CHAPTER 282

IMPROVEMENT OF THE MOST ACCURATE LONGSHORE TRANSPORT FORMULA

by

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ABSTRACT

The ability to predict the longshore sediment transport rate accurately is essential for many coastal engineering applications. Because of the existance of a large number of existing longshore transport formulae, it is important to know which formula to use/apply. Thus, the most universally applicable formula was identified and tested against a comprehensive data set. This formula (Kamphuis formula) was also re-calibrated and guidance is given regarding its use.

INTRODUCTION

The ability to predict the time-averaged longshore sediment transport rate accurately is essential for the design of breakwaters at harbour entrances, navigation channels and their dredging requirements, beach improvement schemes incorporating groynes, detached breakwaters and beach fill as well as for the determination of the stability of inlets and estuary mouths.

Because of the large number of existing longshore transport formula it is important to know which formula to apply in practice. The aim of this paper is therefore to identify the most universally applicable formula and to test this formula against a comprehensive field data database. Finally, this formula is re-calibrated and guidance is given regarding its use.

The data considered in this paper are only for particulate (non-cohesive) sediment (including sand, gravel and shingle) being transported alongshore from the swash zone across the surf zone to deep water. Bulk (total rate calculated perpendicular to the shoreline) as well as local (at a specific point)

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point) transport rates are considered. These bulk rates include both the bedload and the suspended load. Only field data are used because laboratory investigations often contain possible scale effects and/or use regular waves. Furthermore, the ultimate aim is to be able to predict longshore transport accurately in the *field* (Komar, 1988).

It is assumed that if a longshore transport formula is capable of predicting transport rates accurately for the wide ranging data sets described herein, it can be used with reasonable confidence at similar sites to determine the long-term longshore sediment transport budget if representative wave and other input parameter data are available. It would of course be preferable to have site-specific calibration data before calculating average long-term transport rates at a specific site.

Previous studies where longshore transport formulae have been tested against data include Swart (1976), Fleming *et al.* (1986) and Kamphuis *et al.* (1986). These studies entailed a relatively small number of formulae and limited data. Schoonees (1996) evaluated 52 formulae against an extensive data base. This paper reports some of the findings of the above-mentioned study, with specific regard to the Kamphuis formula (Kamphuis, 1991).

FIELD DATA ANALYSIS

Schoonees and Theron (1993) compiled and reviewed almost all the available field data on longshore transport (as recorded up to 1993).

The data were collected at a wide variety of sites around the world, yielding a large number of data-points, of which 273 points give bulk transport rates. (This is considerably more than the 41 data points used in the Shore Protection Manual by US Army, Corps of Engineers, 1984). Included in the database are also 184 points which give local transport rates.

A point rating system was devised whereby the quality of the data could be assessed. The recording method and the accuracy thereof as well as the representativeness of the data were taken into account. It was found that this evaluation was done reasonably objectively and consistently (Schoonees and Theron, 1993).

According to this evaluation, the data sets were assigned to three categories, namely, the lower, middle and higher quality categories. Most of the data sets fell in the middle category which exhibited a very gradual increase in the overall accuracy of the data within this category. Distinguishing between short- and long-term bulk transport data yielded similar trends in the accuracy of the data. The highest score achieved in the data evaluation was only 71%, thus reflecting the difficulty of measuring longshore transport accurately.

EVALUATION OF LONGSHORE TRANSPORT FORMULAE

The method used in this study to evaluate the longshore transport formulae, was to compare the predicted longshore transport rates to the measured rates and to calculate the relative standard error of estimate (σ). (For a definition of σ, see Schoonees and Theron (1994)). The lower σ is, the better the predictions by the particular formula. In addition the residuals (e_i = measured transport rate - predicted rate) and the distribution of the discrepancy ratio (r_d = predicted/measured rate) were also determined. (The residuals were plotted against the predicted rates to check whether there are systematic trends in the residuals - these are not shown here (Schoonees, 1996)). The longshore transport formulae were also tested under as many different conditions and at as many sites as possible.

