CHAPTER 9

Vorticity effects in combined waves and currents.

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Abstract.

This paper concerns the interaction of waves and currents, and in particular the effect of the time-averaged vorticity distribution associated with a sheared current. Laboratory data describing both the "initial interaction" of waves and currents, and the "equilibrium" conditions arising within an established wave-current combination are presented. These results are compared to both the existing irrotational solutions and a multi-layered numerical model capable of describing an arbitrary current profile. The interaction of regular waves and sheared currents is shown to be in good agreement with this latter solution. However, a similar description of random waves on sheared currents is limited by the wave-induced changes in both the mean current profile and the associated turbulent structure.

1. Introduction.

In general, the interaction of waves and currents may be sub-divided into two distinct stages. The first corresponds to the "initial interaction" which arises when a given wave train (specified in the absence of a current) propagates onto a pre-determined current profile. This stage is usually solved in terms of a "gradually varying flow", and describes the initial changes in the wave height, the wave length, and (under some circumstances) the current profile. In contrast, the second stage concerns the description of the so-called "equilibrium conditions" arising from an established wave-current combination. It is this stage which seeks to define the fluid motion appropriate to the wave height, the wave period, the water depth, and the current profile determined in stage 1.

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In its simplest form the combination of waves and currents involves a series of regular waves propagating on a depth-uniform current. In this case the current has no associated vorticity, and laboratory measurements (Thomas, 1990 and Swan, 1990) suggest that the "initial interaction" is well defined by the conservation of wave action and the "equilibrium conditions" are in good agreement with a Doppler shifted solution (Fenton, 1985). Although this case appears very simplistic it is, in fact, valid for a wide range of flow conditions. For example, in the case of a logarithmic current profile (typical of many tidal flows) the vorticity distribution is largely confined to a lower layer adjacent to the sea bed. In this case the vorticity has little effect beyond the near-bed region, and an approximate uniform current provides a reasonable description of both the "initial interaction" and the resulting oscillatory flow. Furthermore, if the current profile is weakly sheared, an "equivalent uniform current" based upon the flow conditions at the water surface (Hedges and Lee, 1992) may be sufficient to define the resulting flow field.

Unfortunately, the success of these simplistic solutions, and in particular the ease with which they can be incorporated within large-scale coastal models, has tended to detract from those situations in which a uniform approximation is inappropriate. For example, in the case of a wind generated current (or, indeed, a wind modified current) offshore measurements suggest that the current profile is strongly sheared in the vicinity of the water surface. In this case a non-uniform vorticity distribution arises (ie. the current is not linearly sheared), and consequently the wave motion may be very different to that which is predicted by the existing irrotational wave-current solutions. Indeed, Tsao (1959) suggested that the wave motion would itself become rotational; while Swan (1992) provided explicit analytical calculations (within a truncated series expansion), and confirmed that the near-surface vorticity distribution altered the wave kinematics over the entire water depth.

The present paper will consider the interaction of waves with depth varying currents, and will examine the importance of the vorticity distribution. Section 2 commences with a brief outline of the experimental apparatus. Laboratory measurements describing the interaction of regular waves and random waves with a variety of current profiles are presented in sections 3 and 4. Section 3 concerns the "equilibrium conditions" and compares the measured data with a variety of solutions including a five-layered numerical model similar to that outlined by Dalrymple and Heideman (1989) and previously discussed by Cummins and Swan (1993). In section 4 this model is, in turn, used within an iterative procedure to solve the energy transfer equation first outlined by Longuet-Higgins and Stewart (1960). Although this latter approach is computationally intensive, it is applicable to both regular and random waves, and (unlike the conservation of wave action) provides an explicit description of the spectral changes arising when random waves propagate onto strongly sheared currents. This is of particular importance from a practical point of view. Finally, some concluding remarks concerning the importance of the vorticity distribution and the applicability of the various solutions are made in section 5.

2. Laboratory apparatus.

The experimental measurements were undertaken in a purpose built wave flume which allows the simultaneous generation of waves and co-linear currents. This facility is 25m long, 0.3m wide, and has a working depth of 0.7m. It is equipped with a numerically controlled random wave paddle located at one end of the wave flume, and a large passive absorber (consisting of poly-ether foam) at the other. The current is introduced via three loops of re-circulating pipework which are pumped individually to give a total volume flow of $0.45m^3/s$. Each loop is fully reversible, and the inlets and outlets are adjustable (in height) to give a variety of both "favourable" (in the same direction as the phase velocity) and "adverse" (in the opposite direction to the phase velocity) current profiles. With this arrangement it is possible to generate a uniform current of approximately 0.2m/s, or a highly sheared current in which the near-surface velocities may be as large as 0.6m/s. A sketch showing the layout of this apparatus is given on figure 1.

