EXPERIMENTAL STUDY ON CRITICAL RESONANT STATE OF UPSTREAM-ADVANCING WAVES

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INTRODUCTION

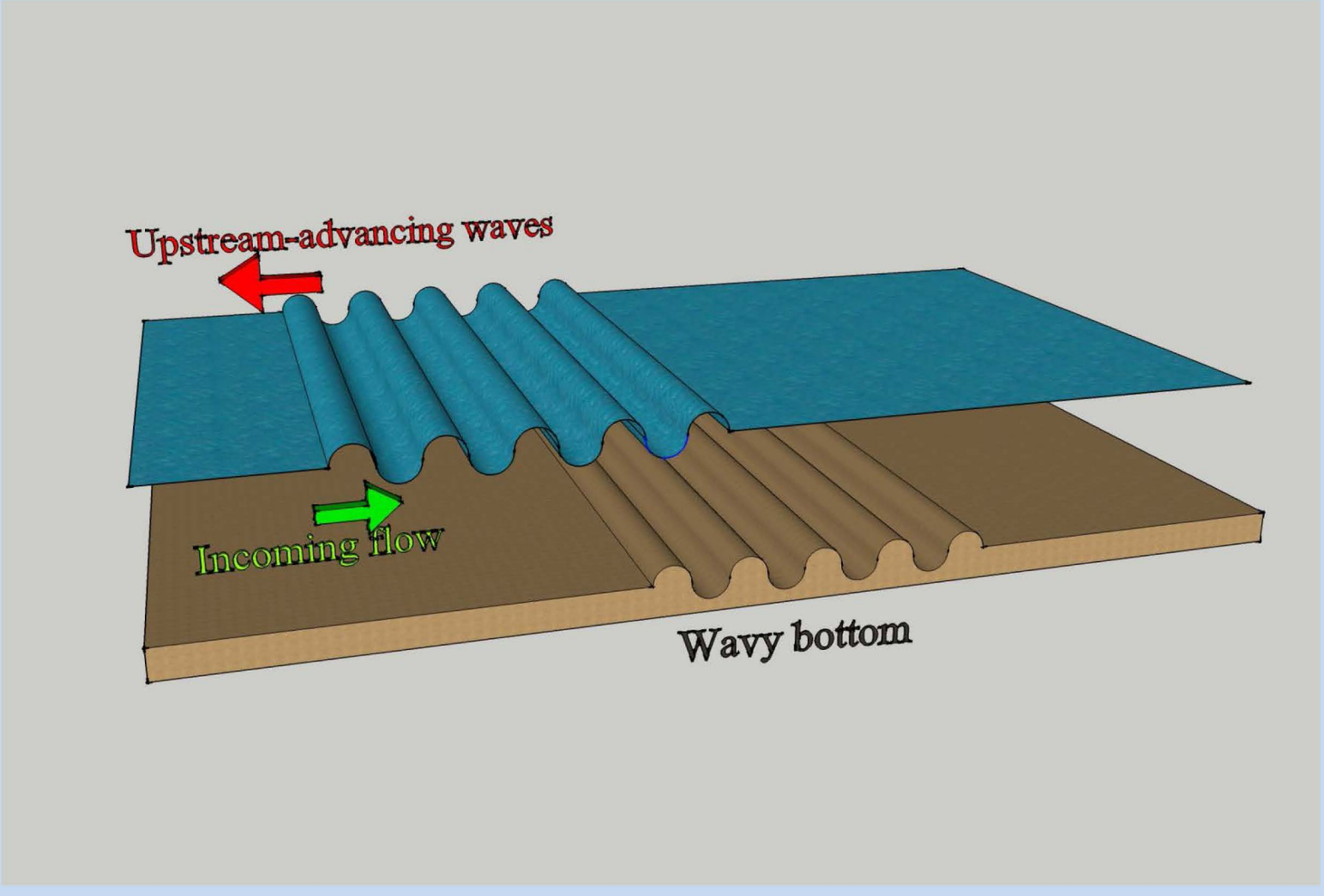


Figure 1 – Generation of the upstream-advancing waves by incoming flow over periodic wavy bottom

