THE IMPORTANCE OF LONG WAVE REFLECTIONS IN TIDAL MODELLING ON A CONTINENTAL SHELF

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This paper investigates the tidal phase error found in a 2-D hydrodynamic model of the Southern Bight of the European continental shelf; an error also found in other models of the region. After identifying the predominant mechanisms controlling the tidal flow in the Southern Bight: bathymetry, bottom friction and coastline reflections, analysis of each mechanism is made. Sensitivity to bathymetry is tested, to bed friction using values based on physical bed properties and to coastline reflection of tidal waves by comparing standing/propagating wave analysis from model results with observations. It was found that using more physically realistic friction values improved the prediction of tidal amplitudes but had minimal effect on tidal phase and an unrealistic reduction in the bathymetry level would be required to correct the phase error. However analysis of tidal reflections revealed that the model under-predicts the amplitude of the reflected wave which may cause the erroneous phase pattern. It is proposed that the reflectiveness of the coastline has increased over time due to the cumulative effect of coastal engineering such as sea walls and barrages reducing energy dissipation. It is concluded that more attention needs to be given to the representation of coastlines in models; something which is currently neglected in model calibrations which focus on changing the bathymetry, bottom friction and boundary conditions to achieve the best model fit. It is suggested that the modelling of this process needs to be improved, especially when calculating long-term sediment transport from the hydrodynamic predictions where residual velocities are important and of small magnitude relative to velocity amplitudes.

Keywords: tidal modelling; reflections; phase error; Southern Bight

1. INTRODUCTION

This study results from unexplained discrepancies between model tidal phase and tide gauge measurements in the Southern Bight. Here a TELEMAC-2D (depth-averaged) model of the European continental shelf is investigated. This model initially showed an error of up to two hours in the timing of high tide. A phase error (of varying magnitude) is consistently found in depth-averaged and 3D models of varying types (Bourban 2012, Heemink 2002, Cazenave 2012) when modelling this region. The aim of this paper is to find the cause of the tidal phase error. The effect of tidal wave reflections on the phase error is considered, something usually neglected in the calibration of shelf sea models.

More generally, this work is part of the Integrating Coastal Sediment Systems (iCOASST) project (Nicholls et al. 2012) which aims to improve predictions of coastline evolution. The continental shelf model was set up to model fine sediment transport which moves over large distances. The aim is to improve coastal management by informing decision makers of the large scale impact of their local options: for example the impact of protecting eroding soft cliffs that may provide a sediment source for a coastal feature further along the coast. The coastal area modelling component of the project aims to determine large-scale residual sediment pathways over the next century and provide boundary forcing for behavioural smaller scale landform models. Hence the aim is to provide hydrodynamic modelling so that accurate residual sediment pathways can be calculated with sediment fluxes determined through sediment transport formulae.

In section 2 the model setup and validation are described followed in section 3 by a summary of the processes controlling tides in the North Sea. Section 4 gives the results of the model sensitivity to bathymetry and bottom friction while section 5 explores the prediction of the reflections. The cause of the phase error and its implications for residual sediment transport modelling are discussed in section 6 and conclusions drawn in section 7.

2. MODELLING

2.1 Model Set Up

This study uses TELEMAC-2D (www.opentelemac.org) a depth-averaged free-surface flow model that solves the Saint-Venant equations using the finite-element method and a computational mesh of triangular elements.

To investigate fine sediment transport a model of the European Shelf was set up; the limit of the continental shelf being roughly defined as the 200 m depth contour. The mesh has variable resolution ranging from 1km at the UK coast to 35 km offshore. The bathymetry is a combination of SeaZone data around the UK (180m resolution) and data from the National Oceanography Centre (NOC)

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elsewhere. The model domain and bathymetry are shown in Figure 1. The offshore boundary is forced by elevation and current time series synthesised at each computational boundary point from 15 tidal constituents: Q_1, O_1, P_1, S_1, K_1, $2N_2$, μ_2, N_2, ν_2, M_2, L_2, T_2, S_2, K_2 and M_4 provided by the NOC. The Thompson method (Thompson 1987) is used at the offshore boundary to minimise artificial reflections. The model does not include atmospheric forcing or wind waves. The constant viscosity turbulence model was used and the method of characteristics for advection of velocities. The model includes tidal forcing within the domain. A uniform value of bottom friction was applied using the Nikuradse formulation. The bottom friction parameter was the only calibrated parameter; its value was chosen to obtain the best fit between model elevations and January 2006 observations from the 44 tide gauges in the UK tide gauge network (National Tidal and Sea Level Facility, NTSLF) .