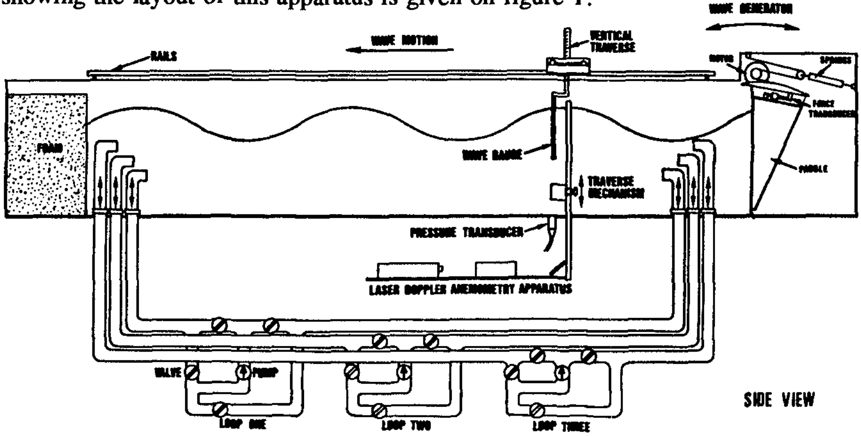
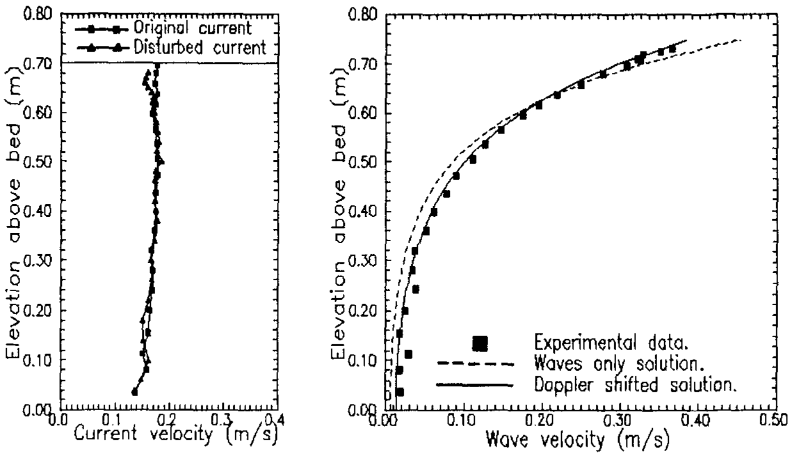


Figure 1. Laboratory apparatus.

Within this study measurements of the water surface elevation were obtained from surface piercing wave gauges which were mounted above the wave flume. Each gauge consists of two vertical wires and provides a time history of the water surface elevation at one point fixed in space. These probes cause a minimal disturbance of the water surface, and have a measuring accuracy of ± 1mm. The velocity field was measured using laser Doppler anemometry. This was based upon a 35mW helium-neon laser, used to create a three beam arrangement with cross polarisation. The intersection of the beams was located along the centre-line of the wave flume, and produced a measuring volume which was estimated to be $0.5mm^3$. This intersection was observed in forward scatter using two photomultipliers positioned on the opposite side of the wave flume. This arrangement provides the optimal signal to noise ratio, with no disturbance of the flow field. After seeding the flow with milk, added in the ratio of 100ppm., a data rate of 2.5 khz was achieved with a measuring accuracy of $\pm 2\%$.

