled following the work of Battjes and Stive (1985). The change in energy flux due to random wave breaking is equated to the time-averaged rate of energy dissipation per unit area, D, as

$$\frac{\partial EC_g}{\partial x} + D = 0, \tag{23}$$

The mean water surface elevation is modelled according to the momentum balance

$$\frac{\partial S_{xx}}{\partial x} + \rho g h \frac{\partial \overline{\eta}}{\partial x} = 0 \qquad \text{where } h = d + \overline{\eta}. \tag{24}$$

The bottom slope equation is not as well established. Here we use the total load cross-shore sediment transport equation of Bailard (1982) to describe the interaction of near-bottom water velocities with the sediment in the nearshore region. The time-averaged form of the expression is

$$\left\langle \vec{i}_x \right\rangle = c_f \rho \left[\frac{\varepsilon_b}{\tan\phi} \left[\left\langle \vec{u} \left| \vec{u} \right|^2 \right\rangle - \frac{\tan\beta}{\tan\phi} \left\langle \left| \vec{u} \right|^3 \right\rangle \right] + \frac{\varepsilon_s}{w} \left[\left\langle \vec{u} \left| \vec{u} \right|^3 \right\rangle - \frac{\varepsilon_s}{w} \tan\beta \left\langle \left| \vec{u} \right|^5 \right\rangle \right] \right] \tag{25}$$

where $\tan\beta = -dd/dx$ is the bottom slope, $\left\langle \vec{u} \left| \vec{u} \right|^2 \right\rangle$, $\left\langle \vec{u} \left| \vec{u} \right|^3 \right\rangle$, $\left\langle \left| \vec{u} \right|^3 \right\rangle$, $\left\langle \left| \vec{u} \right|^5 \right\rangle$ are the total velocity moments in the cross-shore direction and the instantaneous, total cross-shore velocity vector is $\vec{u}_t = (\tilde{u} + \overline{u})\,\hat{i}$, where $\tilde{u}$ is the time-varying velocity component, $\overline{u}$ is the steady current component, and $\hat{i}$ is the onshore unit vector.

The equilibrium condition and thus the bottom slope equation is found by applying the time-averaged no-net sediment transport condition ($\left\langle \vec{i}_x \right\rangle = 0$) of Bowen (1980) to the sediment transport relationship where the bottom slope is expressed as

$$\tan\beta = \left[\frac{\varepsilon_b}{\tan\phi} \left\langle \vec{u} \left| \vec{u} \right|^2 \right\rangle + \frac{\varepsilon_s}{w} \left\langle \vec{u} \left| \vec{u} \right|^3 \right\rangle \right] \left[\frac{\varepsilon_b}{\tan^2\phi} \left\langle \left| \vec{u} \right|^3 \right\rangle + \left(\frac{\varepsilon_s}{w} \right)^2 \left\langle \left| \vec{u} \right|^5 \right\rangle \right]^{-1} \tag{26}$$

This form of the bottom slope equation is most general because the total velocity moment terms are expressed in terms of the total bottom velocity, u. Following R&S, the total velocity moments are expanded and approximated in terms of the steady current, $\overline{u}$ and a time varying current, $\tilde{u}$, where $\tilde{u} \gg \overline{u}$. This produces representations of the total velocity moments as combinations of the steady current, and central odd, ($\left\langle \tilde{u}\,|\tilde{u}|^2 \right\rangle$ and $\left\langle \tilde{u}\,|\tilde{u}|^3 \right\rangle$) and even ($\left\langle |\tilde{u}|^3 \right\rangle$ and $\left\langle |\tilde{u}|^5 \right\rangle$) moments as

$$\left\langle \vec{u}\,|\vec{u}|^2 \right\rangle = \left\langle \tilde{u}\,|\tilde{u}|^2 \right\rangle + 3\overline{u}\left\langle |\tilde{u}|^2 \right\rangle; \quad \left\langle \vec{u}\,|\vec{u}|^3 \right\rangle = \left\langle \tilde{u}\,|\tilde{u}|^3 \right\rangle + 4\overline{u}\left\langle |\tilde{u}|^3 \right\rangle \tag{27}$$

$$\left\langle |\vec{u}|^3 \right\rangle = \left\langle |\tilde{u}|^3 \right\rangle; \quad \left\langle |\vec{u}|^5 \right\rangle = \left\langle |\tilde{u}|^5 \right\rangle \tag{28}$$

Only the first terms of the total even moments are retained, assuming that wave asymmetry does not strongly contribute to these terms.