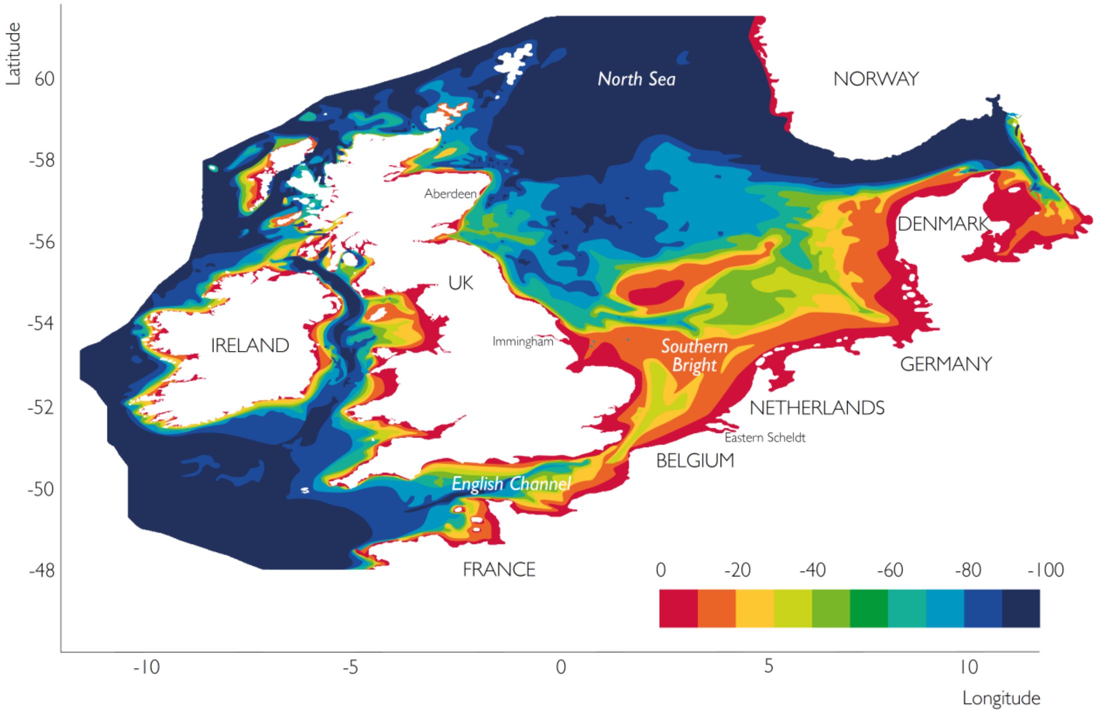


Figure 1. European shelf model domain and bathymetry (meters), values in dark blue are less than or equal to -100m.

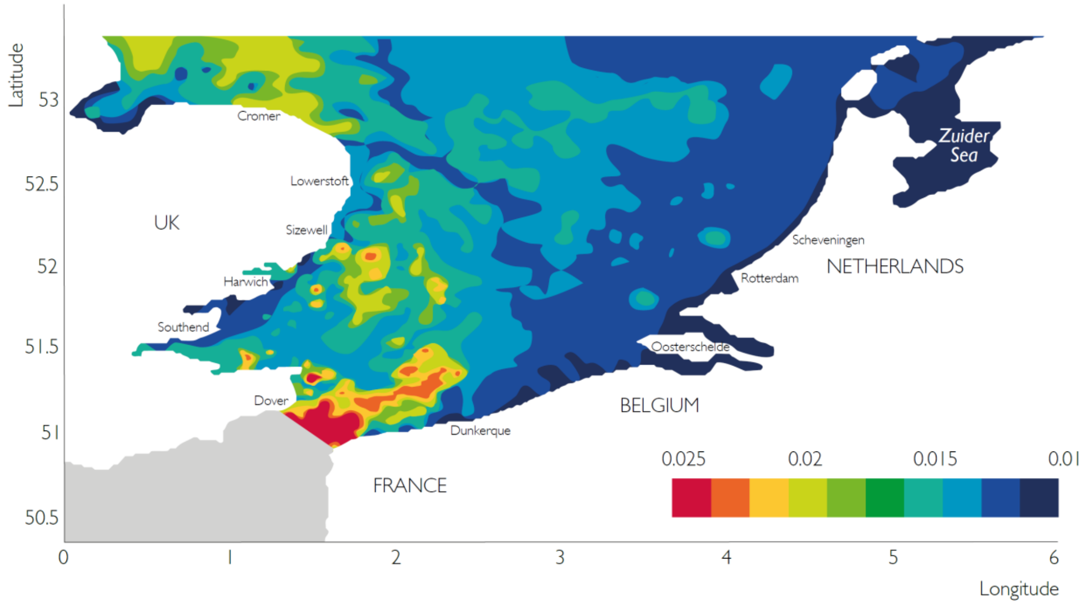


Figure 2. Southern Bight domain and Manning's bottom friction coefficients.

A model of the Southern Bight was used to perform sensitivity testing. The model set up is the same as the shelf model unless stated. The boundary is forced by time series of elevations and currents from the NOC CS3 model (http://noc.ac.uk/f/content/using-science/NOC_ModelDetails_CS3_CS3D.pdf) which includes meteorological surge. The CS3 boundary values of elevation are a good match to the nearest UK tide gauges (Cromer in the north and Dover in the south). The Elder turbulence model is used. Two parameterisations of bottom friction were applied: the first is identical to the shelf model and the second used the Manning's formulation with a variable bottom friction coefficient based on sediment and bedform properties as derived by Cazenave, 2012. The Southern Bight domain and variable bottom friction coefficients are shown in Figure 2.