The interaction among surface waves, current and uneven bottoms is interesting and meaningful when the steady fluid flow travels over the periodic wavy bottom. In coastal regions, this situation may occur when the tidal or river currents pass through the corrugated topography (particularly the periodic sandbars or periodic artificial submerged structures) in shallow water areas such as river inlets and littoral zones. With this interaction, upstream-advancing waves are generated by steady flow over series fixed sinusoidal beds with specific range of the flow velocity, water depth and bottom steepness. The phenomenon of these waves shows the state of instability and the resonant interaction relationship among surface waves, steady flow and periodic wavy bottom. Furthermore, the critical flow velocity and critical relative water depth which stimulate the maximum wave height of upstream-advancing waves were observed through the modified physical experiment. These precise critical values will provide more accurate evidence to reflect the resonant interactive conditions and the most unstable state.

BACKGROUND

STATIONARY WAVES

Linear solution of the stationary wave profiles by flow over sinusoidal bed extending to both far upstream and downstream was presented firstly by Lamb (1932). The profile of the free surface elevation η is given as the equation (1), in which, k is the wavenumber of sinusoidal bed, b is the wave amplitude of sinusoidal bed, h is the mean water depth ($b \ll h$) and U is the steady flow velocity.

$$\eta = \frac{kb\cos kx}{k\cosh kh - (g/U^2)\sinh kh} \qquad (1) \qquad\qquad U_c = \sqrt{\frac{g}{k}\tanh kh} \qquad (2)$$

The profile expression shows that it will exist a critical speed U_c as the equation (2) at which the amplitude of surface elevation becomes unbound or infinite. Besides, the relationship between free water surface profile and wavy bottom profile is out of phase if $U \le U_c$ and in phase if $U \ge U_c$.

It should be noted that the waves mentioned above are stationary wave profiles which do not propagate upstream or downstream. Mei (1969) made a nonlinear analysis and obtained the steady states for the critical case. It shows that the amplitude of the free surface is finite at the critical velocity and the free surface amplitude can be triple-valued near the critical speed.