From the above-mentioned field data database, Data Set 1 containing 123 points was extracted (see Schoonees and Theron, 1994 for a full description of Data Set 1). In Data Set 1 all the parameters required for testing the transport formulae are available. This same data set was used to evaluate existing longshore transport formulae as well as a newly derived formula (Schoonees, 1996) based on the applied wave power concept.

It is important to note that the data ranges of Data Set 1 are:

0,058	<	H_{bs} (significant breaker height, m)	<	3,400
2,32	<	T_p (peak wave period, s)	<	16,60
0,30	<	θ_b (breaking wave angle, °)	<	35,00
0,007 (=1/143)	<	beach slope	<	0,138 (=1/7,2)
0,154	<	D_{50} (median grain size, mm)	<	15,000
600	<	S (longshore transport rate, m^3/year)	<	14 793 000

From the above values it is clear that the data ranges of this data set are quite wide. Most conditions encountered on natural beaches are covered and the data were collected on beaches from a variety of sites from around the world. These factors give credibility to the conclusions drawn in this comparison of predicted versus measured transport rates.

In total, 52 different longshore transport formulae were evaluated (Schoonees, 1996). These formulae were classified into different categories with regard to the theories on which they are based. The following three formulae were found to be the most accurate as tested against Data Set 1:

Order of accuracy	Name of the formula	Relative standard error of estimate (σ)	Category
1	Kamphuis (1991)	0,393	Dimensional analysis
2	Van Hijum, Pilarczyk and Chadwick (1989)	0,417	Energetics (energy flux)
3	Van der Meer (1990)	0,447	Empirical

These formulae are all bulk (total rate) as opposed to detailed predictors. Figures 1, 2 and 3 illustrate the fit of the predicted transport rates against the measured rates (log-log scales).

The formulae were also ranked according to the highest percentage of discrepancy ratios (r_d) between 0,5 and 2 (i.e. under or over prediction by a factor of 2; $r_d = 1$ indicates perfect agreement). A similar ranking of the "best" five formulae was found. However, it was found that σ provides a better way to judge the accuracy of a formula than using the percentage of r_d between 0,5 and 2. This is because, when applying a transport formula to determine a longshore transport budget at a site, a single badly predicted transport rate can distort the calculated budget greatly. At the same time, however, the above-mentioned percentage of r_d can still be very high compared to σ which would be affected greatly by a single badly predicted rate. Therefore σ is a better yardstick.

Dimensional analysis incorporating all relevant variables ensured that the Kamphuis formula contains the most important parameters. The three top formulae (Kamphuis; Van Hijum, Pilarczyk and Chadwick and Van der Meer) are relatively simple. These "simpler" methods performed well probably because a lower degree of inaccuracy is or can be introduced by having fewer (but all the most important) parameters. It is very difficult to acquire accurate input data; and the more parameters incorporated in a formula, the more input data is required (thereby potentially increasing the noise).

It is common practice to compare the predictions from different longshore transport formulae when computing the annual longshore transport regime at a site. Swart and Fleming (1980) advocated the use of a so-called package deal approach. In this approach, the highest and lowest transport rates predicted by six formulae were ignored and the median of the remaining values was determined. The question then remains whether better results can be achieved by means of this or a related method. Three approaches were tried

(Schoonees, 1996). Firstly, by considering the median of the predictions by the five best formulae; secondly, by determining the mean of the three middle values after discarding the highest and lowest predictions; and thirdly, by computing a weighted mean transport of the five predictions. The variation in the transport rates predicted by the five best formulae was investigated. It was found (Schoonees, 1996) that these predictions are reasonably consistent; that is, the individual formulae do not yield excessive outliers. It can therefore be concluded that none of the package deal approaches yield better answers than the best formula (the Kamphuis method) and as such , are not worth pursuing if the above-mentioned five best formulae are used. The reason for this probably lies in the consistency (reliability) of the five best formulae.