2.2 Model Validation

Model validation was in the form of time series and harmonic component comparisons. The observational time series are from 2008, whereas the harmonic constants are calculated from data collected between 1971 and 2008, hence it has been assumed that the change in harmonic constants over time is small. Coastal tide gauges from the UK Tide Gauge Network, France and the Netherlands were used alongside offshore sea floor pressure series supplied by the British Oceanographic Data Centre (BODC).

Initial model validation was performed by time series comparisons with UK tide gauges. Tidal amplitudes were well replicated however the model showed a phase error in the North Sea. The model tide is slightly out of phase at Aberdeen and the phase error grows as the tide propagates south down the North Sea finally becoming greatest at Lowestoft and Harwich where model high tide occurs two hours before the observed high tide. Several other models (depth-averaged and 3D) display a phase error of varying magnitude in the Southern Bight (Bourban 2012, Heemink 2002, Cazenave 2012). To investigate the problem further a tidal harmonic analysis was performed and the model was compared to observations across the shelf; the results for the principal component M_2 are shown in Figure 3.

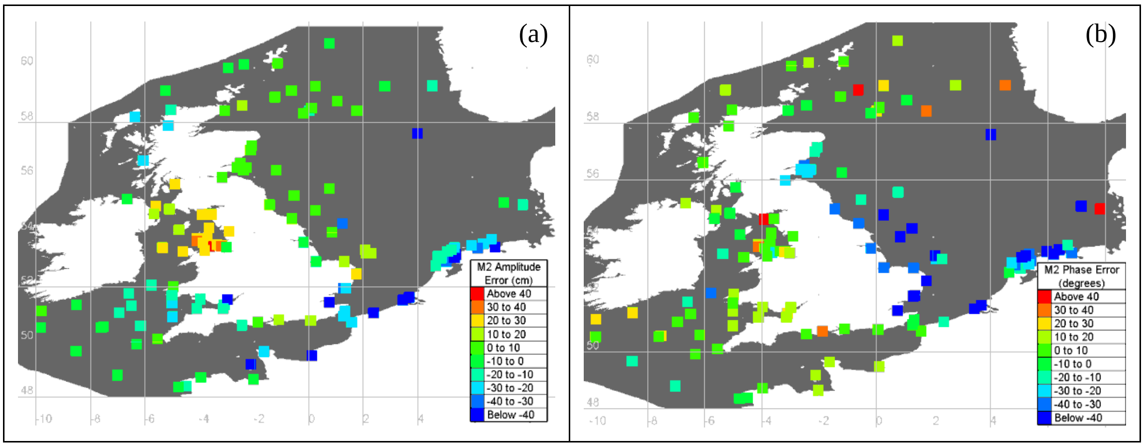


Figure 3. M_2 Errors (Model - Observations).

Figure 3 (a) shows that the model M_2 amplitudes are generally well predicted, although they are overestimated in the eastern Irish Sea (still within 10% of the tidal range) and underestimated in the Southern Bight, especially along the Dutch coast. Figure 3 (b) clearly shows the M_2 phase error along both coastlines of the Southern Bight. In Figure 4 the model M_2 co-amplitude and co-phase are plotted alongside the United Kingdom Hydrographic Office (UKHO) chart 5058 displaying observed mean high water interval and mean spring range. The UKHO chart is based on tidal observations acquired from offshore tide gauge measurements (bottom-mounted pressure gauges); the chart is based largely on historic observations and was last updated in 1996. It should be noted that the UKHO chart tells users to ignore data in the eastern Southern Bight, where the tidal curve is known to be distorted. From Figure 4 it is clear that the model amphidromic point in the Southern Bight is incorrectly positioned: it is too far east and south. Compared to the UKHO observations all three of the North Sea amphidromic points are located too far east, with the northern most amphidromic point being situated on land in the model. The mechanisms controlling the amphidromic pattern in the Southern Bight are discussed in section 3 and in sections 4 & 5 this theory is applied to show how the model can be improved.

