STABILITY OF NEAR-BED RUBBLE-MOUND STRUCTURES

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Abstract

This paper describes the experimental and theoretical analysis carried out to improve our knowledge of the stability of near-bed rubble-mound structures. Laboratory tests have been performed to improve the data base on the stability of this kind of structure. The experimental results are compared with the results of three different approaches to stability: dimensional analysis, Morison forces and forces derived from bed shear stresses. In all cases considered, the influence of the geometry of the structure has not been taken into account, and the characteristics of the flow around the structures have been obtained from linear wave theory. The analysis of the data shows that the mobility parameter, a non-dimensional form of the bed shear stress, gives the best representation of the damage in these structures. From the analysis it can also be concluded that inertia forces are not relevant in the representation of the damage using Morison forces. Based on these approaches, some formulations of damage are presented. These formulations indicate that, for the first stages of damage, damage varies almost linearly with force (Morison or bed shear stress).

Introduction.

Near-bed structures are those whose height is low compared with water depth. The depth of submergence of these structures is enough to assume that wave breaking does not affect the hydrodynamics around the structure. This definition separates them from the low-crested or submerged breakwaters. Rubble-mound, near bed structures are used in coastal engineering for the protection of other structures such as pipelines or outfalls, for scour protection, as toe or lateral beach support or for the construction of artificial reefs for marine life. Stability under wave action and currents is usually the

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design condition while other loads such as those induced by anchors or fishing nets may be of secondary importance.

In recent years, experimental and theoretical efforts have been undertaken to fill the lack of information on the stability of submerged rubble-mound structures. Much of the experimental work has been directed towards low crested or submerged breakwaters, Vidal et al. (1992), van der Meer (1993) and today, some guidelines for the design of these structures are available. In the field of near-bed structures, experimental information is scarce, Lomónaco (1994), Levit et al. (1997), and mostly dedicated to assess the transport of rubble units. Therefore, most engineers are using sediment transport formulas developed for horizontal bottom to determine the stone size.

The approaches that scientists and engineers apply to assess the damage of rubble-mound structures are based on one of the following general methodologies:

- Dimensional analysis: Using the methods of dimensional analysis, the damage is expressed as a function of some non-dimensional parameters. The experimental data are used to obtain the function that gives the better fit between the calculated and the measured damage. This approach is the most widely used in the assessment of damage of rubble-mound breakwaters. This functional will vary any time the geometry of the structure or the type of the armor units changes the hydrodynamics, gravity or interlocking forces involved in the movement of the units.

- Quasi-empirical approach: This approach makes use of the knowledge of the flow around and inside the structure which is possible due to the continuous improvement of the flow models. With a flow model and a formulation for the hydrodynamic forces (similar to those of the dynamical analysis) the quasi-empirical model is based on the assumption that the damage should be a functional of some non-dimensional parameters that represent the hydrodynamic forces acting on the units. This functional will be general for all structures with similar interlocking and gravity forces, that have not been taken into account in the force formulation.

- Dynamic analysis: This approach makes use of some analytical model of the flow around and inside the porous structure. Once the flow is known, the vector forces over the units are expressed, including the interlocking forces. This approach allows the study of the dynamics of the armour stones in real time. As a result, the deformed profile of the structure after the wave attack can be obtained. Although this approach is improving as more knowledge on the flow and interlocking forces is achieved, the state of the art does not allow its use for the design of structures.

In this paper, some work in the frame of the first and second approaches to the stability problem of near-bed rubble-mound structures, carried out at the Universidad de Cantabria is presented. First, some model experimentation has been done to improve the existing data sets on stability, paying special attention to the initial stages of damage. After that, the data analysis is compared with two different parameters for the quasi-empirical model; one based on the Morison drag forces (*drag parameter*) and the other based on a bed shear stress parameter (*mobility parameter*). These two approaches are also compared with a conventional non-dimensional approach using two parameters, the stability number and the relative submergence, that have been found to be the most relevant for the assessment of the damage.

Methodology

Detached near-bed rubble-mound structures are usually very flat, and are deployed in relatively deep waters. Typical ratios of crest height/structure width, are in the range $0.1<h_c/B<0.2$ while the ratios of crest height/water depth are in the range $0.1<h_c/h<0.5$. Waves propagate over these structures with little transformation and reflection is very low. In the case of very steep waves, the breaking takes place after the waves have overtaken the structure and the perturbation of the breaking wave does not affect the flow over the structure. This means that flow around the structure can, in a first approximation, be described by wave theory.