RECALIBRATION OF THE KAMPHUIS FORMULA

The Kamphuis formula can be written as follows:

$$\begin{aligned} S &= (31\ 557\ 600.1.3.10^{-3})\ x_{Kamphuis} \\ &= 41\ 024{,}88\ x_{Kamphuis}\ (m^3/yr) \end{aligned} \tag{1}$$

$$\begin{aligned} x_{Kamphuis} = \frac{1.}{(1-p)\ \rho_s} &\cdot (\rho/T_p)\ L_o^{1{,}25}\ H_{bs}^2\ (\tan\alpha_K)^{0{,}75} \\ &\cdot (1/D_{50})^{0{,}25} (\sin\ 2\theta_b)^{0{,}6} \end{aligned} \tag{2}$$

where p = porosity
ρ_s = density of the sediment grains
ρ = density of sea water
L_o = deep-water wavelength
$\tan\alpha_k$ = beach slope to the breaker line

See Kamphuis (1991) for a more comprehensive definition of all the parameters.

Equation (1), the original Kamphuis formula, is plotted *on linear scales* in Figure 4a and b. Note that Figure 4b shows the detail of Figure 4a for $x_{Kamphuis}$ values up to 80 (instead of 200). The 80%, 90% and 95% confidence intervals for the predicted responses of the original Kamphuis formula are also shown in Figures 4a and b. Despite the fact that the Kamphuis formula fares the best of the 52 formulae tested, it is immediately apparent that the confidence intervals are very wide. For example, at the 80% confidence level, the predicted transport rate for $x_{Kamphuis}$ = 8,7 varies between -1 290 000 m^3/year and + 2 004 000 m^3/year (predicted rate = +357 000 m^3/year) - Figure 4b. The coefficient of determination (R^2) is 0,284.

Illustrated in Figure 5 is the best-fit straight line through all the data (called $S_{Kamphuis}$ recalibrated, 1):

$$S = 88\,248 + 61\,892\, x_{Kamphuis}\ (m^3/year) \tag{3}$$

If Equation (3) is used, it is evident that the lowest transport rate that can be predicted, is 88 248 m^3/year, which is when $x_{Kamphuis} = 0$. This is clearly unacceptable, because the transport rate must be zero if $x_{Kamphuis} = 0$. Furthermore, it can be seen from Figure 4a that there are three main outliers (which fall beyond the 95% confidence limit) and that Equation (3) fits the higher transport rates better than the original Kamphuis formula. Therefore, disregarding the three main outliers and fitting the line through the origin, the following equation is found:

$$S = 75\,549\, x_{Kamphuis}\ (m^3/year) \tag{4}$$

This relationship, $S_{Kamphuis}$ recalibrated 2, is shown in Figure 5a and b (the latter Figure 5b again presents the detail of Figure 5a). Although this formula fits the high transport rates well, it over predicts significantly for $x_{Kamphuis}$ values below 10 (Figure 5b). This is caused by two influential points where transport rates higher than 3 x $10^6 m^3$/year were measured (Figures 5a and b). (Remember that the three main outliers, although shown, have not been used in this regression).

To eliminate this problem all the data points (123) were again considered to yield the third regression line, the $S_{Kamphuis}$ recalibrated, 3:

$$S = 63\,433\, x_{Kamphuis}\ (m^3/year) \tag{5}$$

This formula fits the data reasonably well over the whole range (Figures 5a and b). It gives virtually the same answers at high $x_{Kamphuis}$ values than the first recalibrated formula Equation (3). It also fits the data at lower transport rates quite well. R^2 is 0,620 and thus Equation (5) explains 62% of the variance in the data (which is a 118% improvement compared with Equation (1)). However, the standard error of estimate (σ) for this formulation is 0,405, which is slightly poorer than the 0,393 of the original Kamphuis equation. The reasons for this apparent contradiction are:

- The least squares approach (Equations (3), (4) and (5)) minimizes $\sum (S_{m,i} - S_{p,i})^2$ while the standard error of estimate (σ) uses $(\log S_{p,i} - \log S_{m,i})^2$. (Subscripts m and p denote "measured" and "predicted" respectively while i is the number of the data point).