INSTABILITY BY RESONANCE

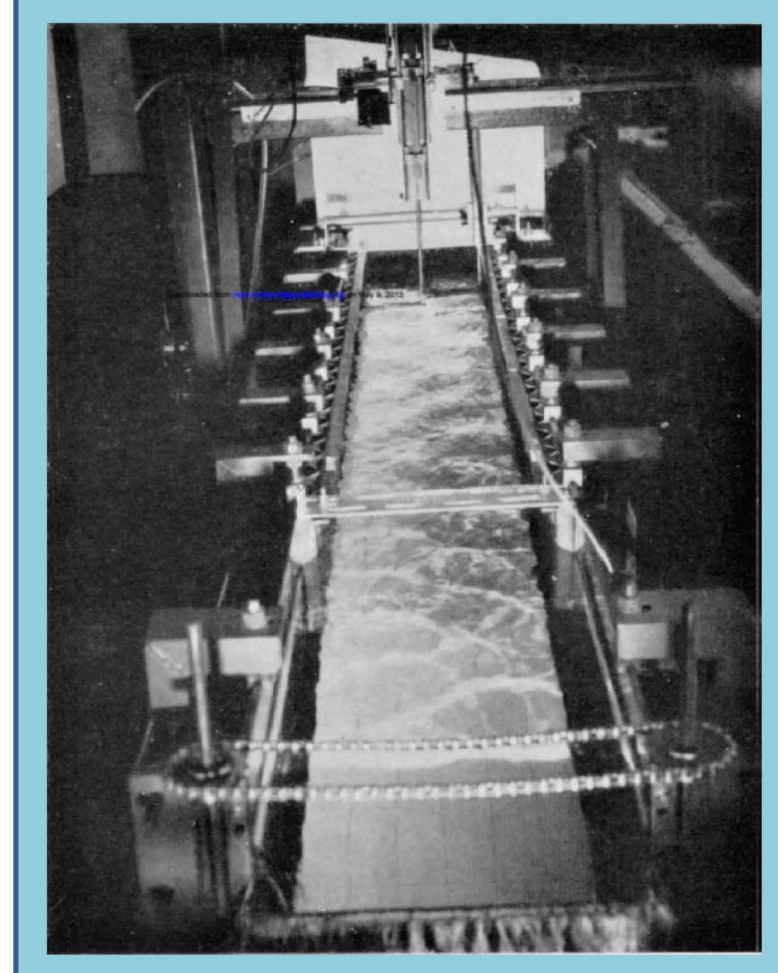


Figure 2-Binnie's experiment in 1960

Binnie (1960) observed the self-induced waves by steady flow through an open channel with vertical corrugated sides. In his experiment, continuous trains of waves were formed and moving steadily upstream on the free surface. Yih (1976) made an analytical study about the stability of the stationary waves. Then the instability was found to be caused by resonance between primary stationary waves and a pair of disturbances which were met with Hasselmann's conditions.

$$\begin{aligned} k_3 &= k_1 + k_2 \\ \sigma_3 &= \sigma_1 + \sigma_2 \end{aligned} \qquad (3)$$