For the implementation of a fully quasi-empirical model of damage, a model of the flow around and in the porous structure will be necessary. But before that, an appropriate flow-force parameter should be chosen in order to relate damage with forces. If enough experimental data is obtained with only one geometry and rubble type, the geometry and porous characteristics of the rubble will not affect the results, and the suitability of different flow-force parameters could be studied using a simple wave theory to describe the flow around the structure. To do this, the next steps will be followed:

1- Stability model tests of a single near-bed rubble-mound structure using regular waves.
2- Dimensional analysis of the experimental data and formulation of damage in terms of the most relevant non-dimensional parameters.
3- Analysis of the relative importance of the drag and inertia terms in the description of the structure's measured damage. Definition of a proper flow-force parameter based on Morison-type forces.
4- Analysis of the observed damage in terms of the Morison drag parameter.
5- Definition of an appropriate bed shear stress flow-force parameter.
6- Analysis of the observed damage in terms of the bed shear stress parameter.
7- Comparison between the three different approaches to damage analysis.

Stability model tests

Experimental set-up

In the wave tank of the laboratory of the Ocean and Coastal Research Group of the Universidad de Cantabria 167 stability tests have been carried out. Regular wave trains of 230 waves have been used to test the stability of a single low-crested, rubble-mound structure. Figure 1 shows the general layout in the wave tank and the geometric properties of the structure. The structure is a detached protection, 150 cm long, 42 cm wide and 6 cm high. The crest width is 6 cm and the slopes are 3/1. The rubble is composed of uniform angular marble stones, with the following characteristics: W_{15} = 0.18 g, W_{50} = 0.23 g, W_{90} = 0.37 g, ρ_s = 2675 Kg/m^3 and n = 0.47, where ρ_s is the density of the stones and n is the porosity of the rubble. With these data, the following cube-equivalent nominal diameters can be calculated: D_{n15} = 4.1 mm, D_{n50} = 4.4 mm and D_{n90} = 5.2 mm.

The structure was installed in the bottom of the wave tank, over a rigid steel plate that can be lifted with the laboratory crane to assess the damage without draining the tank. Between the structure and the wave board there is a mild slope, 32/1, that shoals the waves. Behind the structure, a rubble beach, with a slope 15/1 dissipates the generated waves.

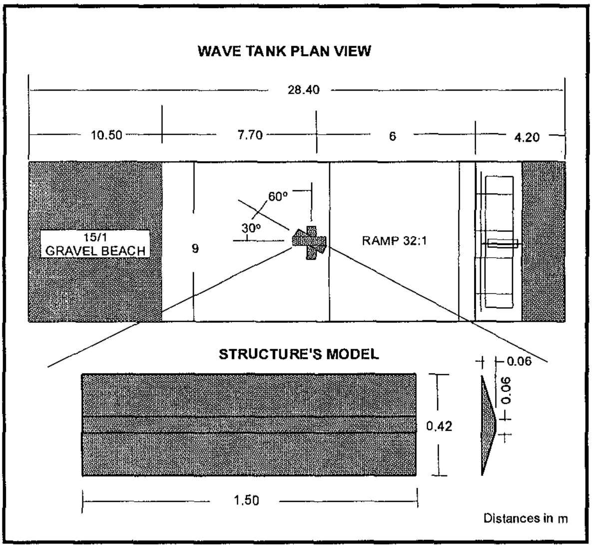

Fig- 1. General layout of the experimental work

For the experiments, three different orientations of the structure axis with respect the wave rays have been considered: 90° (normal incidence), 30° (oblique incidence) and 0° (parallel incidence). In this paper, only the results of the 62 tests carried out with normal incidence will be considered. Three water depths and three wave periods have been selected for the tests. The number of wave heights tested for each water depth and period is variable, depending on the measured damage. Free surface has been measured in four locations, three resistive wave gauges were installed in front of the structure in order to separate incident and reflected waves. Another wave gauge was installed in the rear of the structure, to measure the transmitted waves.

Test methodology.