- The effect of a few data points with high transport rates act as influential points in the least squares approach. On the other hand, the number of data points at low transport rates play an important role in the value of the standard error of estimate (σ).

To investigate the effect of such low rates on the standard error of estimate, certain data points below a cut-off transport rate were temporarily disregarded and σ re-calculated. The result was the following:

Cut-off measured transport rate (m^3/year)	Number of data points	σ for $S = 63433\ x_{Kamphuis}$ (Equation 5)	σ for the original Kamphuis formula (Equation 1)
0	123	0,405	0,393
5 000	115	0,392	0,393
10 000	106	0,368	0,380
25 000	103	0,365	0,383
50 000	87	0,324	0,377
100 000	68	0,299	0,374

These values have been plotted in Figure 6. It is clear from the above table that except when very low transport rates of less than 5 000 m^3/year are included, Equation (5) is superior to the original Kamphuis formulation. Figure 6 shows that the original Kamphuis formula is relatively insensitive to the cut-off transport rate. On the other hand, the standard error of estimate decreases significantly (to only 0,299 compared to 0,374 of the original formula, which is a 20% improvement) for Equation (5), if the cut-off transport rate increases. If a cut-off rate of 50 000 m^3/year is applied, σ reduces from 0,377 to 0,324, a 14% improvement. This finding is important because relatively few storm conditions at any site usually contribute the major part of the longshore sediment transport budget. It is therefore important that the higher transport rates are predicted accurately.

In order to obtain an indication of which wave conditions would cause such cut-off transport rates, the following typical values were chosen:
$T_p = 10$s, $\theta_b = 2°$, $D_{50} = 0{,}3$ mm and $\tan\alpha = 1/25$ (= 0,04). It was also assumed that these wave conditions will occur throughout the year.

Using the *original* Kamphuis formula, the longshore transport was then computed for a range of wave heights:

H_{bs} (m)	Longshore transport rate ($S_{Kamphuis}$) (m^3/year)
0,1	2 007
0,3	18 067
0,5	50 187
0,7	98 366

For this particular (typical) case, it is clear that relatively low wave heights of about 0,5 m and 0,7 m will already cause transport rates of about 50 000 m^3/year and 100 000 m^3/year respectively. For these cut-off rates, the standard errors of estimate are 0,299 and 0,324 respectively (Figure 6). Equation (5) is therefore judged to be "good".

Instead of using the least squares approach, the question may also be asked: What value of K will cause the minimum standard error of estimate (σ) when using *all* the data points? That is, K in:

$$S = Kx_{Kamphuis} \quad (m^3/year) \tag{6}$$

Computations with different K values yielded the following:

K	σ
30 000	0,438
41 025 (Equation (1))	0,393
45 000	0,388
48 000	0,387
49 000	0,387
50 000	0,387
55 000	0,391
64 433 (Equation (5))	0,405
70 000	0,420

These data have been plotted in Figure 7. From this figure it is evident that the minimum standard error of estimate(s) is 0,387. It can also be seen that σ is not very sensitive with regard to the value of K near the turning point: σ = 0,387 even if K varies from 48 000 to 50 000. Taking the accuracy of predicted transport rates during storms into account, the preferred (rounded off) equation is:

$$S = 50\,000\ x_{Kamphuis}\ (m^3/year) \qquad \textbf{(7)}$$
$$\text{with } R^2 = 0{,}397$$

FINAL RECOMMENDATIONS

The least squares as well as the minimum standard error of estimate approaches were used to recalibrate the Kamphuis formula, yielding Equations (5) and (7) respectively. Finally the recommended procedure for calculating longshore sediment transport is as follows:

For obtaining bulk longshore transport rates, it is recommended that **Equation (5)** be applied at sites where the significant wave heights normally exceed say, 0,3 m and where the sediment grain size is usually less than 1 mm; that is, at partially protected and exposed sites (i.e. relatively high transport rates). Only at sites where very calm conditions prevail and/or where the sediment is coarse, is **Equation (7)** expected to yield better answers. The considerably higher R^2 for Equation (5) compared with the corresponding value for Equation (7) supports the preference for Equation (5). A significant improvement of 118% according to R^2 and up to 20% in σ in the predicted transport rates was obtained.