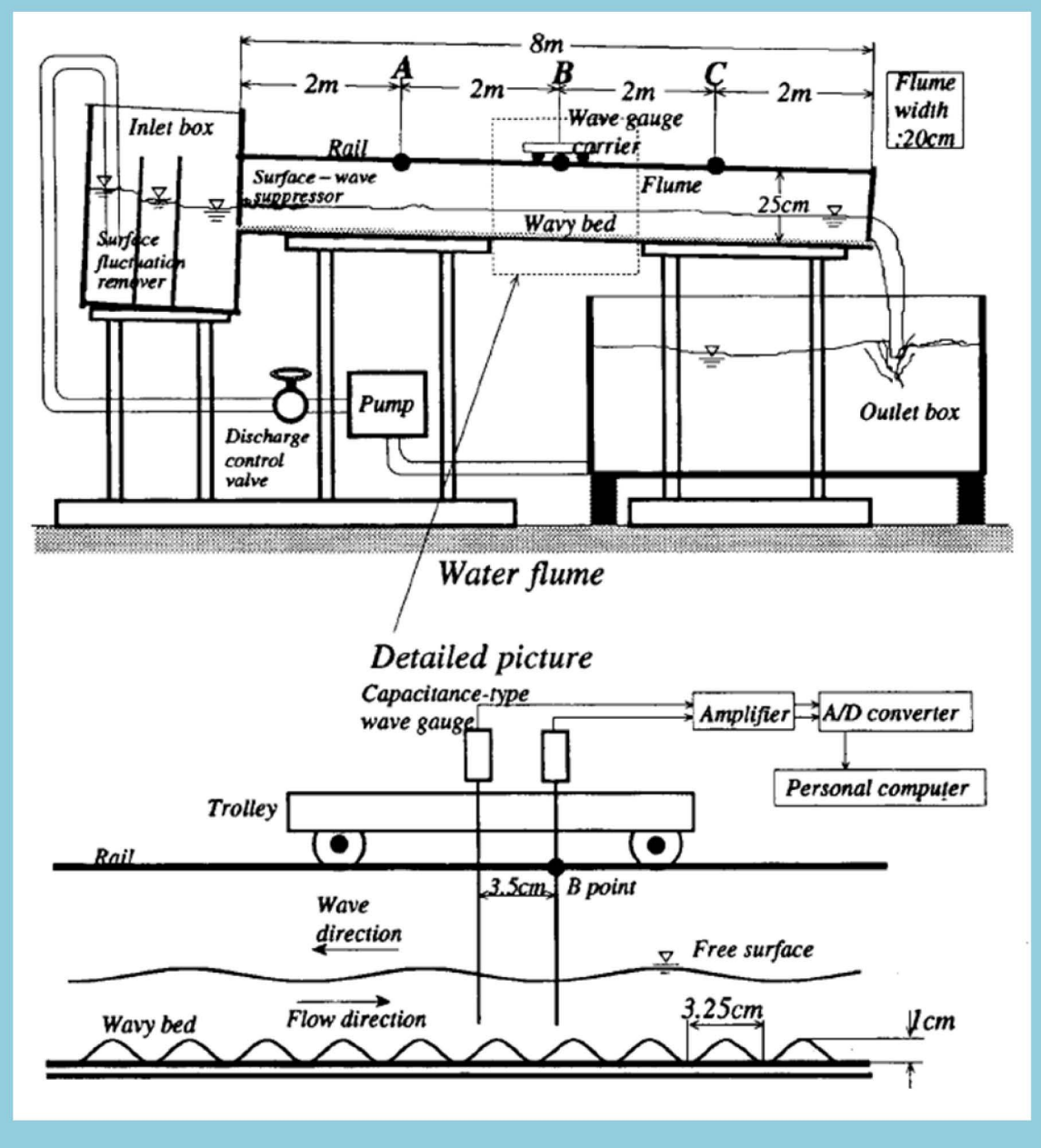


Figure 3-Kyotoh's experiment apparatus

Kyotoh (1997) firstly studied the upstream-advancing waves by open channel flow over a fixed sinusoidal bed. The waves were generated with large amplitude of bottom corrugations when the Froude number was less than the critical value as equation (2). He studied the celerity, the wavelength and the domain of existence of excited upstream-advancing waves by the steady flow over a sinusoidal bed. The amplitudes were just 1-2mm only satisfying the recognition of existence. Then the stability analysis was made by the assumption that the upstream-advancing waves were induced by the Benjamin-Feir-type instability.

However, only a few flow velocities were carried out by Kyotoh because he mainly focused his attention on the influence of the variation of water depth. The reason was that flow was generated by inertia with the declining slope of the flume so that the flow velocity was adjusted by changing the slope. Furthermore, the variation of the amplitude of upstream-advancing waves with the change of current velocity and relative water depth has not been observed due to the limit of his experimental apparatuses' scales.

Therefore, the domain of existence of upstream-advancing waves is too large to discover the exact resonant conditions, so that the amplitude distribution is significant for investigating the mechanism of current-topography resonant interaction in which this unique critical situation will excite the maximum upstream influence.