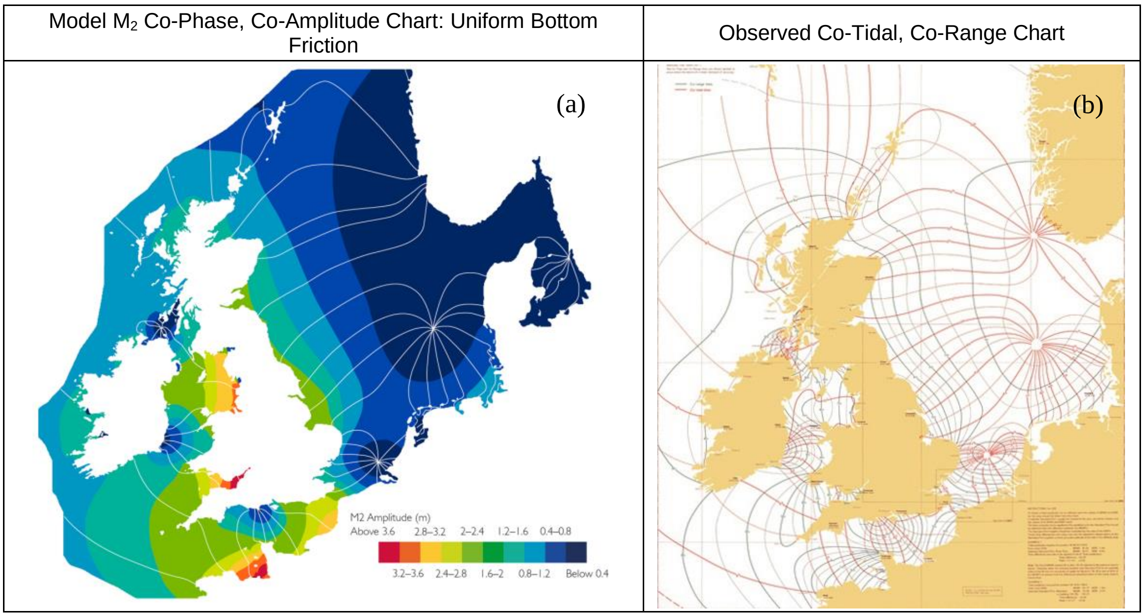


Figure 4. M_2 Co-phase and Co-amplitude model results (a). Observed Co-tidal and co-range chart 5058 (b) © Crown Copyright; reproduced by permission of the Controller of Her Majesty's Stationery Office and the UK Hydrographic Office (www.ukho.gov.uk).

The performance of the Southern Bight model under various setups is discussed in detail in section 4.

3. NORTH SEA TIDES

North Sea tides are dominated by the M_2 harmonic which enters from the North Atlantic and is reflected off the coast as it propagates southwards. This paper will concentrate on the Southern Bight where the tidal pattern has a single amphidromic point, around which the tides rotate anticlockwise. Figure 4 (b) shows the observed co-range and co-tidal pattern.

The tidal wave can be expressed as a Kelvin wave which after entering from the North Atlantic propagates down the UK coast before being reflected at the end of the basin and then travelling north along the continental European coast. Taylor (1921) gives a simplified model of tides in the North Sea by considering the reflection of a Kelvin wave from the closed end of a rotating channel which is infinite in one direction. The amphidromic point, nearest to the reflecting coastline is located a quarter of a wavelength away and since the tidal wave travels as a shallow water wave, ignoring the role of friction, the amphidromic point is located a distance L from the reflecting coastline

$$L = \frac{\sqrt{gh}}{4}T \qquad (1)$$

where g is gravity, h is the water depth and T is the period of the tidal component. Concentrating on the principal harmonic M_2, the period is constant and hence it can be seen that the latitude of the amphidromic point (because in the Southern Bight the reflecting coast is to the south) is controlled by the bathymetry.

Hendershott and Speranza (1971) expanded on Taylor's problem by considering a dissipative coastline and showed that the weaker the reflection from the coastline (i.e. the more wave energy is dissipated at the coast) the further the amphidromic points move towards the wall guiding the reflected wave. So in the Southern Bight the strength of the reflection affects the longitude of the amphidromic point. Taylor's problem was studied again by Rienecker & Teubner (1980) who showed that the symmetry of the co-amplitude lines is lost as the value of bottom friction is increased. With increasing bottom friction the along channel spacing of the amphidromic points is little affected, but cross channel the amphidromic points move towards the wall guiding the reflected wave. Unlike the scenario of an energy-absorbing barrier where the cross channel movement of all amphidromes is the same, because the presence of bottom friction causes the reflected Kelvin wave to decay up-channel, amphidromes move closer to the wall with increasing distance from the reflecting boundary.

The assumption of a closed basin is removed in the work of Brown (1987) which considers an oscillating boundary such as found in the Southern Bight in the form of the English Channel, the presence of which also influences the location of the amphidromic point by potentially over or under reflecting the Kelvin wave.

To summarise, tides in the Southern Bight rotate anticlockwise around one amphidromic point; the latitude of the amphidromic point is principally controlled by the bathymetry while its longitude is principally controlled by the bottom friction and the strength of tidal reflections. The initial model validation against UK observations showed the largest phase error at the Lowestoft and Harwich tide gauges. Importantly the co-phase lines are especially close near Lowestoft and Harwich and hence a small error in the amphidromic pattern may lead to large errors in the phase at these locations.

An ensemble of models using different numerical techniques all produce a phase error in the Southern Bight hence it is assumed that the error is not caused by the numerical formulations. This paper will consider how the errors are affected by the three controlling mechanisms identified above: bathymetry, bottom friction and coastline reflection.

