CHAPTER 19

A Regression Model for Estimating Sea State Persistence

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Abstract

This paper deals with regression models for estimating probability distributions of long term wave height, sea state persistence above a wave height threshold and peak wave height during a storm, and yearly-averaged occurrence rate of peak wave height event. The models, in which input condition is the variance of long term wave height at a concerned sea area, were constructed on the basis of statistical analysis of the long term wave height data obtained around the Japanese coast. First, the effect of observation interval on the above-mentioned wave climate statistics was investigated and a method to remove the effect by use of FFT was proposed. Second, Applicability of each regression model was confirmed from close agreement between estimation and observation for the wave climate statistics. Finally, return periods of extreme wave height with prescribed sea state persistence and wave height threshold at wave observation points around the Japanese coast were estimated with a combination of the present regression models.

1. Introduction

Sea state persistence is an important factor to be considered in maritime activity and design of coastal structures. Although many studies on its statistical properties (Lawson and Abernethy, 1975; Graham, 1982; Takahashi et al., 1982; Kuwashima and Hogben, 1984; Smith, 1988; Yamaguchi et al., 1989, 1993; Teisson, 1990; Mathiesen, 1994) have been conducted using the observed

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wave data, probability distribution of sea state persistence of high waves which is crucial to the design of coastal structures and a statistical model for estimating sea state persistence of high waves still remain open to question, because of the lack of long term observed wave data with good quality.

The aim of this study is to present a regression model for estimating return period of extreme wave height with prescribed sea state persistence and wave height threshold on the basis of statistical analysis for distributions of significant wave height and sea state persistence and peak wave height during a storm using long term data of coastal waves around Japan acquired over several years by the Japan Meteorological Agency and the Bureaus of Harbor Construction, Ministry of Transport.

2. Wave Data and Analysis Method

Wave data used in the analysis are those measured for 20 minutes every 1 to 3 hours with a sonic-type wave gauge installed in water of 30 to 50 m depth. Fig. 1 shows location of 14 wave observation points around the Japanese coast. The longest and shortest observation periods are 14 years at Irouzaki in the Pacific coast and Kyougamisaki in the Japan Sea coast and 5 years at Shirihamisaki in the Pacific coast of North Japan. The

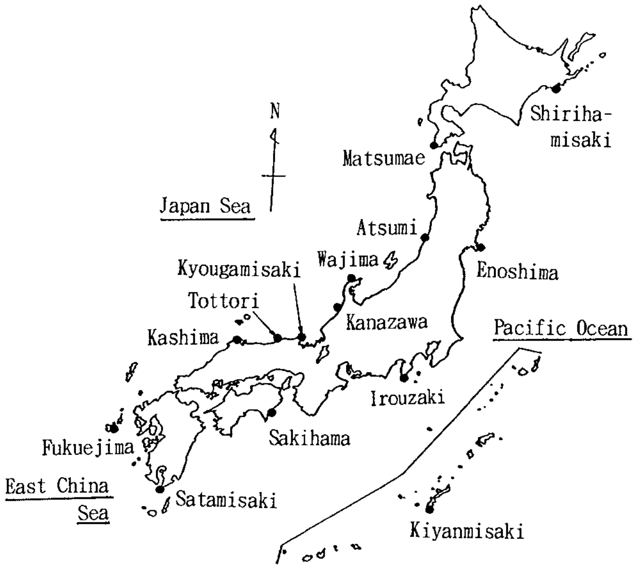

Fig. 1 Location of wave observation points.

most prominent feature of the wave data is that breaks in the data record are rare, in which even the highest ratio of breaks is less than 5 % of the total run at Sakihama in the Pacific coast. One or two missing data points in the time series of wave height were filled using linear interpolation but larger gaps due to breaks were not filled.

Fig. 2 illustrates the definitions of sea state persistence τ and peak wave height H_p. Individual sea state persistence τ is defined as linearly-interpolated time span above a wave height threshold $H_{1/3c}$. Wave height threshold is prescribed every 0.25 m for the range of 0.5 to 5 m. Peak wave height H_p is obtained by fitting of parabolic curve to the three largest wave height data to recover the reduction of wave height associated with observation of discrete interval and application of FFT.