3. Equilibrium conditions.

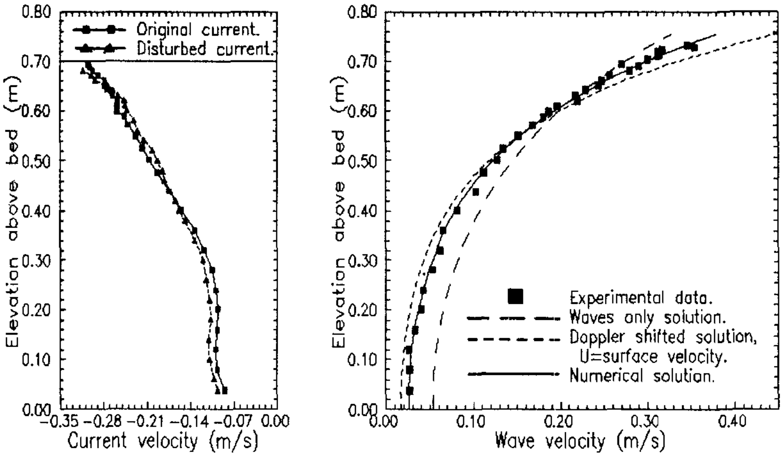
The interaction of regular waves (T=0.75s and H=0.083m) with a "favourable" uniform current is considered on figures 2a and 2b. The first of these figures describes the current profile measured both before and after the interaction with the wave train; while the second figure describes the depth variation in the horizontal component of the wave velocity measured beneath a wave crest. In this case, and indeed in all other cases with zero vorticity, there is virtually no change in the current profile, and the measured wave kinematics are in good agreement with the fifth order Doppler shifted solution proposed by Fenton (1985).



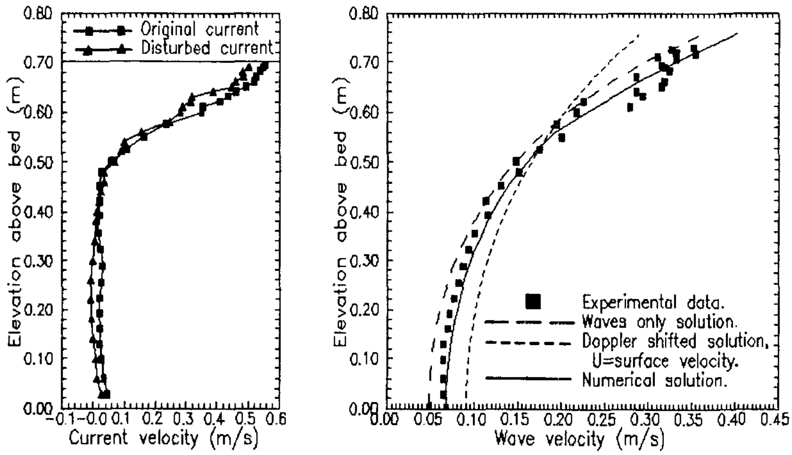
Figures 2a-2b. Regular waves on a "favourable" uniform current.

Figures 3a-3b present a similar sequence of results describing the interaction of regular waves on an "adverse" sheared current; while figures 4a-4b correspond to waves on a "favourable" sheared current. In both of these cases there is some evidence of a change in the current profile (ΔU). This is particularly apparent in the "favourable"case (figure 4a) where the interaction with the wave motion appears to reduce the time-averaged near-surface vorticity (ie. the current becomes less sheared). In contrast, figure 3a suggests that in an "adverse" current the magnitude of the near-surface vorticity increases.

If the disturbed current profile (or that measured in the presence of waves) is used to calculate the wave-induced kinematics, a Doppler shifted solution based on the magnitude of the near-surface current is in poor agreement with the laboratory data (figures 3b and 4b). However, a non-linear numerical model, which incorporates the effects of the vorticity distribution, provides a good description of the measured data. The numerical solution referred to in figures 3b and 4b is based upon a five layered approximation in which the measured current profile is described by five linear segments of variable depth. This approach represents an extension of the bi-linear model originally proposed by Dalrymple (1974), and provides a satisfactory compromise between the description of the current profile (particularly the vorticity distribution) and the computational effort required for convergence. This model has been rigorously tested against other wave-current models (Chaplin, 1990), and is described in detail by Cummins and Swan (1993). Figures 3b and 4b confirm the importance of the vorticity distribution, and suggest that this must be taken into account if the wave kinematics are to be correctly predicted.



Figures 3a-3b. Regular waves on an "adverse" sheared current.



Figures 4a-4b. Regular waves on a "favourable" sheared current.