EXPERIMENTAL APPROACH

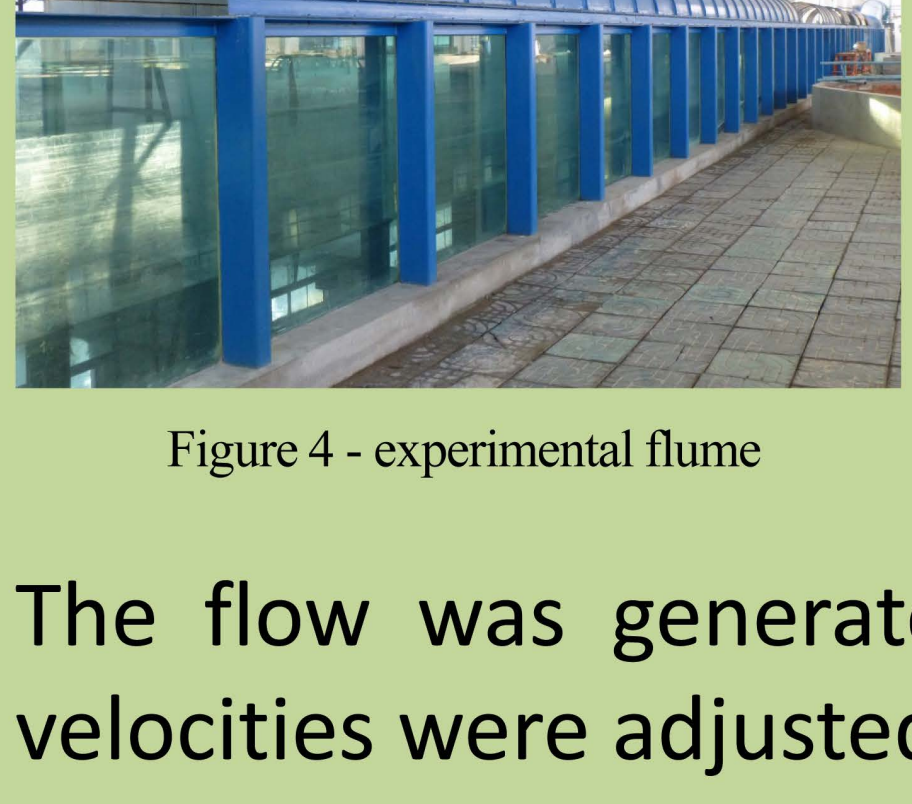


Figure 4 - experimental flume

The experiment has been carried out in a large scale wind-wave-current flume. The flume is 80m long, 1.0m wide and 1.5m deep along with eight fixed standard wooden sinusoidal bottom corrugations which are of 24cm wavelength and 8cm wave height.

The flow was generated by bump and its velocities were adjusted with an interval of 1 cm/s. The water depth relative to the bottom wavelength varied from 0.6 to 1.4. Then the water wave profiles were measured by 14 capacitive wave gauges along the length of the flume.