For each test, the model was rebuilt over the steel plate, outside the water tank. The model shape was obtained using a steel plate that glided longitudinally over two rails, see Figure 2-a. A gap at the top of the plate allowed the input of more stones, if necessary. When the model was finished, no single stone of the model touched the gliding plate. With the laboratory crane, the steel plate that supported the model was lifted to its position in the wave tank. The model was moved carefully to avoid any movement of stones when the model was submerged in the water, Figure 2-b. Each test was defined by the wave incidence angle, water depth, wave period and wave height. After the wave run, the model was carefully lifted again and moved to a working table to assess the damage, which was measured counting the number of stones in the accreted area of the structure and those removed from the structure. The test area of the model was a section 50 cm long, that in the case of normal incidence was located in the center of the model. To obtain the accreted stones, the gliding plate is moved along the model. All the stones of the test area touching the gliding plate were collected and added to those stones removed from the test area.

Fig. 2- Experimental methodology. (a) Model over steel plate. (b) Submerging the model.

Test results

The wave gauge data were analyzed to obtain incident, reflected and transmitted wave trains. Due to the low h_c/h ratio, wave reflection from the structure was very low, less than 5%. For the longer periods, wave reflection from the dissipating beach was higher (about 10%) than the reflection from the structure. Conventional up-crossing analysis of the incident waves was carried out to determine the statistical properties of the waves.

The measured water depth, incident wave period and wave height as well as the number of displaced stones are presented in table 1.

Water depth (m)	**Wave period (s)**	**Measured incident wave heights** (mm)-Number of displaced stones
0.2	1.2	**54**-15 **63**-51 **90**-96 **110**-356 **143**-630
	2.0	**61**-31 **83**-55 **110**-131 **129**-301 **149**-1293
	2.8	**49**-33 **68**-65 **89**-151 **120**-391 **124**-857
0.4	1.2	**95**-21 **118**-42 **139**-117 **163**-99 **172**-109 **186**-143 **207**-196 **212**-209
	2.0	**83**-31 **100**-34 **118**-56 **135**-72 **155**-122 **175**-236 **192**-334 **222**-848 **241**-1996
	2.8	**82**-22 **97**-25 **118**-66 **132**-70 **151**-124 **178**-228 **220**-361 **242**-729 **263**-1984
0.61	1.2	**183**-59 **210**-69 **217**-52 **235**-50 **236**-62
	2.0	**116**-12 **143**-32 **159**-49 **181**-41 **201**-46 **221**-70 **250**-177 **278**-562 **290**-1457
	2.8	**113**-26 **154**-44 **190**-66 **211**-127 **236**-234 **267**-602 **334**-1814

Table 1. Measured wave parameters and number of displaced stones for normal incidence

Although the waves are regular there are always slight differences in wave heights that have to be taken into account in the statistical analysis. The average of the 100 biggest waves, H_{100}, of the incident wave train has been used as the wave height parameter for the following stability analysis.

Data analysis

As stated earlier, the objective of this research is the comparison between the suitability of two flow/force parameters (Morison forces and bed shear stress forces) to explain the observed damage. A conventional non-dimensional approach will be used as a reference.

Dimensional analysis of stability

The number of stones in the accreted area is the measured damage. In a conventional dimensional analysis, this damage should be a functional of all the flow and geometry parameters involved:

$$N = f(H, T, \theta, h, h_c, B, \alpha, D, \rho_s, \rho, \mu) \quad (1)$$

where:

- -H: Wave height
- -T: Wave period
- -θ: Angle between wave ray and the structure axis
- -h: Water depth

-h_c: Crest height of the rubble protection
-B: Width of the rubble protection
-α: Slope angle of the rubble protection
-D_{n50}: Nominal diameter corresponding to the W_{50} stones of the rubble
-ρ_s: Density of the rubble stones
-ρ: Density of the water
-μ: Dynamic viscosity of water.

In this work the geometrical characteristics of the structure will be held constant, and only normal incidence results will be used, so the expression (1) can be simplified to:

$$N = f(H, T, h, h_c, D, \rho_s, \rho, \mu) \qquad (2)$$

The measured damage is defined as the number of stones, N, counted in the accreted area. This accreted area is assumed to be the same as the eroded area, so N is also the number of stones lost in the eroded area. Once N and the mound porosity is known, the average eroded area, A_e, in a section of length X, can be estimated using the non-dimensional parameter S expressed as:

$$S = \frac{A_e}{D_{n50}} = \frac{ND_{n50}}{(1-n)X} \qquad (3)$$

It is well-known that the non-dimensional parameter that relates the stone size and density with the wave height is the stability number or Hudson numbcr:

$$N_s = \frac{H_s}{\Delta D_{n50}}; \quad where\ \Delta = \frac{\rho_s}{\rho} - 1 \qquad (4)$$

The freeboard of the structure has a great influence on the damage. Its influence can be taken into account using the non-dimensional parameter h_c/h. Other non-dimensional parameters that could be important for the damage are the relative water depth, h/L and the wave stcepness, H/L, whcre L is the wave length.