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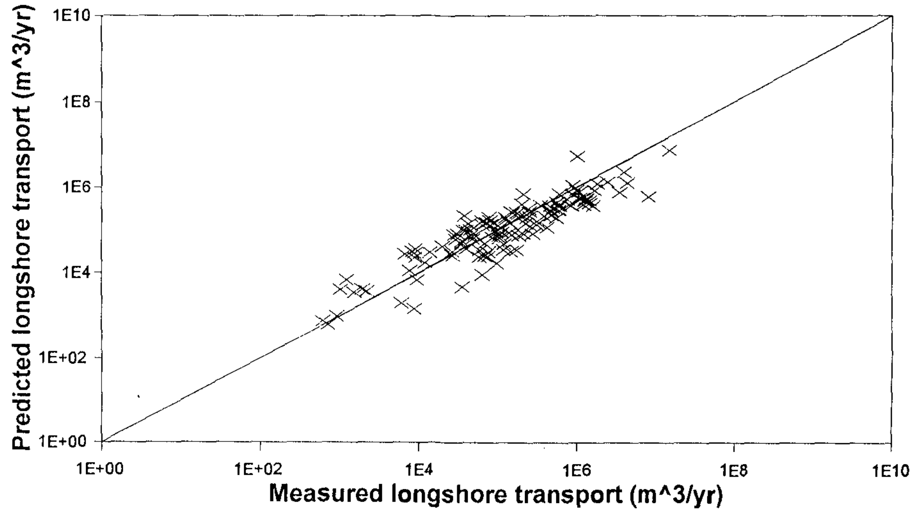

Figure 1: Predicted versus measured longshore transport rates for the Kamphuis formula

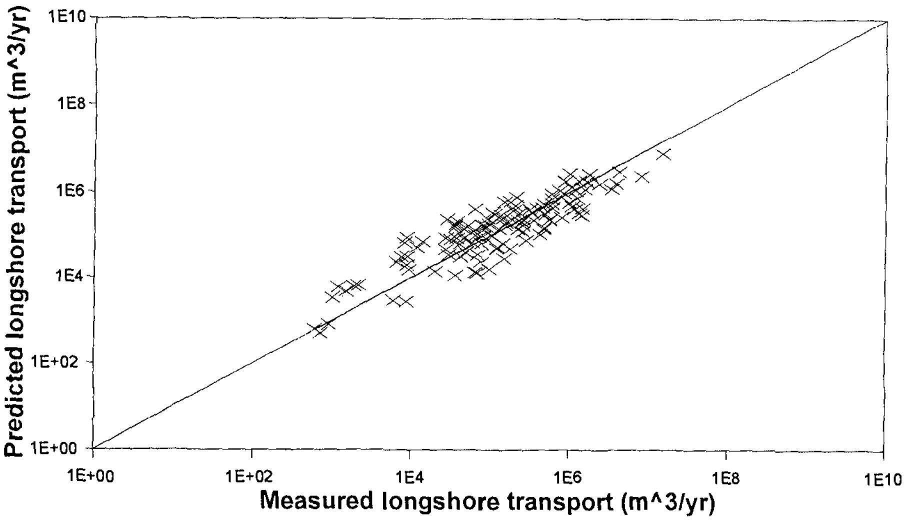

Figure 2: Predicted versus measured longshore transport rates for the Van HIjum, Pilarczyk and Chadwick formula