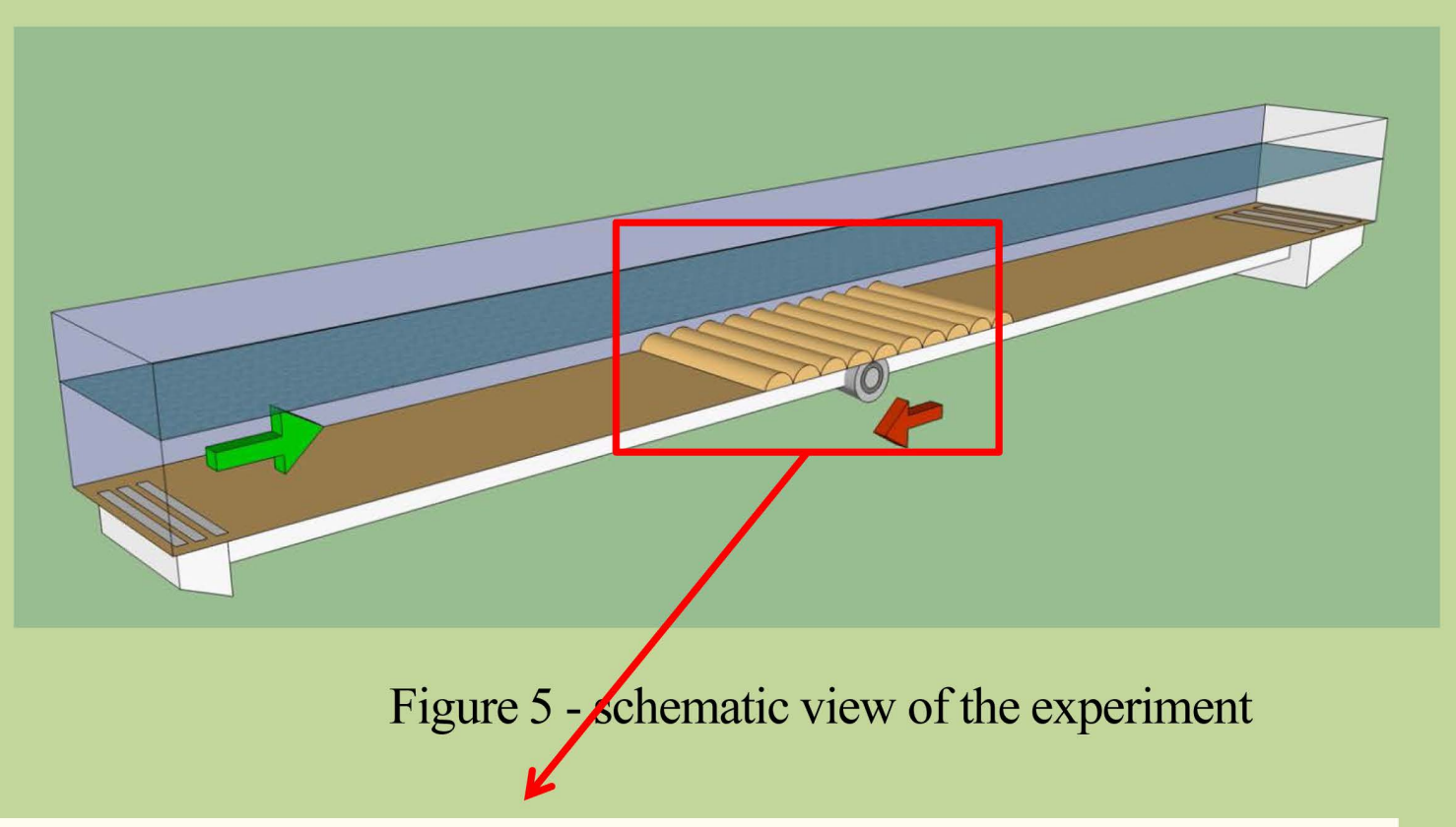


Figure 5 - schematic view of the experiment

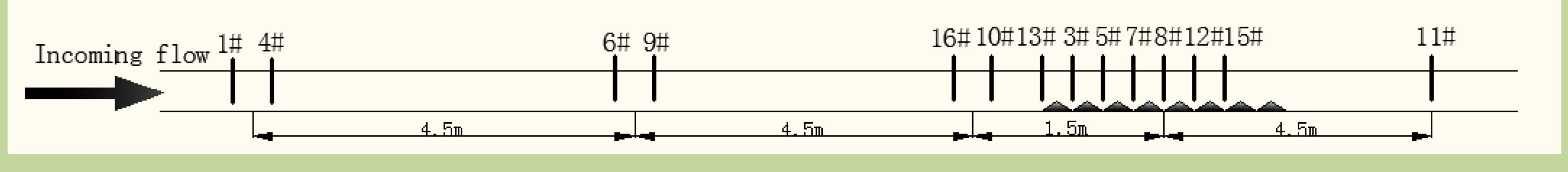


Figure 6 – Wave gauge settings

The experiment was divided into eight groups. In each group, the water depth was fixed and the flow velocity was increased from low values with small increment intervals. Then the water depth was adjusted by different groups. It should be noted that the steepness of the sinusoidal wavy bottom is 0.33 (the maximum slope is 1.05 correspondingly).

Group No.	Average water depth above the bottom (cm)	The ratio of water depth to bottom wavelength	Velocity range cm/s (with interval 1cm/s)
1	14.4	0.6	25-42
2	16.8	0.7	20-38
3	19.2	0.8	27-41
4	21.6	0.9	28-42
5	24.0	1.0	28-40
6	26.4	1.1	29-45
7	28.8	1.2	31-43
8	33.6	1.4	30-45