Previous work by Hedges and Lee (1992) suggests that the interaction of waves with sheared currents may be described by an "equivalent uniform current". This is defined as the uniform current which produces the same wave number as the measured current for a given wave period, wave height, and water depth. In other words, it ensures that the dispersive characteristics of the waves are correctly modelled. However, this does not imply that the underlying kinematics will be correctly predicted. Indeed, Hedges and Lee comment that the solution may be inappropriate if there are regions of very strong shear; while Swan (1992) suggests that if this is indeed the case, an additional rotational term arises within the description of the wave kinematics. The measured data appears to confirm this effect. Figure 5 compares the measured kinematics on a "favourable" sheared current with the numerical model (discussed above), and a Doppler shifted solution based upon an "equivalent uniform current". In this case (and indeed, in several other cases involving layers of strong current shear) the "equivalent uniform current" does not provide a good description of the wave kinematics.

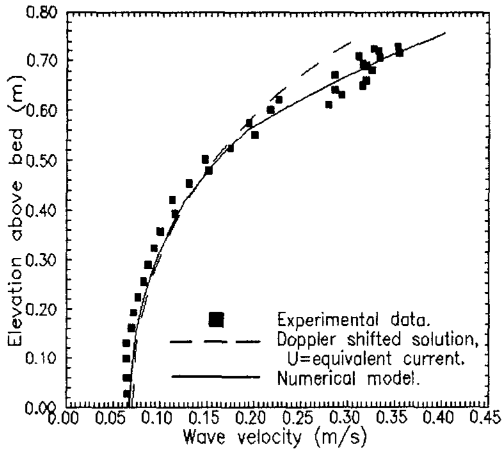


Figure 5. Comparison with an "equivalent uniform current".

4. Initial interaction.

When a wave train first propagates onto a current there are changes in the wave number (k), the wave height (H), and under some circumstances the current profile, U(z). The numerical model (discussed above) is able to predict the wave number for a given wave period, wave height, water depth, and current profile. If we assume that the water depth is known, and (at present) that the current profile remains unchanged, the energy transfer equation first identified by Longuet-Higgins and Stewart (1960) may be applied in conjunction with the numerical model to define an iterative solution for the overall change in the wave train (ie. Δk and ΔH).

If R defines the mean rate of energy transfer across a fixed surface (S), Longuet-Higgins and Stewart (1960) give:

$$R = \int_S \left(P + \frac{1}{2}\rho \underline{u}^2 + \rho g z\right) \underline{u} \cdot \underline{n} \; dS \tag{1}$$

where $\underline{u}$ is the velocity vector, P is the pressure, ρ is the density, g is the gravitational constant and $\underline{n}$ a unit vector normal to the surface S.

If we consider a control volume, and assume that there is no reflection of wave energy, then the sum of the mean energy transfer rates due to the current (acting alone) and the wave train (also acting alone), must exactly balance the total mean rate of energy transfer associated with the combined wave-current motion. This approach is entirely consistent with the original analysis outlined by Longuet-Higgins and Stewart (1960) in which they introduced the concept of radiation stress. Indeed, Longuet-Higgins and Stewart suggested in a footnote (page 574) that the vorticity may be taken into account by supposing U to be dependent upon z (ie. U(z)). This is exactly what we have done in the present analysis.

Regular waves.

Having coupled the energy transfer equation with the numerical model the interaction of regular waves with a uniform current was considered. In this case the present calculations were shown to be in good agreement with the fifth order solution (based upon the conservation of wave action) proposed by Thomas (1990). In a second test-case the interaction of regular waves with a linearly sheared current was considered, and the results compared with the second order solution outlined by Jonsson et al. (1978). In this case there was again good agreement between the two solutions provided (as one might expect) the comparisons were restricted to small amplitude waves. However, in those cases involving larger wave amplitudes, the difference between these solutions emphasises the importance of the higher order non-linear interactions. These results appear to be consistent with the laboratory observations presented by Swan (1990) and, in particular, the comparison with the third order kinematics predicted by Kishida and Sobey (1988). Further comments concerning the importance of the non-linear interactions are provided by Cummins and Swan (1993).

The experimental measurements presented on tables 1a and 1b concern a total of seven cases involving the interaction of regular waves and depth varying currents. Cases 1-4 correspond to a "favourable" sheared current (table 1a); while cases 5-7 correspond to an "adverse" sheared current (table 1b). In each case the initial wave properties (measured in the absence of a current) are denoted by H_o and T_o; while the wave height measured in the presence of a current is indicated by H. Three comparative solutions are al.o presented. The first corresponds to a uniform current based upon the near surface velocity ($U=U_s$); the second corresponds to an "equivalent uniform current" ($U=U_e$); and the third represents the present solution. In each case the numerical model provides the best description of the measured data.