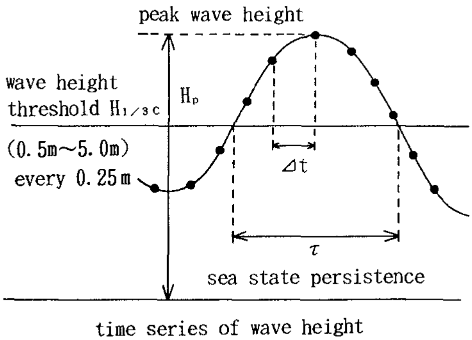

Fig. 2 Definitions of sea state persistence and peak wave height.

3. Fitting of Probability Distribution to Wave Data

Four kinds of probability distributions such as 3-parameter and 2-parameter Weibull distributions, 3-parameter lognormal distribution and hypergamma distribution (Suzuki, 1964) or generalized gamma distribution (Ochi, 1992) were used for data fitting. As a result of goodness of fit test, the 3-parameter Weibull distribution was chosen for fitting to the long term wave height data and peak wave height data, and the 2-parameter Weibull distribution was used for fitting to persistence data above a wave height threshold given every 0.25 m. The method applied for the estimation of the parameters is the maximum likelihood method. Each parameter is distinguished with suffix "H", or " τ " or "e". Non-exceedance probability of the 3-parameter Weibull distribution

F(x) is written as

$$F(x)=1-\exp[\{(x-b)/(x_0-b)\}^k], \quad (1)$$

in which k, b and x_0 are the shape parameter, location parameter and scale parameter respectively. If the location parameter b is set to zero, Eq. (1) reduces to 2-parameter Weibull distribution.

Fig. 3 shows an example of the probability distributions fitted to the wave height data. Goodness of fit of the 3-parameter Weibull distribution to the wave height data is satisfactory in this case. The other distributions produce poor fit in the higher or lower wave height region and the parameters estimated for hypergamma distribution are not always statistically stable.

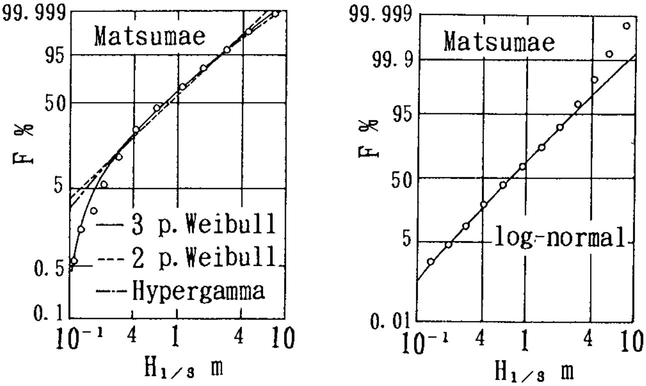

Fig. 3 Fitting of probability distributions to wave height data.

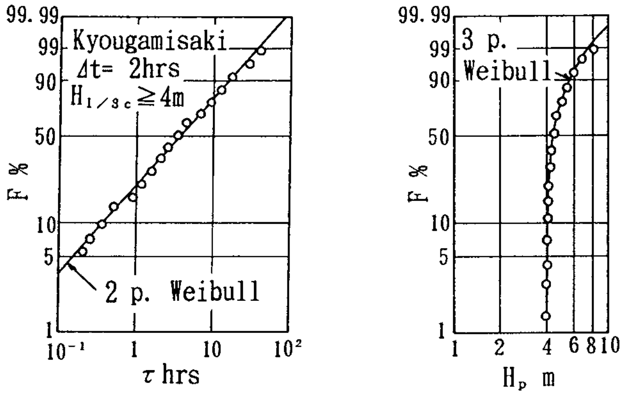

Fig. 4 Fitting of 2-parameter Weibull distribution to sea state persistence data and fitting of 3-parameter Weibull distribution to peak wave height data.