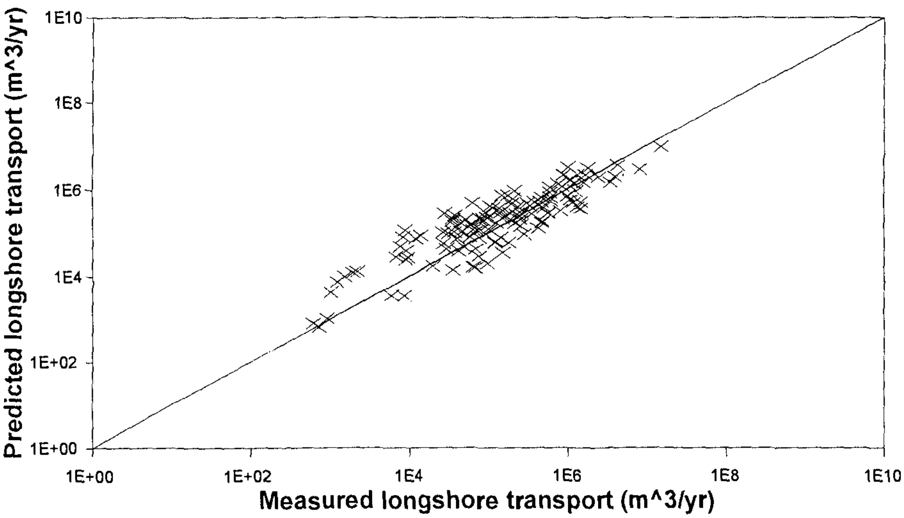

Figure 3: Predicted versus measured longshore transport rates for the Van der Meer formula