The viscosity of the water affects the flow around the stones. This influence is taken into account using the Reynolds number. In the hypothesis that model tests are such that the flow above the rubble is rough turbulent, the Reynolds number would not affect the drag over the stones and the influence of the water viscosity on the resulting damage can be neglected. After the previous statements, eq. (3) can be simplified to the following non-dimensional form:

$$S = f(N_s, \frac{h_c}{h}, \frac{h}{L}, \frac{H}{L}) \qquad (5)$$

Figure 3 shows the influence of N_s on damage. The points clearly show the exponential increase of damage with N_s. It is also clear from the figure that damage is decreased when the crest submergence, h_c/h, decreases. Although the general trend is an exponential growth of damage with N_s, if only low values of S are taken into account, S<2, the growth of S with N_s can be considered to be linear.

The influence of wave steepness and relative water depth is not that clear. Following the results shown in Figure 3, for a water depth of 0.2 m and a period of 1.2 s, damage increases slightly faster than for a period of 2.0 s. However, for a period of 2.8 s the evolution of damage is similar again to that of the former period of 1.2 s. This behavior is different for the two other water depths considered, indicating that the influence of these two parameters is small and that it cannot be easily evaluated with independent potential fits of any of them. For this reason, and as a first approach, the influence of these two parameters is discarded in the analysis, and only the two main parameters will be taken into account:

$$S = f(N_s, \frac{h_c}{h}) \quad (6)$$

Figure 3 shows also the best fit lines for the function f(), using an exponential form for the influence of N_s and a potential form for the influence of h_c/h.

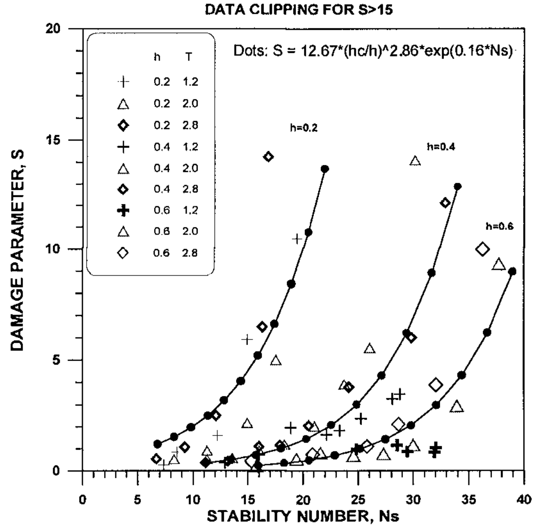

Fig. 3. Damage parameter, S, versus stability number, Ns and crest sumergence h_c/h. Experimental data and best fit lines.

Morison forces analysis

The Morison equation relates the flow properties with the parallel and normal forces generated by that flow above a solid object:

$$F_D = \frac{1}{2} \rho \, C_D D_{n50}^2 \, \bar{u}|\bar{u}| + \rho \, C_M D_{n50}^3 \, \frac{D\bar{u}}{Dt} \quad (7)$$

$$F_L = \frac{1}{2}\rho\ C_L D_{n50}^2\ \bar{u}\left|\bar{u}\right| \tag{8}$$

where $\bar{u}$ is the velocity vector and C_D, C_M and C_L are drag, inertia and lift coefficients, respectively, that depend on the shape of the solid and on the characteristics of the flow. These coefficients should be obtained experimentally.

Tørum (1994) measured forces over individual armour units of a rubble- mound breakwater laboratory model and determined the drag, inertia and lift coefficients. Among other conclusions, two are enhanced here:

- Maximum parallel (to the slope) forces are in phase with maximum velocities. This implies that maximum parallel forces are dominated by drag.
- Maximum normal forces are a combination of drag, lift and inertia forces. Their magnitude is less than half the maximum parallel forces.