Approximating the odd moments with Stokes second order theory, the even moments with linear theory and the mean current with Longuet-Higgins' streaming velocity solution recovers Bailard's results. Building on these results, the following models for the wave and current fields will be applied to determine the effects of more accurate models to an equilibrium beach profile solution.

Steady Current Component - Undertow Solution– Across the nearshore region the mean cross-shore flow is known to move onshore outside the surf zone (where wave breaking is not prevalent) and offshore inside the surf zone. This dominant mean cross-shore flow inside the surf zone is the undertow. To model this transition in mean flow from onshore to offshore we follow Stive and de Vriend (1987), who model the mean flow across the nearshore area as a linear combination of the flow from the unbroken and broken fractions of the wave field across the surf zone. For the unbroken wave field, the Longuet-Higgins streaming velocity solution is applied. For the broken wave field, a three layer approach is adopted. This result provides a continuous solution for the mean current across the nearshore region different than that of a monochromatic breaking wave.

It is of interest to note that a change in the direction of the mean bottom velocity has been hypothesized (Dyhr-Nielsen and Sorensen 1970) to contribute to longshore bar formation. In support of this hypothesis, Dally (1987) observed that the dominant mechanism in the establishment of the longshore bar is the breaking induced mean return flow where, as wave breaking begins, the mean current changes from onshore to offshore. In light of this, the model of Stive and de Vriend seems to be a suitable mean flow model to establish one component of the probable flow conditions required to form a longshore bar in a beach profile.

Central Odd and Even Velocity Moments–From the expansion of the total time-averaged flow moments, the terms $\left\langle \tilde{u}\,|\tilde{u}|^2 \right\rangle$ and $\left\langle \tilde{u}\,|\tilde{u}|^3 \right\rangle$ are found to contribute to the representation of the total odd moments. As a first approximation,

the group-induced long wave flows are assumed to be negligible and thus we represent the time-averaged central, odd moments with only the short wave flows under nonlinear wave forms as

$$\left\langle \tilde{u}\,|\tilde{u}|^2 \right\rangle = (1 - Q_b)\left\langle u_s\,|u_s|^2 \right\rangle; \quad \left\langle \tilde{u}\,|\tilde{u}|^3 \right\rangle = (1 - Q_b)\left\langle u_s\,|u_s|^3 \right\rangle \qquad (29)$$

where u_s is the bottom-velocity obtained from 6^{th} order Stream Function theory. The contributions to the odd moments from the unbroken waves are represented by considering the fraction of waves breaking, Q_b. These expressions represent the unbroken short wave contributions to the central odd velocity moments. The input monochromatic wave height needed by Stream Function theory is approximated with the H_{rms} wave height from the breaking model.

The results of Guza and Thornton (1985) are adopted to model the even moments. The even moments $\langle|\tilde{u}|^n\rangle$ are nonzero for symmetric wave forms and thus do not require the high order nonlinear wave solutions to produce contributions to the mean flows in the surf zone. For the case of a random wave field, Guza and Thornton suggest as a first approximation representing the wave field using a Gaussian description of the wave heights in the surf zone of a linear random sea.

Model Summary–The governing equations for the GEBP are the energy, momentum and bottom slope equations. These first order ordinary differential equations are solved numerically as a coupled initial value problem where initial wave height and water depth are given outside the surf zone.

Results–The input wave conditions are the deep water incident root-mean-squared wave height, H_{rmso}, the peak wave period, T_p. The mean water surface displacement, $\overline{\eta}$, is taken to be zero far offshore from the surf zone. The selection of the input depth, d, is arbitrary as long as wave breaking has not been initiated. Given the deep water incident wave conditions, the effective H_{rms} wave height, H_{rmse}, is determined at the starting depth by linear shoaling theory.