Examples of the fitting of the 2-parameter Weibull distribution to the persistence data and the fitting of the 3-parameter Weibull distribution to the peak wave height data are indicated in Fig. 4. Goodness of fit of these distributions is satisfactory. When sea state persistence data are well approximated with the 2-parameter Weibull distribution, use of the 2-parameter distribution rather than the 3-parameter distribution is preferable, because the estimates of parameters in this case are statistically more stable.

4. Effect of Sampling Interval on Wave Climate Statistics

In order to investigate the effect of sampling interval on wave climate statistics, time series of wave height data were resampled with sampling intervals of 2, 3, 4, 5, 6, 8, 10 and 12 hours from those observed every 1 hour and with sampling intervals of 4, 6, 8, 10 and 12 hours from those observed every 2 hours. Then, wave climate characteristics were analyzed using the above-mentioned methods. The left side of Fig. 5 shows an example of the relation between shape parameter k_τ of the 2-parameter Weibull distribution fitted to sea state persistence data and wave height threshold $H_{1/3c}$ in the case of sampling time of Δt=2 hours, and the right side of Fig. 5 is the relation between shape parameter k_e of the 3-parameter Weibull distribution fitted to peak wave height data and wave height threshold $H_{1/3c}$. Relations between the Weibull parameters and wave height threshold are approximated well with power functions such as $k_\tau = a_{k\tau}(H_{1/3c})^{b_{k\tau}}$ and $k_e = a_{ke}(H_{1/3})^{b_{ke}}$. These results hold for the other parameters $x_{0\tau}$, b_e and x_{0e} of the Weibull distribution.

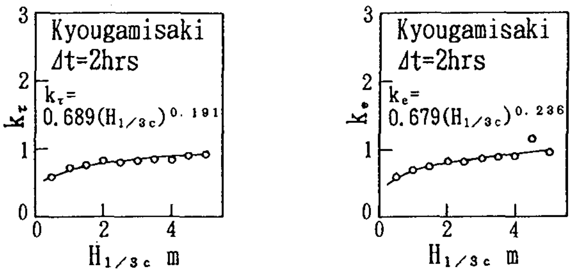

Fig. 5 Relation between Weibull parameter and wave height threshold.

The relations between the coefficients $a_{k\tau}$, a_{ke} in the power function and sampling interval Δt are indicated in Fig. 6. These coefficients, and consequently, the probability distributions of sea state persistence and peak wave height are strongly dependent on sampling interval Δt. But the parameters of the 3-parameter Weibull distribution fitted to long term wave height data are almost independent of sampling interval.

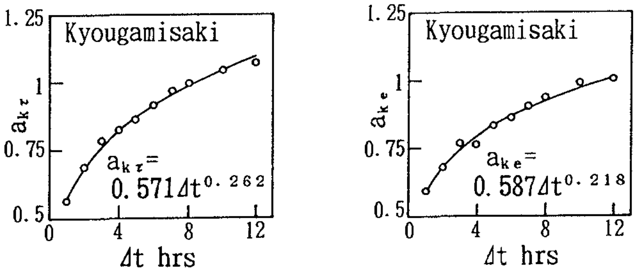

Fig. 6 Dependence of coefficient in power function on observation interval.

In order to remove the effect of the sampling interval on wave climate statistics, higher frequency components in time series of wave height data were filtered out by making use of FFT, and then the same methods as before were applied for the estimation of the wave climate statistics. Fig. 7 describes the effect of cut-off period on the coefficient in the power function, and an almost constant value of the coefficient can be found for the range of sampling interval shorter than half of a prescribed cut-off period. But it should be noted that filtering of time series of wave height data produces longer sea state persistence data and lower peak wave height data. In the following analysis, the cut-off period of Δt=6 hours is chosen in order to avoid the smoothing effect as much as possible and to make quality of wave height data similar with respect to the sampling interval, when the sampling interval used in actual observation is taken into account.