Initial state.		Lab. data	Predicted wave height		
T_0	H_0	H	$U=U_s$	$U=U_e$	Model.
.75	.083	.079	.051	.061	.077
.90	.088	.082	.055	.066	.081
1.1	.102	.096	.065	.083	.091
1.2	.107	.104	.071	.093	.096

Table 1a: Wave height change on a "favourable" sheared current.

Initial state.		lab. data	Predicted wave height		
T_0	H_0	H	$U=U_s$	$U=U_e$	Model.
0.9	.075	.086	.107	.111	.094
1.0	.075	.096	.111	.110	.098
1.1	.075	.097	.113	.108	.098

Table 1b: Wave height change on an "adverse" sheared current.

In figures 6a-6b the wave lengths associated with cases 1-7 are again compared to three potential solutions. The first corresponds to a waves only solution and thus describes the waves in the absence of the current. The second corresponds to a Doppler shifted solution based upon the near surface current ($U=U_s$), while the third represents the present numerical results. In each case the latter solution provides the best description of the measured data. In general, the measured changes in both the wave height and the wave length suggest that the vorticity distribution acts to reduce the effect of the Doppler shift associated with the surface current. This is consistent with the second order approximation presented by Swan (1992), but because of the non-linearity of the wave-current interaction a higher order numerical solution is required to provide a good fit to the measured data.

Random waves.

Having demonstrated that the present approach is able to describe the changes in a regular wave train propagating onto a strongly sheared current, a similar approach will be applied to investigate the spectral changes in a random wave train. The laboratory apparatus described in section 2 was used to generate a random wave train with a Pierson-Moskowitz input spectrum. Both the water surface elevation ($\eta(t)$) and the horizontal velocity component ($u(t)$) at $z=-0.1$m were sampled at 25Hz for 200 minutes, and the resulting data was analysed using a ten point moving average. Figures 7a-7b concern the interaction with a "favourable" uniform current. In figure 7a the uppermost curve (indicated by a dashed line) describes the spectrum of the

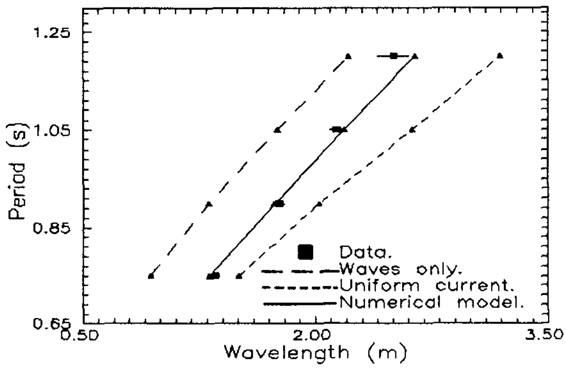


Figure 6a. Wave length on a "favourable" sheared current.

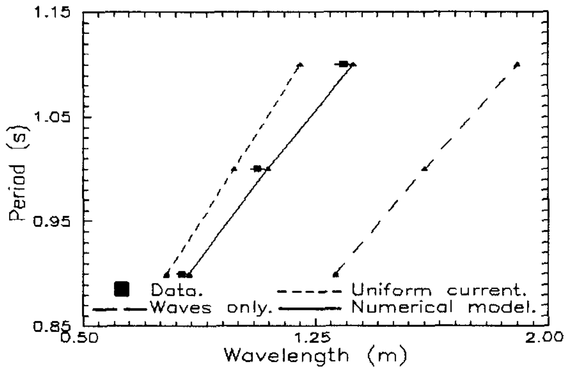
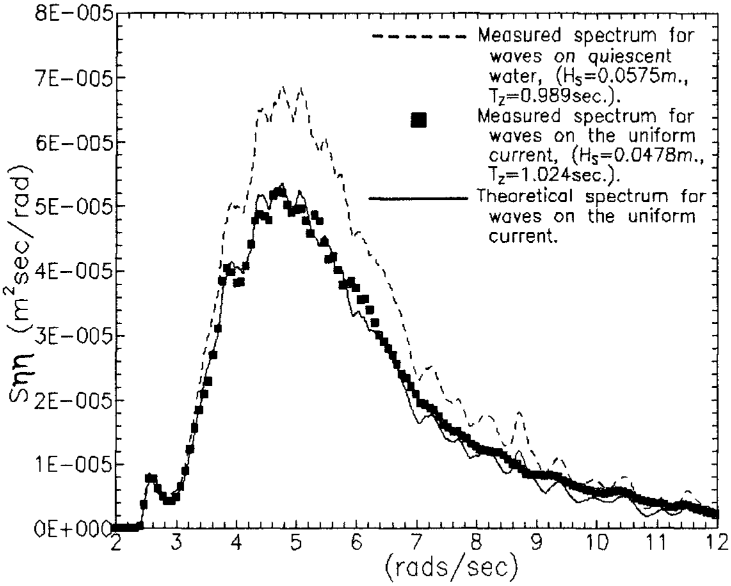