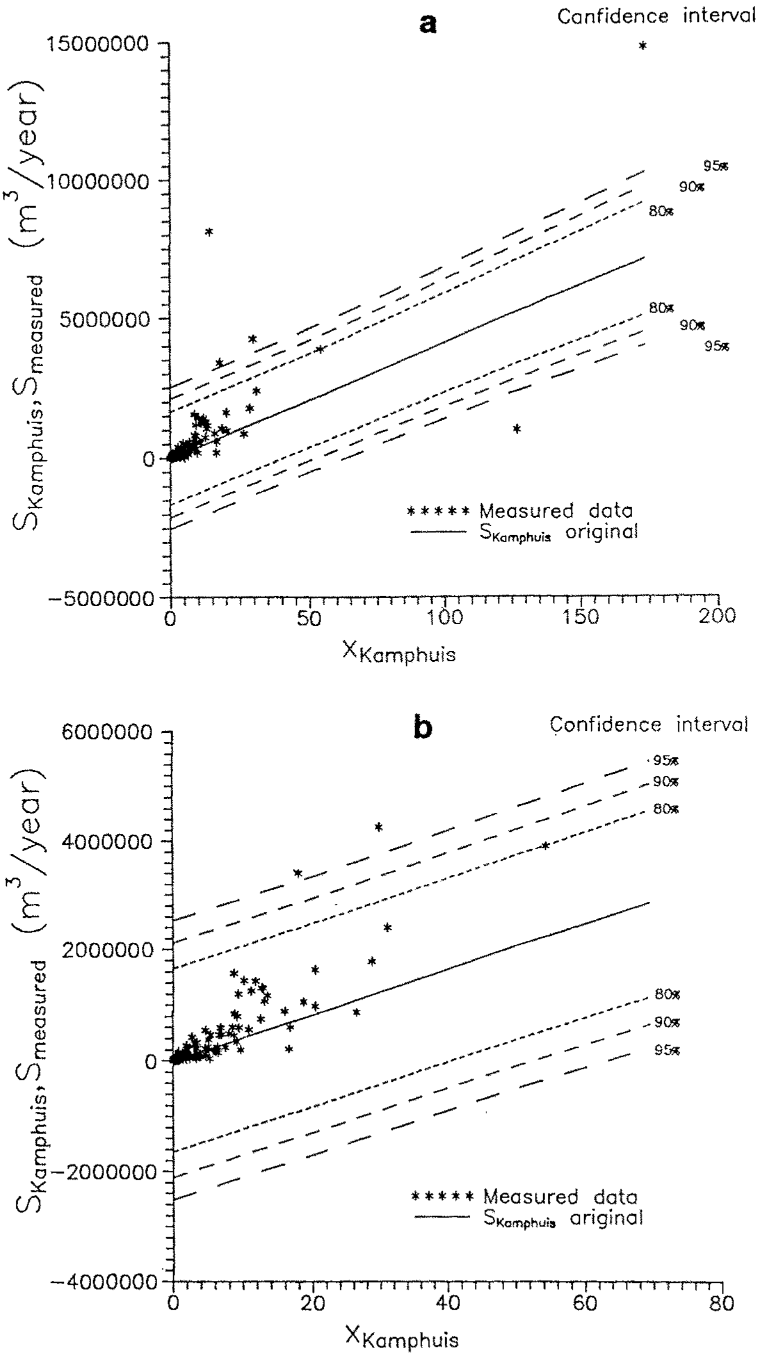

Figure 4: Confidence intervals for the original Kamphuis Formula

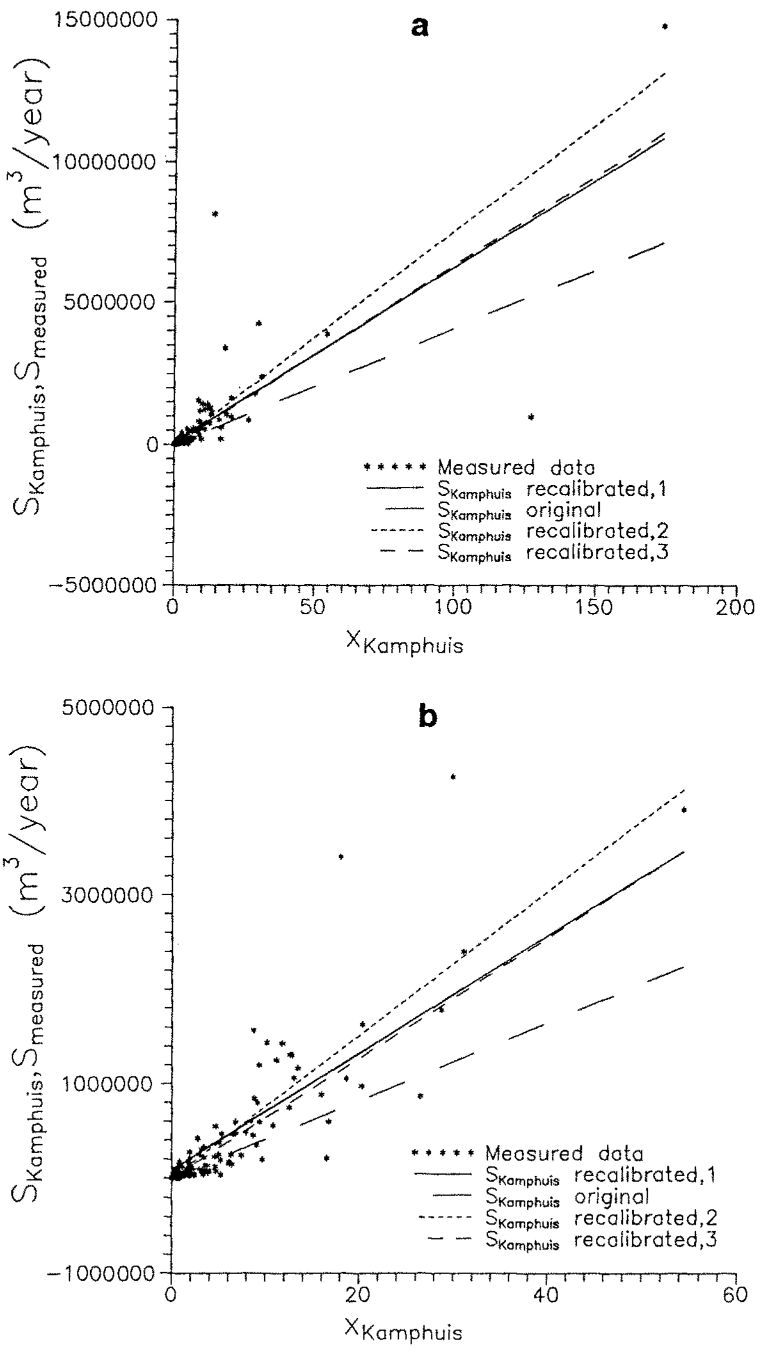

Figure 5: Recalibration of the kamphuis formula