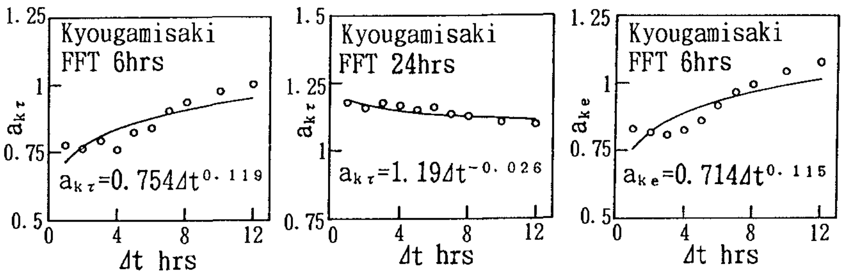

Fig. 7 Effect of cut-off period on coefficient in power function.

5. Construction of Regression Models

(1) Regression model for wave height distribution

Fig. 8 shows the relations between parameters of the 3-parameter Weibull distribution fitted to long term wave height data and variance $H_\sigma{}^2$ of wave height data in each wave observation point. The relations are approximated well by power functions such as

$$k_H=1.10(H_\sigma{}^2)^{-0.246},\ b_H=0.088(H_\sigma{}^2)^{-0.894},\ x_{0H}=1.19(H_\sigma{}^2)^{0.088} \qquad (2)$$

These relations are approximate ones, because the parameters of the Weibull distribution theoretically depend on mean, variance and skewness of the population. But long term wave height distribution can be approximately estimated from these regression equations by giving wave height variance in the sea area as an input condition.

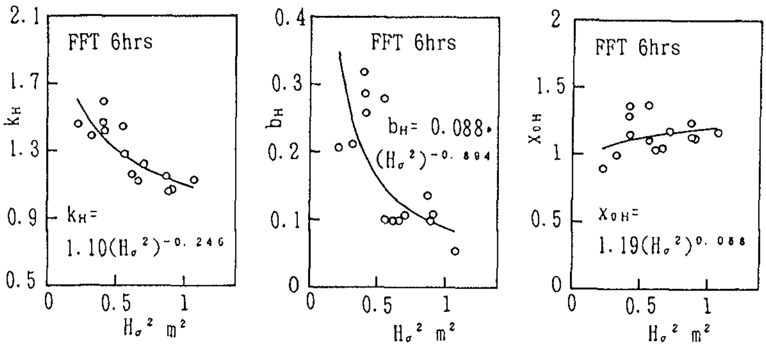

Fig. 8 Relation between Weibull parameter for wave height distribution and variance of wave height.

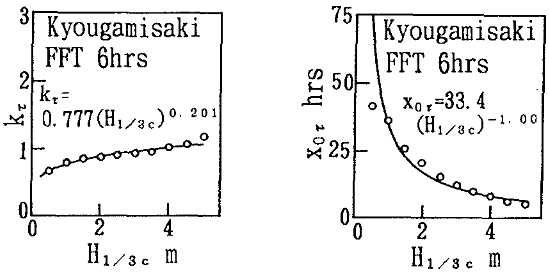

Fig. 9 Relation between sea state persistence parameter and wave height threshold.

(2) Regression model for sea state persistence

Relations between sea state persistence parameters and wave height threshold are indicated in Fig. 9. The relations are approximated well by regression equations such as

$$k_\tau = a(H_{1/3c})^b,\quad x_{0\tau} = c(H_{1/3c})^d \qquad (3)$$

Fig. 10 shows the relations between coefficients in the above regression equations and variance of wave height at 14 wave observation points around the Japanese coast. Relatively high correlation is found between these variables and the relations are approximated with power functions such as

$$\begin{aligned} a &= 0.778(H_\sigma^2)^{0.088}, & b &= 0.205(H_\sigma^2)^{-0.284} \\ c &= 35.1(H_\sigma^2)^{0.555}, & d &= 0.955(H_\sigma^2)^{0.324} \end{aligned} \qquad (4)$$