In the threshold of initiation of damage, the armor units will be displaced only by the maximum forces, that are dominated by drag. If that is true, damage will only be a function of the maximum parallel velocities affecting the units. Maximum parallel forces over the crest of the rubble protection can be represented as a function of the oscillatory velocity amplitude at the crest level, U_c, by the following non-dimensional drag force parameter:

$$F_{dp} = \frac{\rho_w U_c^2}{\rho_s g D_{n50}} \tag{9}$$

where,

$$U_c = \frac{\pi H}{T}\frac{\cosh kh_c}{\sinh kh} \tag{10}$$

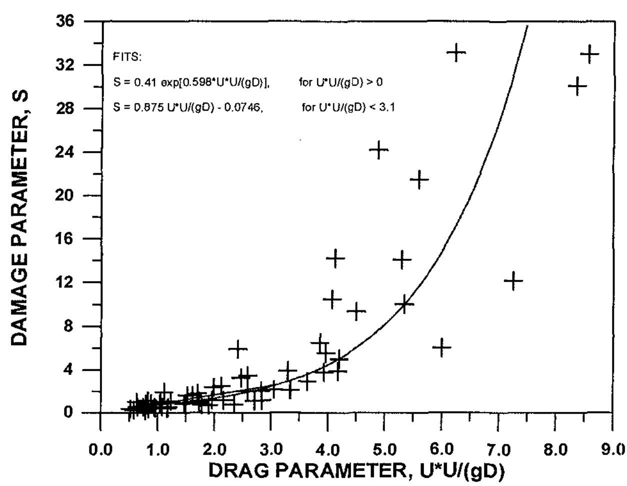

Fig. 4. Damage parameter vs. drag parameter. Data and best fit.

Figure 4 shows the damage data as a function of the drag force parameter (9) with the best exponential fit. As can be seen, the fit for low levels of damage, S<4, is fairly good, but the spread increases above that value of S, indicating that above that level, transport of stones is not well related with drag forces.

Figure 5 shows a zoom of the data below S=4. As can be seen the data trends for different periods and water depths can still be separated, indicating that there should be more flow information in the force parameter. Also shown in the figure is that, for S<2, a linear relation between the drag force parameter and damage is nearly as good as the exponential fit.

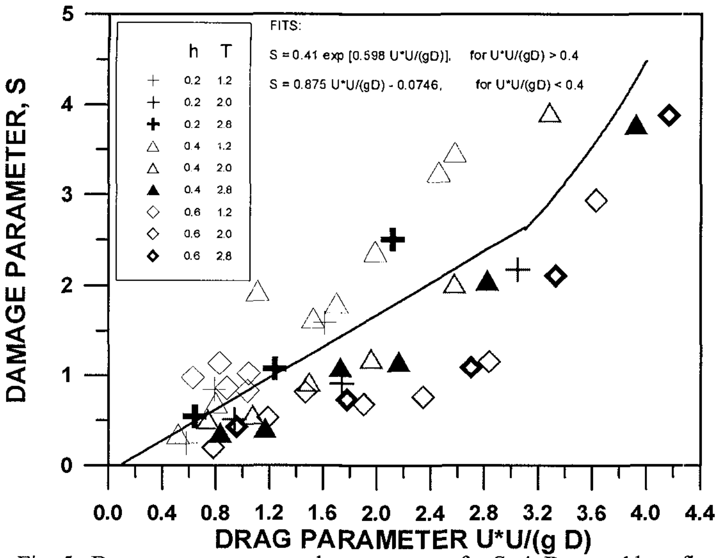

Fig. 5. Damage parameter vs. drag parameter for S<4. Data and best fit.

<u>Bed shear stress forces analysis.</u>

Formulations of sediment transport make use of the Shields parameter or mobility parameter, M_p, that relates the shear stress at the crest, τ_{cw}, with the submerged weight of the sediment grains:

$$M_p = \frac{\tau_{cw}}{\Delta g D_{n50}} \tag{11}$$

The shear stress over the crest due to the oscillatory motion of waves can be expressed as a function of the oscillatory velocity amplitude at the crest level, U_b, using the following quadratic expression:

$$\tau_{bw} = 0.5\rho f_w U_c^2 \tag{12}$$

where f_w is a wave friction factor, that can be expressed following Swart (1976) as follows:

$$f_w = \exp\left[-6 + 5.2\left(\frac{A_c}{k_s}\right)^{-0.19}\right], \quad for\ \frac{A_c}{k_s} > 1.57 \tag{13}$$

$$f_w = 3, \quad for\ \frac{A_c}{k_s} \leq 1.57 \tag{14}$$

where A_c is the amplitude of the oscillatory horizontal displacement of the water particles under the wave motion at the crest level that for linear wave theory is given by:

$$A_c = \frac{H}{2}\frac{\cosh kh_c}{\sinh kh_c} \tag{15}$$

and k_s is the bed roughness that can be related to the stone size trough the expression given by Kamphuis (1975):

$$k_s = 2D_{90} \tag{16}$$

In Figure 6 the damage results as a function of the mobility parameter is presented. Again, for damage levels S>4, the dispersion increases, indicating that the mobility parameter alone is not appropriate to explain high levels of damage, where formulations of transport could be more suitable. Figure 7 shows a zoom of the damage data in the range S<4. It can be seen that dispersion is lower than for the drag force parameter case. Again, in the first stages of damage, the evolution of damage with the mobility parameter is very linear; in particular in the range of S<1. For damages S>1, dispersion increases, giving some information about when transport starts.

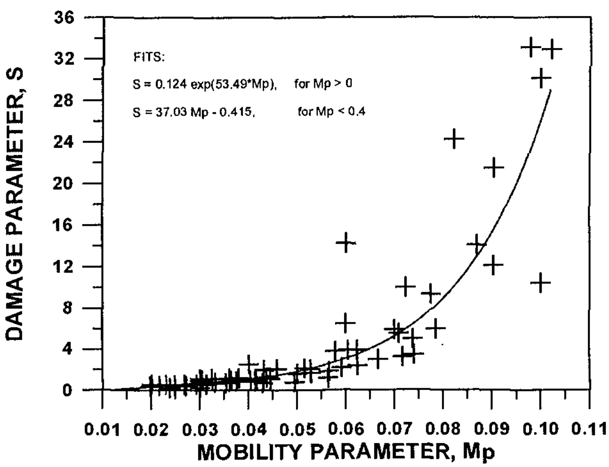

Fig. 6. Damage parameter vs. mobility parameter. Data and best fit.

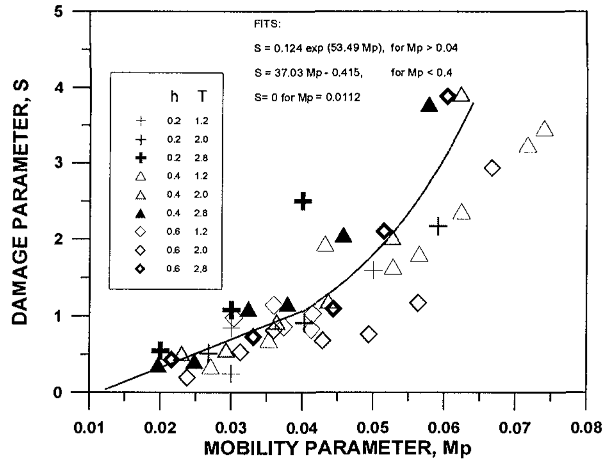

Fig. 7. Damage parameter vs. mobility parameter for S<5. Data and best fit.

Comparison between the three approaches.

Figure 8 is a plot of the measured damage parameter against the calculated one, using the three approaches and fits indicated above: non-dimensional with two parameters, drag force parameter and mobility parameter. Only damage data in the range S<4 have been taken into account. S calculated versus S measured has been also drawn in the figure. In the legend, the mean square error (*mse*) between the measured and the calculated damage is also shown. As can be seen from Figure 8, the mobility parameter gives the lowest *mse*, while the drag force parameter and the biparametric formulas give similar results for the *mse*. For S<1, the suitability of the mobility parameter is still better than the other approaches.

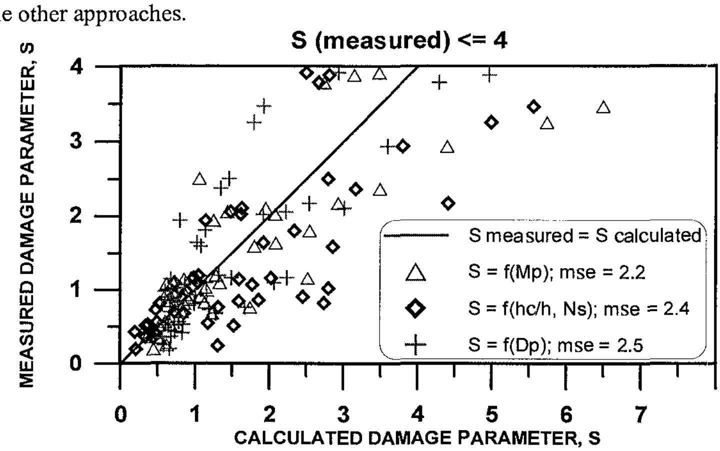

Fig. 8. Measured vs. calculated damage parameter.