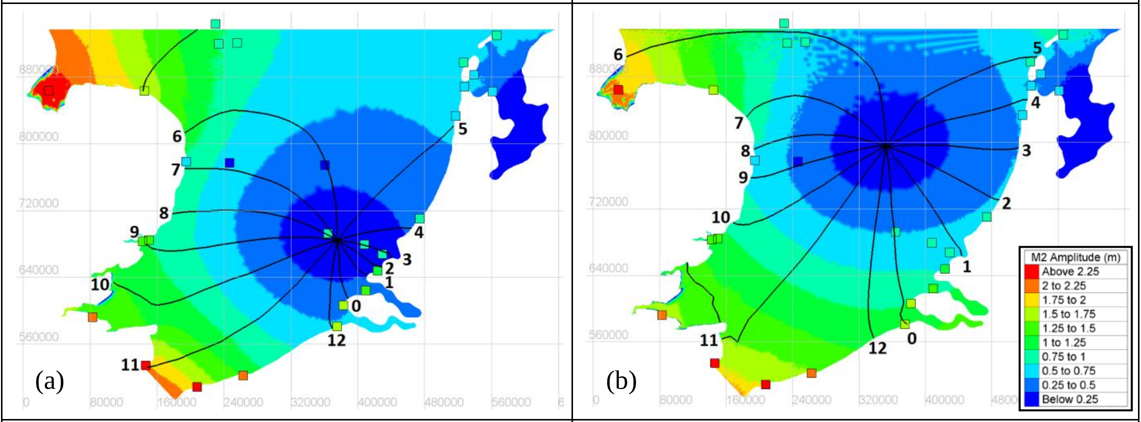
4. BATHYMETRY AND BOTTOM FRICTION

As described in section 3 the amphidromic pattern is dependent on the strength of reflections from the coastline, the value of bottom friction and the bathymetry. To test the relative importance of each of these processes a model of the Southern Bight was set up as described in section 2.1. Figure 5 (a) shows the original M_2 co-phase and co-amplitude results for the shelf model mapped on to the Southern Bight domain for comparison. Figure 5 (b) shows the M_2 co-phase and co-amplitude results for the Southern Bight model using the same coastline, bottom friction value and bathymetry as the original shelf model. The amphidromic point is further north in the Southern Bight model and the amphidromic pattern is less asymmetric (due to the different boundary conditions). In Figure 5 the observed M_2 amplitudes are plotted in black squares. Along the east coast the shelf model greatly under-predicts the amplitude, this is also evident in Figure 6 (b) and (d). In the Southern Bight model the M_2 amplitude is well replicated along the Dutch coast south of Rotterdam but again under-predicted north of Rotterdam. Both models under-predict the amplitudes in the south of the domain, more so in the Southern Bight model. It can be seen from Figure 6 that phase is more accurately predicted in the Southern Bight model than in the shelf model, however the Southern Bight model still exhibits a phase error in the south-west and east of the domain.

Initially the bottom friction value used in the shelf model was chosen to calibrate the model amplitudes in comparison to measurements at UK tide gauges and as such was not constrained to be physically realistic; the value chosen was very high. Given that all three amphidromic points in the North Sea are located too far east it is likely that the bottom friction and or model reflections are incorrect. To quantify the effect of the bottom friction choice the Southern Bight model was run with a spatially varying value of bottom friction based on the physical properties of the bed (described in section 2.1). The results in the form of M_2 co-phase and co-amplitude plots are shown in Figure 5 (c).

Shelf Model Results Mapped to Southern Bight domain, Uniform Bottom Friction

Southern Bight Model with Uniform Bottom Friction



Variable Bottom Friction

Variable Bottom Friction and Bathymetry * 0.7

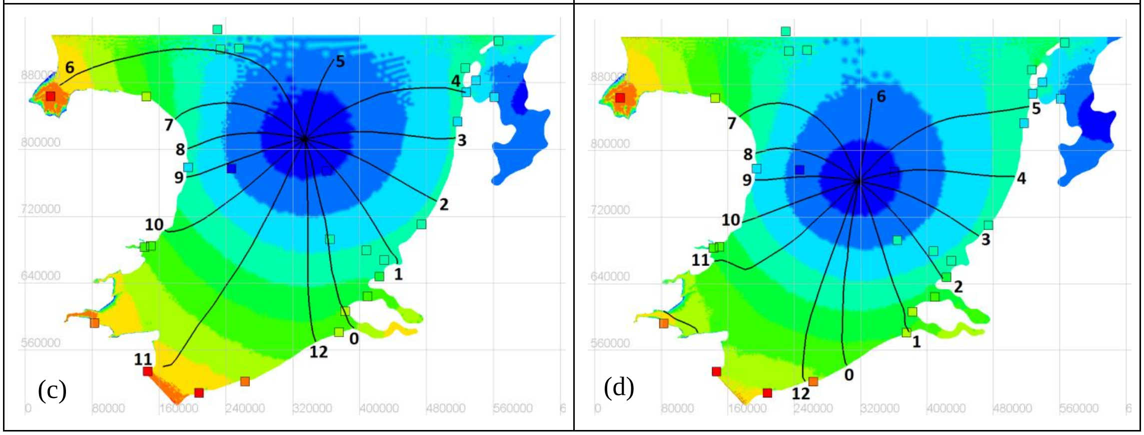


Figure 5: M2 co-phase (black lines) and co-amplitude (colours) patterns for different bathymetries and values of bottom friction. The colours in black boxes are the M2 amplitude from the tide gauge measurements at that location.