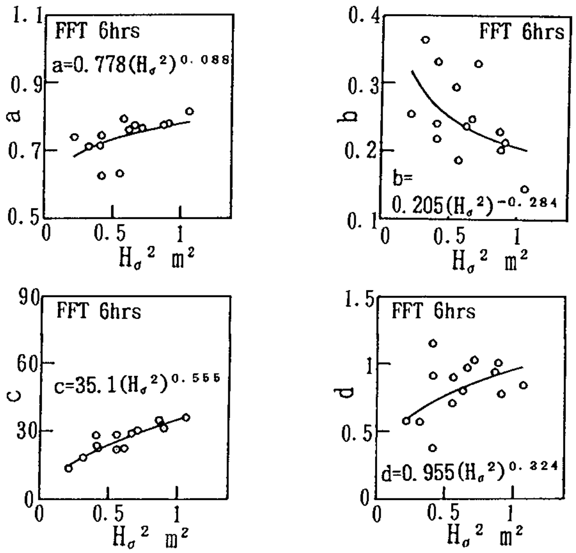

Fig. 10 Relation between coefficient in regression equation and variance of wave height.

Thus, probability distribution of sea state persistence above a prescribed wave height threshold can be estimated by using these regression models, when wave height variance in the sea area is given as an input condition. Fig. 11 illustrates the comparison between estimation and observation for mean $\bar{\tau}$ and standard deviation τ_σ of sea state persistence and probability distribution. Good correspondence is observed between estimation and observation.

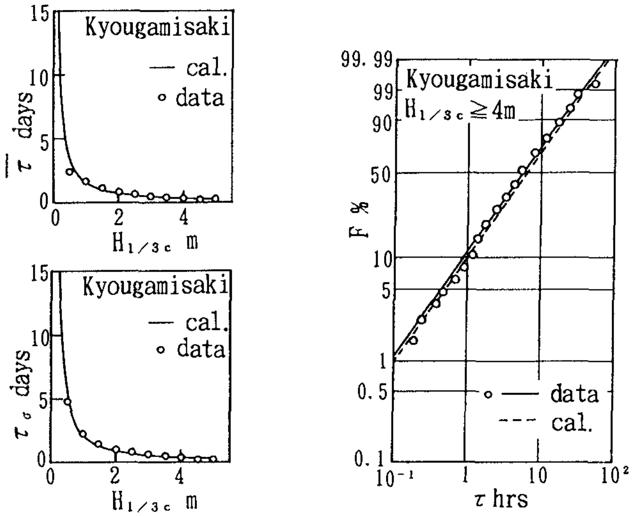

Fig. 11 Comparison between estimation and observation for sea state persistence.

(3) Regression models for peak wave height and its occurrence rate

Fig. 12 shows the relation between peak wave height parameters in the Weibull distribution k_e, b_e, x_{0e} and yearly-averaged occurrence rate of peak wave height $\bar{N}$, and wave height threshold $H_{1/3c}$. The former three relations are approximated with power functions such as

$$k_e=a(H_{1/3c})^b,\quad b_e=c(H_{1/3c})^d,\quad x_{0e}=e(H_{1/3c})^f \tag{5}$$

and the last relation is approximately expressed with exponential function as

$$N=p.\exp\{-q(H_{1/3c})^{1.5}\} \tag{6}$$

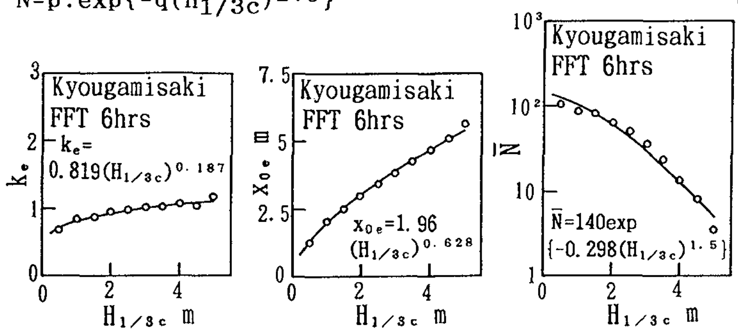

Fig. 12 Relation between peak wave height parameters and occurrence rate, and wave height threshold.