Figure 6b. Wave length on an "adverse" sheared current.

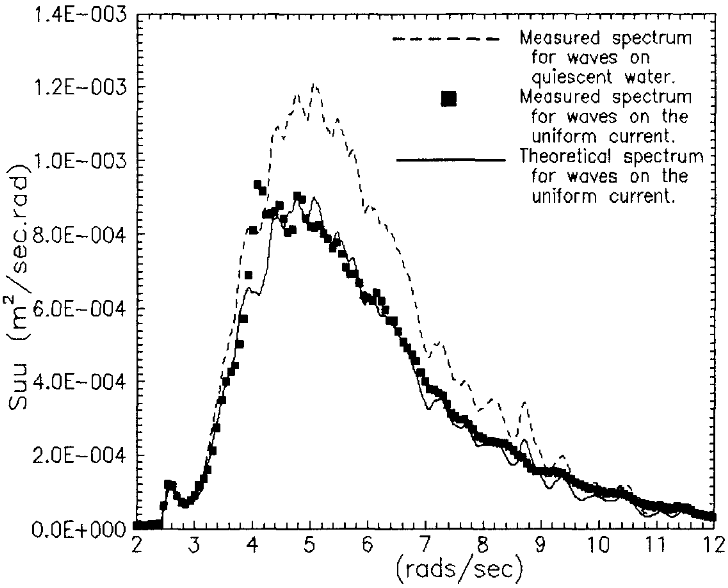
water surface elevation ($S\eta\eta$) measured in the absence of a current; the solid squares represent the data measured in the combined wave-current motion; and the solid line corresponds to the present numerical model. A similar sequence of results is presented on figure 7b, but in this case they correspond to the spectrum of the measured horizontal velocity (Suu) at z=-0.1m. Together, these figures suggest that the proposed numerical model provides a good description of the changes in both the water surface elevation and the underlying kinematics.

Figures 8a-8b present a similar sequence of plots concerning the interaction with an "adverse" uniform current. In this case the steepening of the high frequency components induces wave breaking, and consequently the experimental measurements within the range ($\omega > 7$ rad/s) diverge from the predicted behaviour. This result was also noted by Hedges et al. (1985), and an experimental correction (referred to as the equilibrium range constraint) was proposed. Having incorporated this correction, the present model once again provides a good description of the laboratory data.

(a) Water surface elevation.

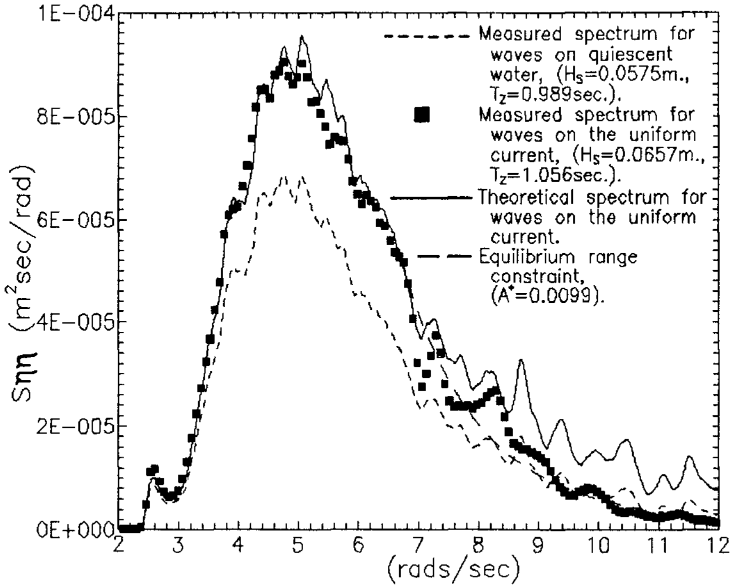


(b) Horizontal kinematics (z=-0.1m.)