As an example of the use of the GEBP, the 1981 average wave and profile data from the U.S. Army Corps of Engineers Field Research Facility (Miller *et al.* 1981) are used. The H_{rms} wave height is 1.02 m, the peak wave frequency, T_p is 10.0 sec and the starting depth is 20 m. The sediment fall velocity is assumed to be 0.04 cm/s and constant across the nearshore region. Bailard's (1981) bedload and suspended load sediment transport efficiency factors of $\epsilon_b = 0.025$ and $\epsilon_s = 0.01$, respectively, are used. The internal angle of friction sand, $\tan\phi$, is taken as 0.63, the fluid density, ρ, is 1000 kg/m^3 and the gravitational acceleration, g, is 9.81 m/s^2.

Figure 2a shows the results for the H_{rms} wave height across the equilibrium beach profile, where x is positive onshore. As the waves shoal, the H_{rms} wave height first increases up to a point where the percent of waves breaking is such that energy dissipation is more dominant than wave shoaling. At this point the

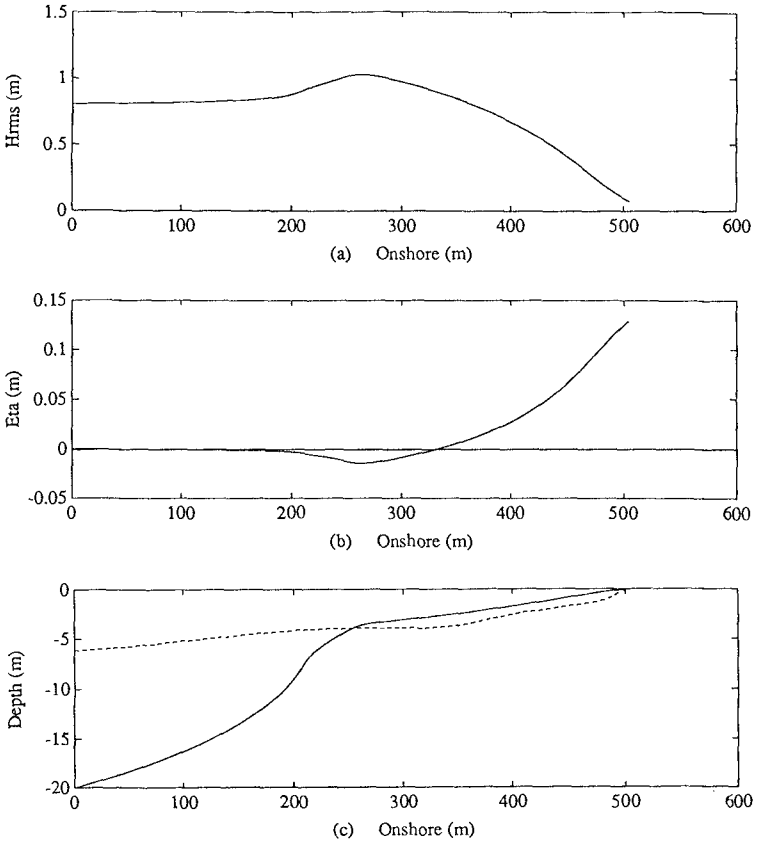

Figure 2: GEBP Solutions; (a) H_{rms}, (b) $\overline{\eta}$, (c) d (*solid*) and Mean Profile FRF (1981) (*dashed*)

H_{rms} wave height will decrease until a stable water depth is reached for all waves or until all energy is dissipated from the waves at the shoreline.

Figure 2b shows the solution for the mean water surface, $\overline{\eta}$, across the equilibrium solution. It can be seen that set-down is most pronounced as the waves shoal to their maximum height and set-up increases as wave breaking intensifies.