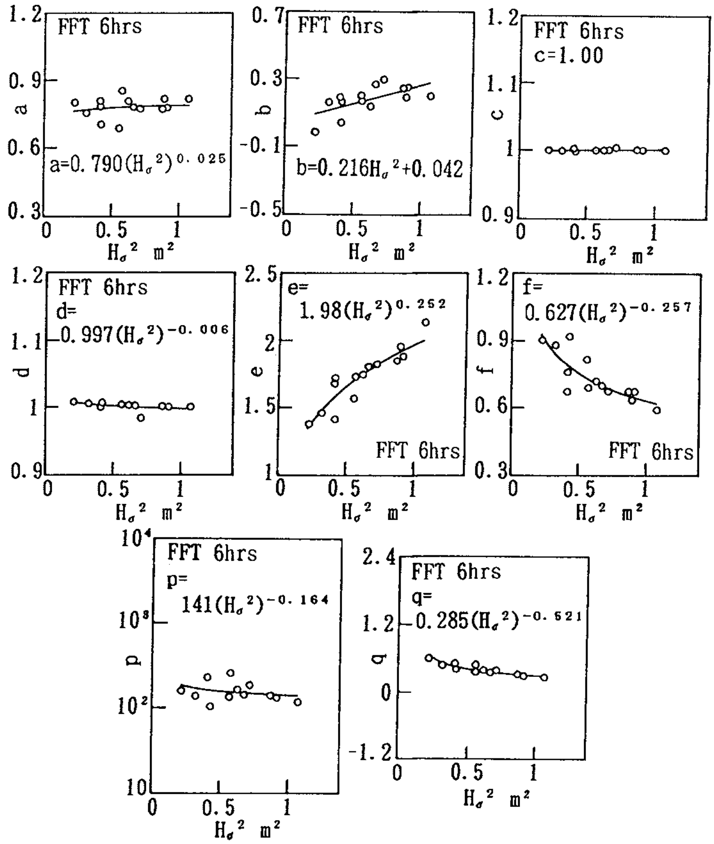

Fig. 13 Relation between coefficients in regression equations and wave height variance.

Then relations between coefficients in regression equations and variance of wave height estimated individually at 14 wave observation points are shown in Fig. 13. These relations are also well approximated with power functions as

$$
\begin{aligned}
&a=0.790(H_\sigma{}^2)^{0.025}, && b=0.216H_\sigma{}^2+0.042 \\
&c=1.000, && d=0.997(H_\sigma{}^2)^{-0.006} \\
&e=1.98(H_\sigma{}^2)^{0.252}, && f=0.627(H_\sigma{}^2)^{-0.257} \\
&p=141(H_\sigma{}^2)^{-0.164}, && q=0.285(H_\sigma{}^2)^{-0.521}
\end{aligned} \tag{7}
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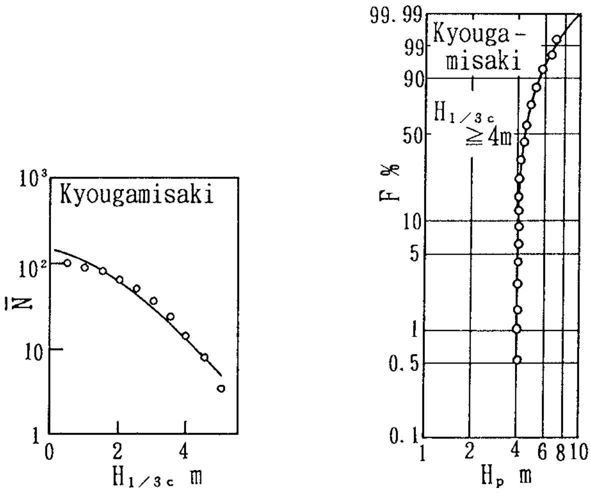

Fig. 14 Comparison between estimation and observation for yearly-averaged occurrence rate of peak wave height and probability distribution of peak wave height.