Reducing the bottom friction to more physically plausible values causes the amphidromic point to move north and west, as also found by Pelling (2013). It makes the amplitude pattern more symmetric and increases amplitudes in the south of the domain, bringing model amplitude results closer to observations, particularly along the east coast of the domain. It changes the tidal phase in the north east corner of the domain however reducing the bottom friction has minimal effect on the phase in the rest of the Southern Bight as illustrated in Figure 5 and Figure 6.

Our knowledge of the true bathymetry of the Southern Bight is incomplete. The bathymetry used in this study is a composite of survey data from different times, some of which may be very old due to a lack of more recent surveys and, as with any data set, there are errors associated with the measurement techniques as well as with the interpolation of irregularly spaced data. Further errors may have been introduced in the interpolation of the bathymetry data to the model mesh. Some calibrations incorporate bathymetric uncertainty (Verboom 1991; Cea and French 2012), allowing the bathymetry to be altered within its range of uncertainty. To investigate if bathymetric errors are a likely cause of the phase error a sensitivity test was performed. The variable bottom friction values were used as these are based on physical properties of the seabed and the model bathymetry was changed uniformly for simplicity. The best fit to the observations was found by reducing the water depth by 30%: the M_2 co-phase and co-amplitude results for this scenario are shown in Figure 5 (d). Reducing the water depth moves the amphidromic point south and west minimising the average phase error at the tide gauges but also increasing the M_2 amplitude under-prediction in the south of the domain. Whilst the average phase error at the tide gauges is minimised it can be seen that compared to observations the co-phase lines shown in Figure 5 (d) are too evenly spaced in the observations lines 2-6 are closer together as are lines 7-11. A 30% reduction is far too large to be accounted for by bathymetric errors and hence it is concluded that the bathymetry alone cannot be the cause of the phase error.

Figure 6 compares model results to observations at six locations around the coast of the Southern Bight. It should be noted that the observations include surge but the shelf model does not and the Southern Bight models only propagate surge from the boundary conditions: for this reason results are displayed for midday 21st to 23rd September 2008 when absolute residuals at Cromer, Lowestoft and Harwich are all below 20 cm.