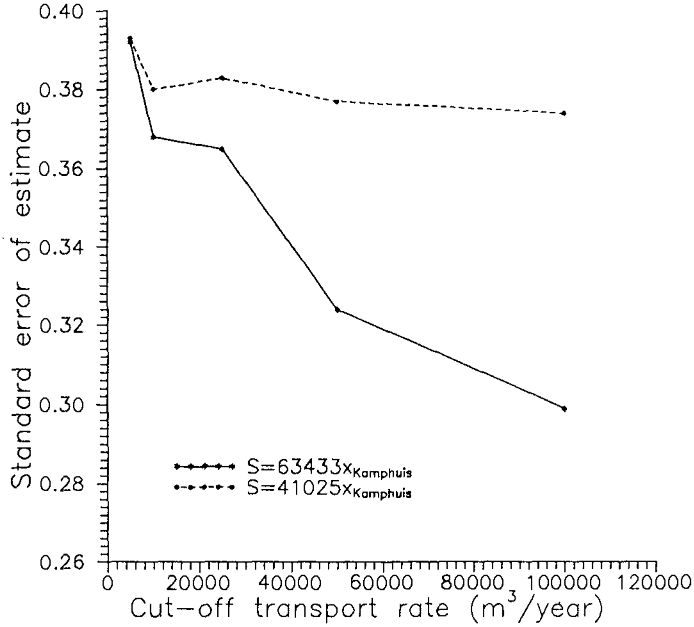

Figure 6: Standard error of the estimate for different cut-off rates

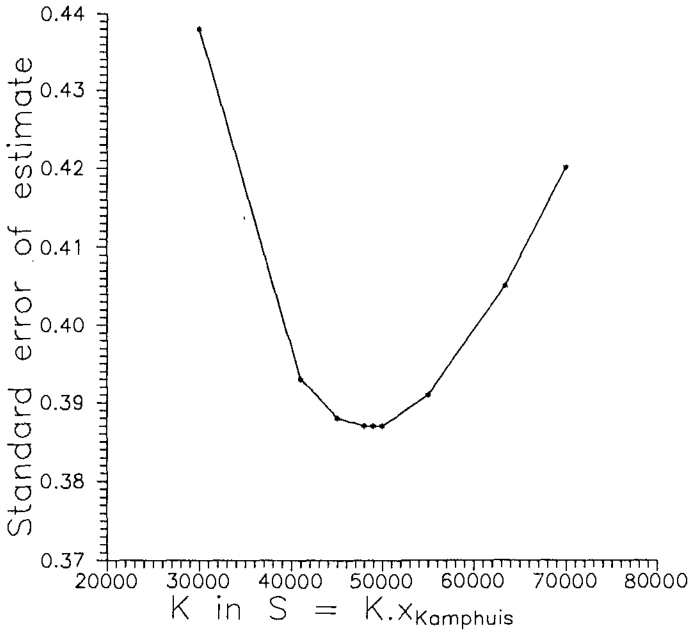

Figure 7: Minimizing the standard error of estimate for the Kamphuis formula