Figure 2c shows the equilibrium beach profile as a function of onshore distance. The point of interest is the change in slope in the equilibrium solution between $x = 200\ m$ and $x = 250\ m$ where the slope changes from a steep offshore slope before breaking to a milder foreshore slope after breaking has initiated. This is the result of a change in the direction from onshore to offshore of the steady current, $\overline{u}$, as wave breaking dissipation increases with the decrease in depth. In figure 3, the resulting mean flow across the surf zone changes direction at approximately $x = 200\ m$. As seen in Eqn. 26, a change in the steady

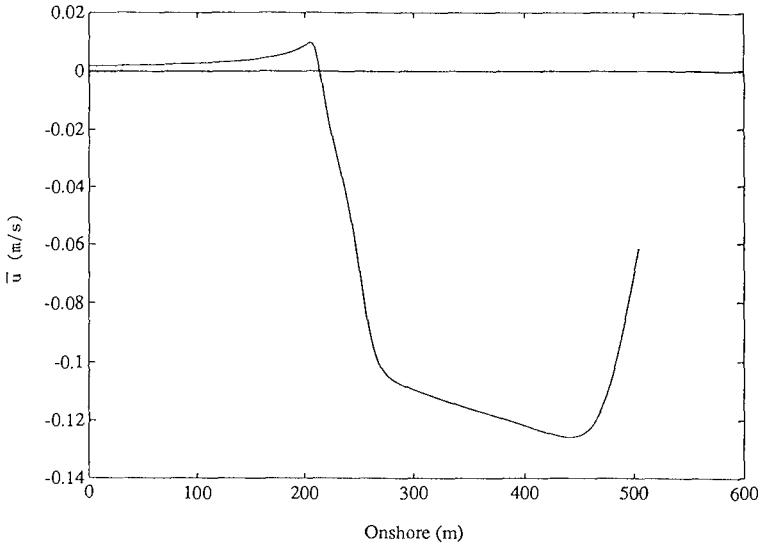

Figure 3: Steady Current, $\overline{u}$, Across an Equilibrium Beach Profile.

current, $\overline{u}$, is directly related to the bottom slope equation through the total odd velocity moments, and a decrease in magnitude or change in sign of $\overline{u}$ results in a decrease in bottom slope. In other words, the bottom slope is influenced by a balance of forces between the flow field and gravity. In the offshore region, the flow forcing is entirely in the onshore direction where the mean wave-induced current and asymmetric flows are onshore. The combination of onshore flows works against the effects of gravity and produces a steep bottom slope. This result is equivalent to the findings of Bowen (1980) and Bailard (1981). As wave breaking begins and the mean return flow is established, the onshore flow forcing is reduced as the result of the offshore directed mean current opposing the asymmetric onshore flows and the slope becomes milder. The resulting change in slope due to the change in direction of the steady current is considered to be a longshore bar based on the findings of Dyhr-Neilsen and Sorensen and Dally. The steep slope of the depth solution outside the surf zone is attributed to the inclusion of Longuet-Higgins' streaming velocities for the mean flow in the unbroken wave field. As expected, these slopes are on the same order of magnitude as those found by Bailard.

Figure 4a shows the total odd moments across the equilibrium profile. In the equilibrium state, the magnitude of the total odd moments, $\langle u|u|^2\rangle$ and $\langle u|u|^3\rangle$ in Eqn. 26, is forced by the relative magnitude of the central moments and the steady current. As seen in figure 4a, the total odd moments are predominately positive across the entire surf zone. This is a result of the net balance of the flow forcing in the onshore direction. The balance is established through the