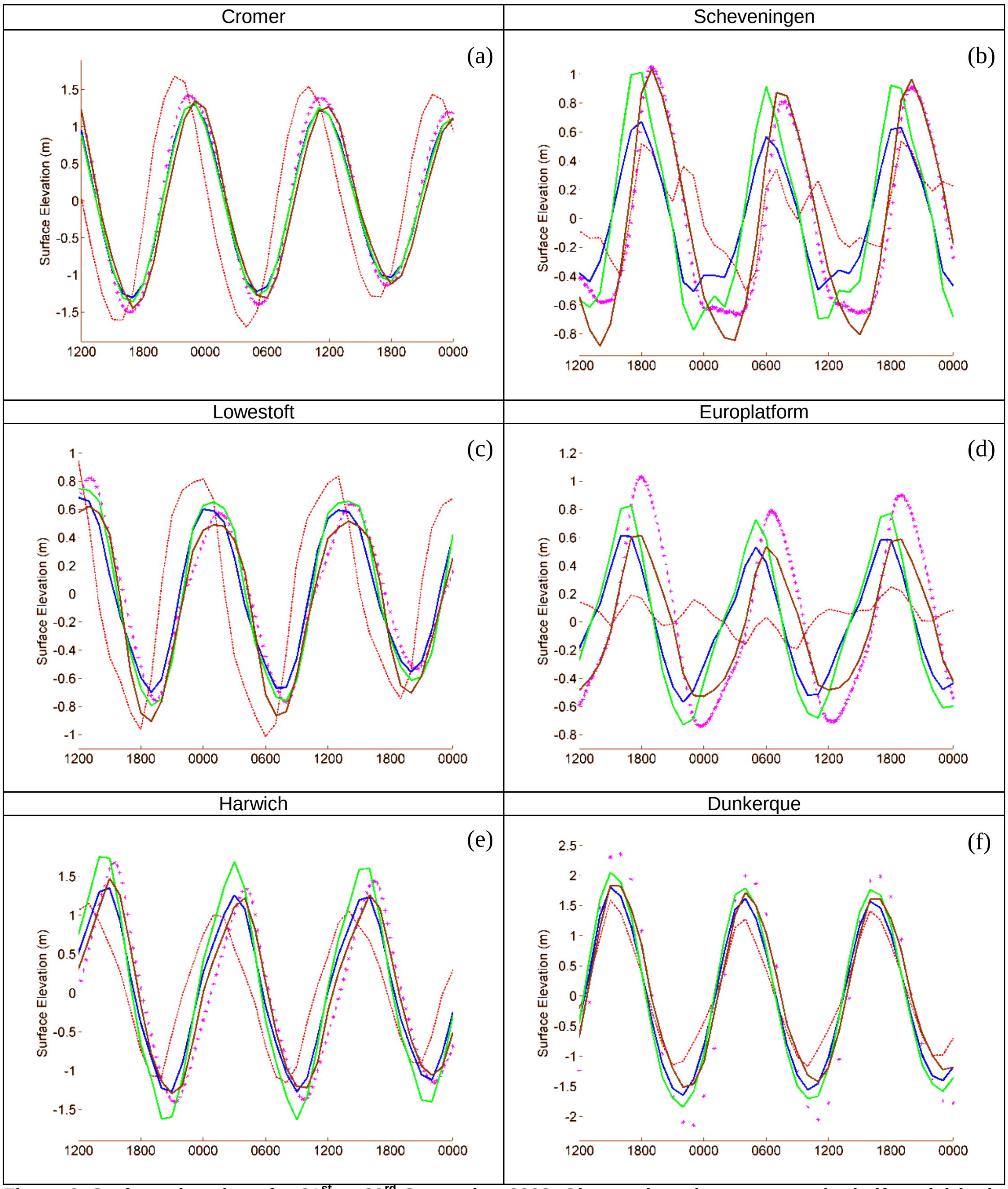


Figure 6. Surface elevations for 21st to 23rd September 2008. Observations (magenta, star), shelf model (red, dashed), uniform bottom friction (blue), variable bottom friction (green) and bathymetry*0.7 (brown).

5. REFLECTIONS

To investigate if the phase error was caused by incorrect model reflections a standing/ propagating wave analysis was performed on observations and in the model at the corresponding locations. Three observation time series covering 27th September to 16th November 2008 from near Sizewell on the UK Suffolk coast were made available by EDF via CEFAS. The objective of the standing/ propagating wave analysis was to quantify the ratio of reflected to incident wave amplitude so the model reflection could be compared to the observed reflection. Firstly a harmonic analysis was performed on the elevation and current time series and then only the M_2 wave is considered. The M_2 wave was separated into the standing and propagating wave components using the method of Pugh and Vassie (1976). The technique separates the components by utilising the fact currents and elevations are in phase for a progressive wave and 90^0 out of phase for standing waves. The results of the analysis are shown in Figure 7.

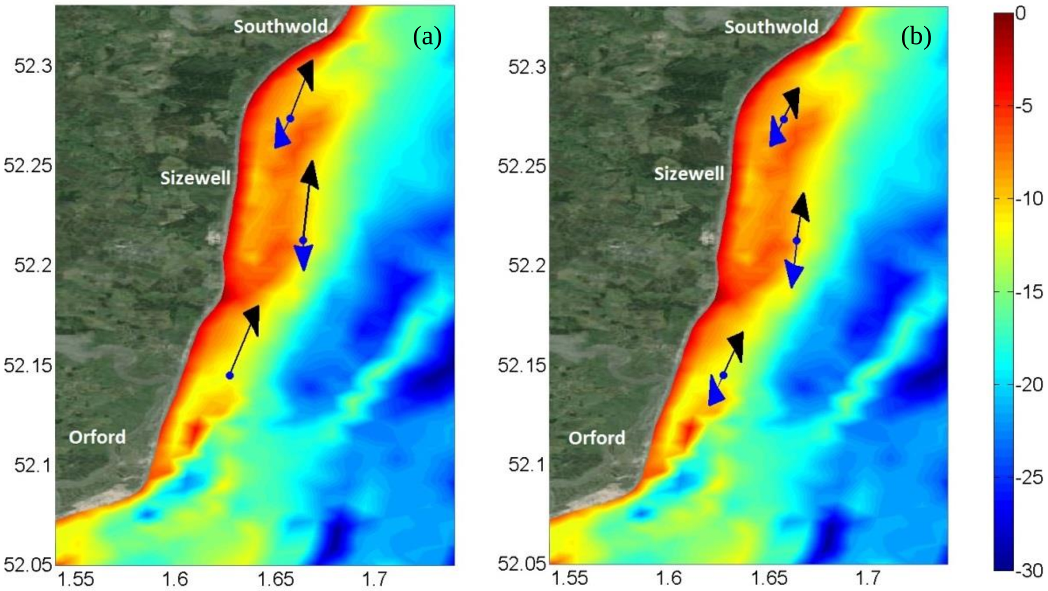


Figure 7 Propagating/ standing wave analysis results for observations (a) and model results (b). Standing (black) and progressive (blue) wave amplitudes and direction of propagation for the latter, the standing wave arrow shows the direction of streaming on the falling tide. The length of each arrow is proportional to the amplitude of the wave. Arrows are plotted on top of the model bathymetry: colourbar given in meters.

Twenty three long term mid-water observations in the same area were made over a four and a half year period spanning 1975 – 1979. These observations were analysed using the same technique (Lees 1983) and it was found that over half of the M_2 amplitudes could be attributed to the reflected standing wave. The ratios of reflected to standing waves found from the modern (2008) observations (Figure 7 (a)) are similar to those previously observed but the reflected wave amplitude is proportionally larger in the modern observations particularly at the southernmost station (comparing nearest observation points, as observations are not collocated). Analysis of observations from a long-term mooring (2.5 years) at Sizewell showed that the standing and progressive wave proportions appear to vary during the year (Lees 1983); therefore the model analysis was performed on model results covering the same time period as the observations, although the shelf model does not include atmospheric forcing.