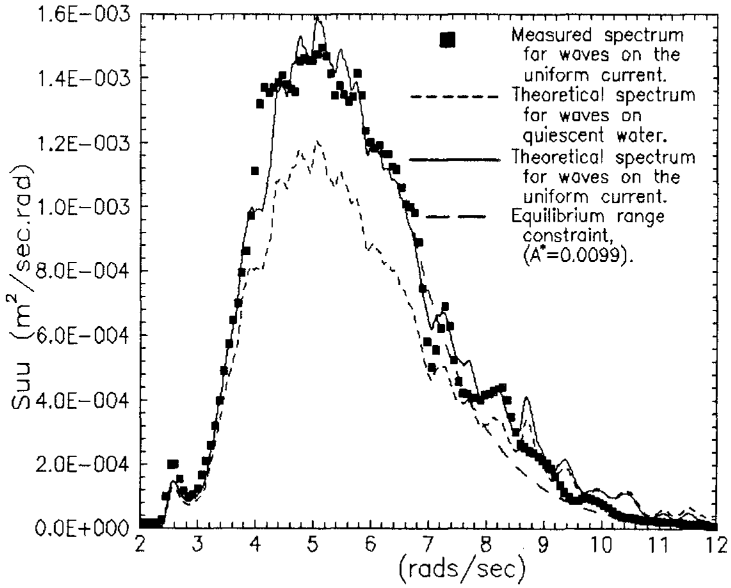


Figures 7a-7b. Interaction of random waves with a "favourable" uniform current.

(a) Water surface elevation.



(b) Horizontal kinematics (z=-0.1m).



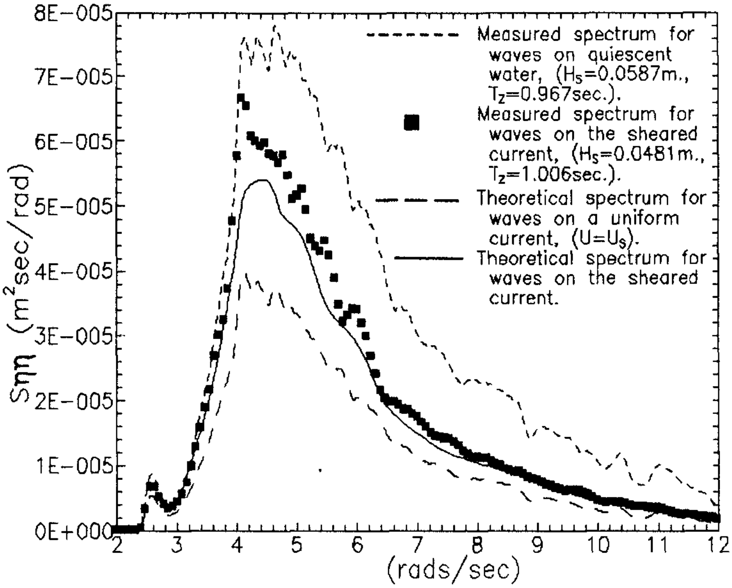
Figures 8a-8b. Interaction of random waves with an "adverse" uniform current.

These examples (involving the interaction with a uniform current) provide further validation of the proposed numerical scheme. However, in practice, the existing analytical solutions are also appropriate to these cases, and provide a simpler calculation procedure. Indeed, if the individual waves are linear, the first order approximation originally proposed by Longuet-Higgins and Stewart (1960) may be applied. Alternatively, if the waves are steeper (and, in particular, if the interaction involves an "adverse" current) the fifth order solution proposed by Thomas (1990) will be appropriate.