Probability distributions of peak wave height and yearly-averaged occurrence rate of peak event at a prescribed wave height threshold can be estimated by using these regression models for given wave height variance. Fig. 14 is an example of the estimation for yearly-averaged occurrence rate of peak wave height and probability distribution of peak wave height. High correlation between estimation and observation can be seen.

6. Estimation of Return Period of Extreme Wave Height

Assuming independence between sea state persistence and peak wave height, return period R of peak wave height H_p with prescribed sea state persistence τ above a wave height threshold $H_{1/3c}$ can be estimated from the following equation

$$1/R=N(H_{1/3c})\{1-F(H_p;H_{1/3c})\}\{1-F(\tau;H_{1/3c})\} \quad (8)$$

in which $F(H_p;H_{1/3c})$ and $F(\tau;H_{1/3c})$ are non-exceedance probabilities of peak wave height H_p and sea state persistence τ above a wave height threshold $H_{1/3c}$. When variance of long term wave height at a location around the coastal area of Japan is given as an input condition, the Weibull parameters and yearly-averaged occurrence rate of peak event above a prescribed wave height threshold required in Eq. (8) are determined by using all regression equations mentioned before.

Table 1 illustrates examples of return periods of peak wave height H_p=8 m with sea state persistence τ=12 hours above a wave height threshold $H_{1/3c}$=4 m which are estimated around the coastal area of Japan. The return periods of extreme wave height under these conditions range from about 160 to 270 years. The longest return period is obtained at Atsumi facing Northern Japan Sea. This is due to low exceedance probability of a prescribed extreme wave height and the resulting longer return period. The second longest return period is evaluated at Satamisaki facing Western Pacific Ocean. This is due to geographical situation around Satamisaki. That is to say, two islands, Tanegashima Island and Yakushima Island are located near Satamisaki, which tends to be sheltered from incoming waves by these islands.

Table 1 Return period R for peak wave height H_p with sea state persistence τ above wave height threshold $H_{1/3c}$ for H_p=8 m, τ=12 hours and $H_{1/3c}$=4 m.

location	H_σ^2 (m^2)	$\bar{N}(H_{1/3c})$	$1-F(\tau)$	$1-F(H_P)$	R (years)
Matsumae	.620	8.2	.246	.0029	169
Atsumi	1.067	15.4	.276	.0009	269
Kyougamisaki	.885	12.7	.268	.0014	211
Kashima	.663	9.0	.251	.0026	173
Fukuejima	.565	7.2	.240	.0035	165
Shirihamisaki	.406	4.3	.214	.0066	165
Enoshima	.555	7.0	.239	.0036	164
Irouzaki	.407	4.3	.215	.0066	164
Sakihama	.320	2.7	.194	.0105	179
Satamisaki	.216	1.1	.157	.0223	249
Kiyanmisaki	.415	4.4	.216	.0064	164
Wajima	.869	12.4	.267	.0015	207
Kanazawa	.906	13.0	.269	.0013	216
Tottori	.708	9.8	.255	.0023	178

7. Conclusions

Main results obtained in this study are summarized as follows;

(1) Effects of wave observation interval on the distributions of sea state persistence and peak wave height during a storm are relatively strong, whereas the effect on the long term wave height distribution is negligibly weak.

(2) Effect of observation interval on wave climate statistics can be removed by use of FFT.

(3) Probability distribution of sea state persistence above a prescribed wave height threshold can be estimated with a proposed regression model, when variance of long term wave height is given as.an input condition.

(4) At the coastal sea area of Japan, return period of extreme wave height with prescribed sea state persistence and wave height threshold can be estimated with a combination of regression models for probability distributions of sea state persistence and peak wave height, and yearly-averaged occurrence rate of peak wave height, when wave height variance in the sea area is given as an input condition.

8. Acknowledgment

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