Figure 7 (b) illustrates the analysis of the shelf model results: showing a larger propagating wave amplitude and smaller reflected wave amplitude compared to the observations. The results imply the model coastline does not reflect as strongly as the real coastline. The high value of bed friction used in the shelf model was clearly incorrect, to check if this influenced the standing/ propagating wave analysis the shelf model was rerun using the variable bottom friction values. The analysis showed the standing and propagating waves both have higher amplitudes using the lower variable bottom friction coefficients but the ratios remain very similar and the conclusion that the model over predicts the propagating wave amplitude and under predicts the reflected wave amplitude is unaffected.

6. DISCUSSION

This work arose from discrepancies between a 2-D model of the European continental shelf and tide gauge measurements in the Southern Bight, an error commonly seen (with varying magnitude) in other models. Analysis is made of each of the predominant mechanisms controlling the tide in this region: bathymetry, bottom friction and coastline reflections. It was found that using more physically realistic friction values improved the replication of tidal amplitudes but had minimal effect on tidal phase and an unaccountable reduction in the bathymetry would be required to correct the phase error alone. However analysis of tidal reflections revealed the model under-predicts the amplitude of the reflected wave which may be causing the erroneous phase pattern.

The European continental coastline represented in the model is the natural coastline; the reason tidal reflections may be under-predicted compared to observations is that this does not represent the actual coastline. Extensive coastal engineering has taken place along these coasts; including in the Southern Bight the construction of the Eastern Scheldt storm surge barrier, part of the delta works in the southern Netherlands which reduces flow into the Oosterschelde and the closing of the Zuider Sea, both of which are open in the model and represent large areas for energy dissipation. As well as barrages, attempts have been made along much of the coastline to fix the position of the shoreline, for example by building sea walls where naturally the shoreline is eroding. This leads to a reduction in the beach in front of the structure and hence the energy dissipation at the coast is reduced. Coastal engineering impact studies tend to focus on a single piece of engineering and only investigate the impact on the local area, however there is already evidence that cumulatively coastal engineering work can alter tidal patterns at the shelf sea scale; Pelling (2013) shows how the model definition of the European coastline (if new areas are allowed to flood or if the shoreline is fixed) can affect the M_2 response to sea level rise and Jung (2014) shows how the tidal regime in the Yellow Sea has been changed due to the construction of sea-dikes and seawalls with changes in the M_2 phase of up to 0.4 hours.

The idea that the modern day coastline is more reflective than the natural coastline is supported by the increase in the ratio of the standing wave in the modern observations at Sizewell compared to observations from the late 1970's. Additionally if the tidal reflections have got stronger one would expect the amphidromic point to move west and for tidal amplitudes to increase on the east of the basin and decrease on the west. There is some data to support this; an unexplained increase in Dutch and German tidal amplitudes from the period 1955 – 1980 (Hollebrandse 2005) and a smaller increase in tidal range on the UK coast at Southend (1934-1966) and Immingham (1956 – 1988) than observed on the Dutch and German coasts (Woodworth et al 1991). Unfortunately most UK tide gauges in the Southern Bight were not installed until after 1975, hence it is not possible to do a comparison for 1955 – 1980. The start of the time period, 1955 correlates well with the onset of significant coastal protection work following the devastating flood of 1953.

It is unlikely that the original two hour model phase error is due entirely to the incorrect representation of the coastline; however this paper suggests that more accurate representation of the coastline, including flood defences and the associated reduction in dissipative area, to accurately replicate tidal reflections needs to be part of the solution, something which is neglected in most calibrations which focus on changing the bathymetry, bottom friction and boundary conditions to achieve the best model fit. In the case of finite difference models this may require a reassessment of which grid cells along the water-land boundary should be active or inactive and if significant changes have occurred, an update of the active cell depths.

7. CONCLUSIONS

The conclusions of this work are limited by the small number of time series available for standing/ propagating wave analysis. The sensitivity analysis is also limited by the necessity to assume the boundary conditions are correct, as mentioned in section 3 the phase at the English Channel also affects the position of the amphidrome (Brown 1987) however this does not affect the shelf model results and the boundary conditions do match the phase of the elevations observed at Dover on the UK side of the English Channel boundary. Further work should compare bathymetry sources to gauge the potential size of this error and analyse standing/ propagating waves at more locations covering a larger spatial domain to increase confidence in the results and to improve understanding of how the magnitude of the reflections has changed over time. A more accurate coastline should be implemented to see if the correct reflected wave amplitude can be achieved.

This work is part of the iCOASST project which aims to improve predictions of coastline evolution. The hydrodynamic shelf model was set up to drive a sediment transport model, which was to be used to determine residual sediment transport. Initial calculations of residual bed load transport show divergence in sediment pathways off the Suffolk coast in agreement with previous work (HR Wallingford 2002). The latitude of the divergence in the sediment pathways is roughly the same as that of the amphidromic point. The interaction between M_2 and M_4 is essential in determining the direction of sediment transport and the mean stress distributions associated with these flows diverge from the M_2 amphidromic point in the Southern Bight (Pingree and Griffiths 1979): hence to model accurately the divergence of the sediment pathways it is necessary to model accurately the location of the amphidromic point.

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