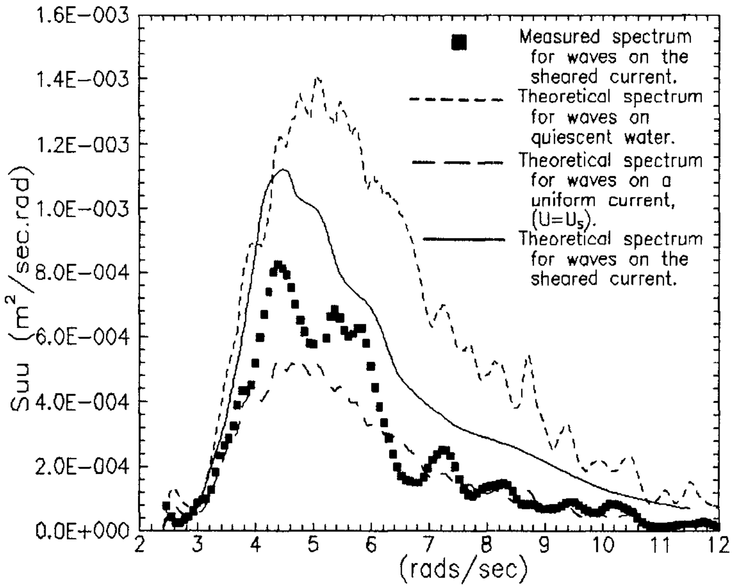
In contrast, if the current profile is strongly sheared and involves a non-uniform vorticity distribution, the analytical solutions are invalid. In this case the present numerical scheme provides the only method of determining the change in the wave spectra. Figures 9a-9b concern exactly this case and compare the measured data with both a uniform current approximation and the numerical solution. The power spectrum of the water surface elevation ($S_{\eta\eta}$) is considered in figure 9a. Although, in this case, the numerical model provides the best description of the measured data, there remain significant differences between the observed and predicted behaviour. These discrepancies are probably associated with a change in the current profile similar to that noted on figures 3a and 4a. The present formulation assumes that the current velocity is unchanged by the wave-current interaction. Indeed, if the fluid is inviscid and the flow laminar, the vorticity must remain constant along a streamline. However, in the present investigation the turbulent intensity (or the root-mean-square velocity fluctuations expressed as a ratio of the mean current velocity) was of the order of 8-10%. This provides an effective transport mechanism capable of re-distributing the vorticity profile. Without a detailed description of the turbulent structure, the change in the current profile remains indeterminate. This represents an important limitation to the present model. Nevertheless, figure 9a suggests that the numerical calculations provide a significant improvement over the uniform current approximation based upon the near-surface velocity.

Figure 9b describes the spectrum of the horizontal velocity at $z=-0.1m$. In this case the measured data, including an apparent bi-modal peak, does not correspond to either of the existing solutions. Indeed, the data lies mid-way between the uniform current approximation ($U=U_s$) and the numerical predictions. These results may, once again, reflect the importance of the current change (ΔU). However, the wave spectrum indicated on figure 9b, has been derived by subtracting the power spectrum of the turbulent fluctuations (measured in the absence of waves) from the total velocity spectrum measured in combined waves and currents. If, as discussed above, the interaction of waves and currents produces a modification of both the vorticity distribution and the turbulent structure, the present results may also reflect the uncertainty in the turbulence spectrum arising in combined waves and currents.

(a) Water surface elevation.



(b) Horizontal kinematics (z=-0.1m).



Figures 9a-9b. Interaction of random waves with a "favourable" sheared current.

5. Concluding remarks.

The present paper has considered the interaction of waves and currents, and has presented the results of a new experimental study. This has considered both regular and random waves, and has sought to identify the importance of the time-averaged vorticity distribution. Preliminary measurements concerning the interaction with a uniform current (zero vorticity) confirm that the initial changes in both the wave height and the wave length are consistent with the fifth order solution proposed by Thomas (1990). Furthermore, in the absence of vorticity, the underlying kinematics are in good agreement with the Doppler shifted solution proposed by Fenton (1985).

In contrast, if the current profile is strongly sheared, and in particular if there is significant vorticity at the water surface, the present data suggest that the resulting flow field cannot be predicted by an "equivalent uniform current". However, a multi-layered numerical model, capable of describing an arbitrary current profile, provides a good description of the kinematics beneath a regular wave train. Furthermore, if the solution is coupled with an energy transfer equation the initial change in both the wave height and the wave number can be satisfactorily predicted. Finally, a similar approach was applied to the propagation of random waves on a strongly sheared current. In this case the predicted power spectrum of the water surface elevation was in reasonable agreement with the laboratory data. However, there were important differences between the observed and predicted spectrum of the underlying wave motion. At present, these discrepancies are believed to reflect the wave-induced changes in both the mean current profile and its associated turbulent structure. Until these changes are clarified, the exact nature of these important wave-current interactions will remain indeterminate.

Acknowledgements.

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