The sharp increase of dispersion between the calculated and the observed damage parameters could be an indication of a change in the transport modes: for S<1 the stone jumps or rolls once until a more stable position in the mound is achieved. In this case, only the least stable units are affected and the number of waves (regular waves) should not influence the measured damage. For 1<S<2, units can be moved many times until they find a stable position on the rubble. In that case the number of waves necessary for the stabilization of damage increases with the damage level but the damage continues its linear increase with the mobility parameter. If the total number of waves is maintained, dispersion increases. For S>2 stones are not able to find a stable position in the rubble and can be transported away. In that case, damage never stabilizes and depends on the number of waves. As the transport is not well described by the mobility parameter alone, dispersion increases again. The change from linear to exponential in the relation between the damage and the mobility parameter is a clear indication that new parameters should be added to describe the influence of the transported stones.

Engineers are not comfortable with the idea of having the stones of their protection rolling away (and perhaps appearing in the nearby beach). For that reason, it seems reasonable to design these structures for damages in the S<1 region.

Conclusions.

The results of the analysis show that the approach considering the forces generated by the bed shear stresses gives a better agreement between the observed and the calculated damage than the conventional approach using Morison forces on the units. If dimensional formulas of damage are restricted to a maximum of two parameters, the more relevant parameters for damage are the stability number and the relative submergence of the structure. In that case, the bed shear stress gives a better fit to damage than the dimensional approach.

In the first stages of damage, for damage parameters in the range S<1, the evolution of damage with the mobility parameter is very linear. In the range 1<S<2, the evolution of damage with the mobility parameter is also linear, but dispersion increases. In the range S>2, the evolution of damage with the mobility parameter is exponential, with dispersion increasing as the mobility parameter increases.

The sharp increase of dispersion between the calculated and the observed damage parameters could be an indication of a change in the transport modes: for S<1 the stone jumps or rolls once until a more stable position in the mound is achieved. In that case, only the least stable units are affected and the number of waves (regular waves) should not influence the measured damage after the first tenths of waves have stabilized the damage. For 1<S<2, units can be moved many times until they find a stable position on the rubble and the number of waves necessary for the stabilization of the damage increases with the damage level. If the total number of waves is maintained, dispersion is increased. For S>2 stones are not able to find a stable position in the rubble and can be transported away. In that case, damage never stabilizes and depends on the number of waves. As the transport is not well described by the mobility parameter alone, dispersion increases again. The change from linear to exponential in the relation between the damage and the mobility parameter is a clear indication that new parameters should be added to describe the transported stones.

It can be concluded that it seems reasonable to design near-bed rubble mound structures considering damages in the S<1 region.

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References

Kamphuis, J.W., (1975). Friction factors under oscillatory waves. Proc. A.S.C.E. J. Waterw. Harbors Coastal Eng. Div. 101, (WW2): 135-144.

Levit, M., van Gent, M.R.A. and Massie, W.W, (1997). Stability of pipeline covers under waves and currents. Proc. of the 8th Int. Conf. on the Behaviour of Offshore Structures (BOSS'97). Vol. 1 pp. 195-210.

Lomónaco, P. (1994). Design of rock cover for underwater pipelines. Master of Science Thesis HH 197. International Institute for Infrastructural, Hydraulic and Environmental Engineering. Delft.

van der Meer, J.W. (1993). Conceptual design of rubble mound breakwaters. Delft Hydraulics Publication, no. 483.

Swart, D.H. (1976). Coastal sediment transport. Computation of longshore transport. Delft Hydraulics Laboratory. Rep. R968, Part I.

Tørum, A. (1993). Wave-induced forces on armour unit on berm breakwaters. J. of Waterw., Port, Coastal and Ocean Eng., Vol 120, No. 3, May/June, 1994, pp251-267.

Vidal, C., Losada, M.A., Medina, R., Mansard, E.P.D and Gómez-Pina, G. (1992). A universal analysis for the stability of both low-crested and submerged breakwaters. Proc. 23th Int. Conf. on Coastal Eng. ASCE. Vol. 2, pp. 1679-1692.