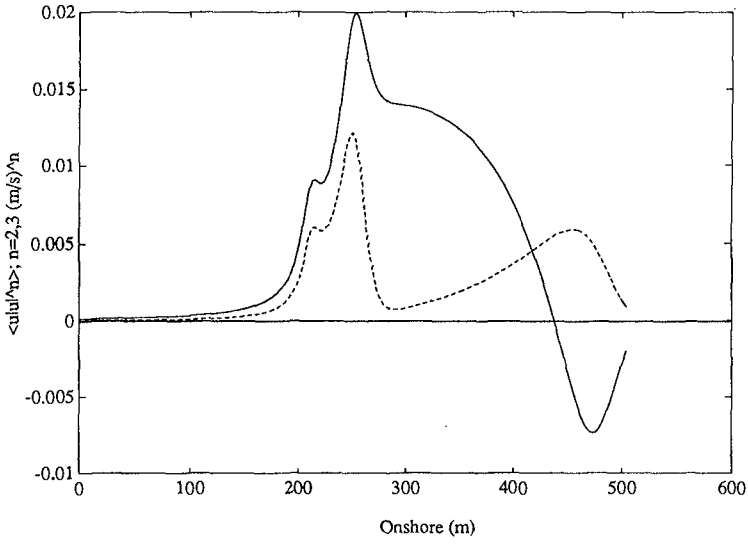

Figure 4: Total Moments Across an Equilibrium Beach Profile (a) Odd: $\langle u|u|^2 \rangle$ (*solid*), $\langle u|u|^3 \rangle$ (*dashed*), (b) Even: $\langle |u|^3 \rangle$ (*solid*), $\langle |u|^5 \rangle$ (*dashed*).

combinations of the central odd and even moments and the steady current where net onshore flows are required to balance the system and maintain a negative sloping bottom, reducing the depth from offshore to onshore.

Figure 5 shows the central odd moments, which are the result of the asymmetric onshore flow of the nonlinear wave form in the nearshore region to maintain positive values across the entire surf zone. The relative magnitudes of these terms in the GEBP model are governed by the balancing of the equilibrium solution between the suspended load and bedload.

For this case, the positive values for the total odd moments across the surf zone exhibit different characteristics as those measured and modelled by R&S. R&S present negative values for the total odd moments across the surf zone. A source of difference possibly lies in the equilibrium or average wave field representation in this example as opposed to the erosive sea state used by R&S. A comprehensive study of the model's response to variations in wave climate and sediment characteristics was conducted by Creed (1992).

Conclusions

Several approaches to modelling equilibrium beach profiles assuming a random sea state have been addressed. The simple and complete models advance the models of Dean and Larson, but still only offer a monotonic representation

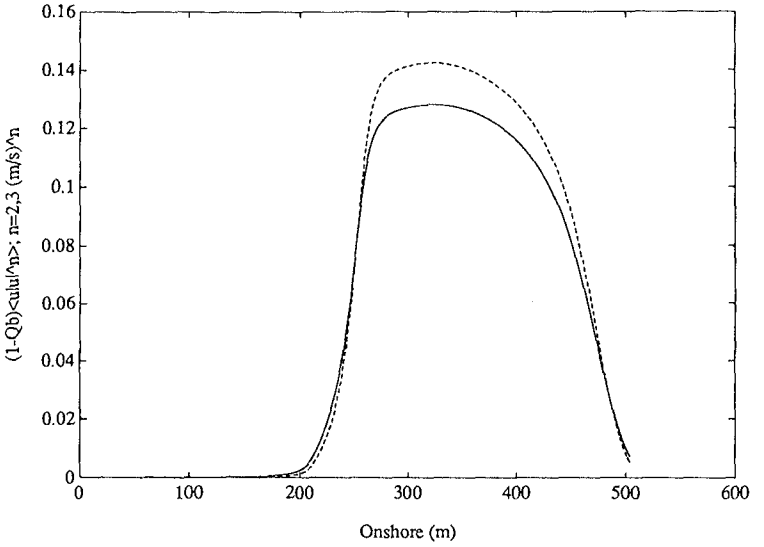

Figure 5: Central Odd Moments Across an Equilibrium Beach Profile: $\langle \tilde{u}|\tilde{u}|^2 \rangle$ (*solid*), $\langle \tilde{u}|\tilde{u}|^3 \rangle$ (*dashed*).

of the beach profile.

The GEBP can produce non-monotonic equilibrium solutions of a beach profile when a time-averaged no-net sediment transport condition is applied.The formation of a bar in the equilibrium beach profile is attributed to a change in the direction of the near-bottom mean current from onshore to offshore as wave breaking establishes a mean return flow inside the surf zone. The model does not produce realistic bottom slopes outside the surf zone, but inside the surf zone, where the sediment transport characteristics are better quantified, the equilibrium shape of the beach compares favorably with time-averaged field data.

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