Table 1 –Arrangement of water depths and flow velocities

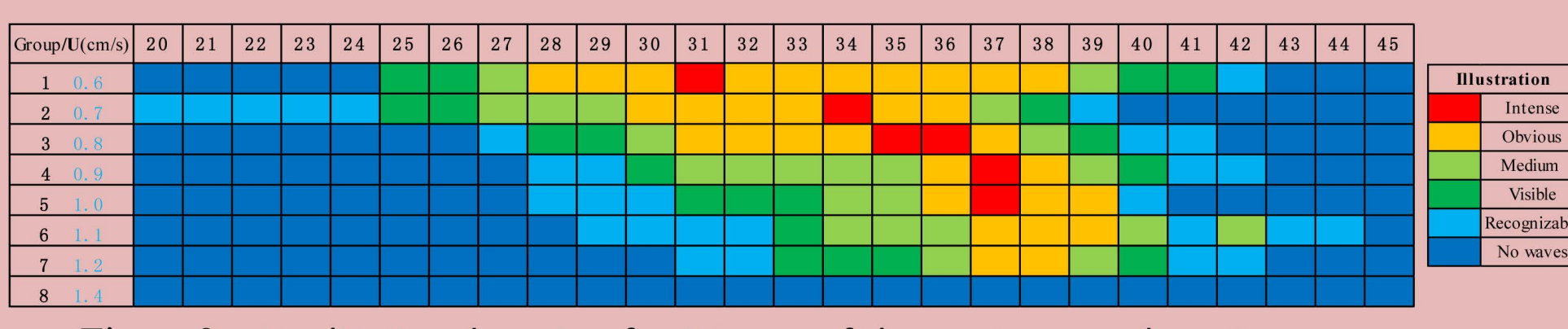
KEY RESULTS

Figure 7 – Upstream-advancing waves observed by experiment

The free-surface waves were generated and propagating upstream with a small range of current velocities along with the strong oscillation of the water body above the wavy bottom.

I. The distribution of the wave amplitudes with the change of flow velocity

The amplitudes of upstream-advancing waves surge to a maximum value rapidly at a critical flow velocity and decline sharply during the growth of the flow velocity (subcritical). The wave gauges also measured small wave components with the same frequency of upstream-advancing waves in the downstream area.



Group/U(cm/s)	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
1 0.6	🟦	🟦	🟦	🟦	🟦	🟢	🟢	🟩	🟧	🟧	🟧	🟥	🟧	🟧	🟧	🟧	🟧	🟧	🟧	🟩	🟢	🟢	🔵	🟦	🟦	🟦
2 0.7	🔵	🔵	🔵	🔵	🔵	🟢	🟢	🟩	🟩	🟩	🟧	🟧	🟧	🟧	🟥	🟧	🟧	🟩	🟢	🔵	🟦	🟦	🟦	🟦	🟦	🟦
3 0.8	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🔵	🟢	🟢	🟩	🟧	🟧	🟧	🟧	🟥	🟥	🟧	🟩	🟢	🔵	🔵	🟦	🟦	🟦	🟦
4 0.9	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🔵	🔵	🟢	🟩	🟩	🟩	🟩	🟩	🟧	🟥	🟧	🟩	🟢	🔵	🔵	🟦	🟦	🟦
5 1.0	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🔵	🔵	🔵	🟢	🟢	🟢	🟩	🟩	🟧	🟥	🟧	🟧	🔵	🟦	🟦	🟦	🟦	🟦
6 1.1	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🔵	🔵	🔵	🔵	🟢	🟩	🟩	🟩	🟧	🟧	🟧	🟩	🔵	🟩	🔵	🔵	🟦
7 1.2	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🔵	🔵	🟢	🟢	🟢	🟩	🟧	🟧	🟩	🟢	🔵	🔵	🟦	🟦	🟦
8 1.4	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦

Illustration	
🟥	Intense
🟧	Obvious
🟩	Medium
🟢	Visible
🔵	Recognizable
🟦	No waves

Figure 9 – Qualitative domain of existence of the upstream-advancing waves

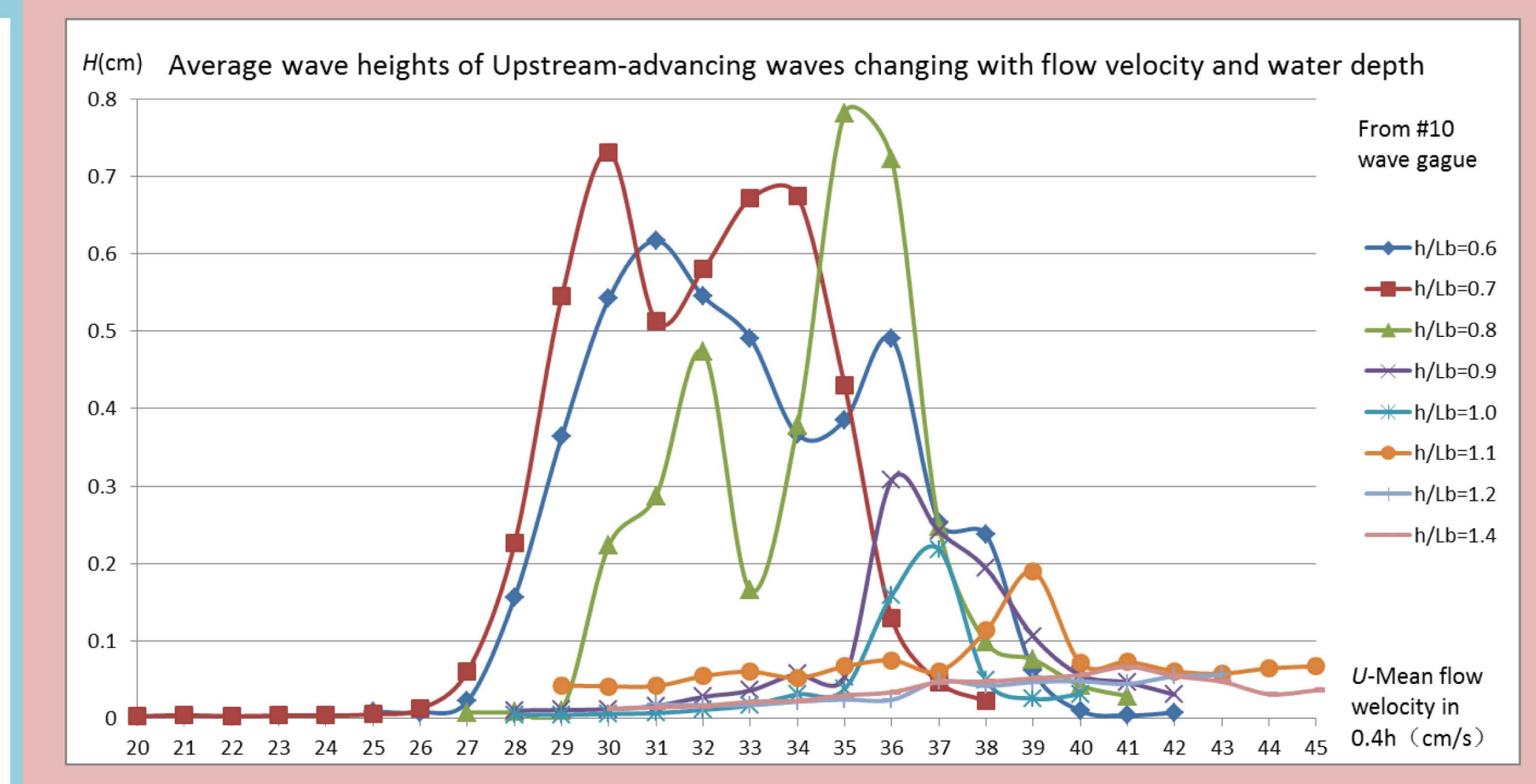


Figure 10 – Amplitude distribution of the upstream-advancing waves

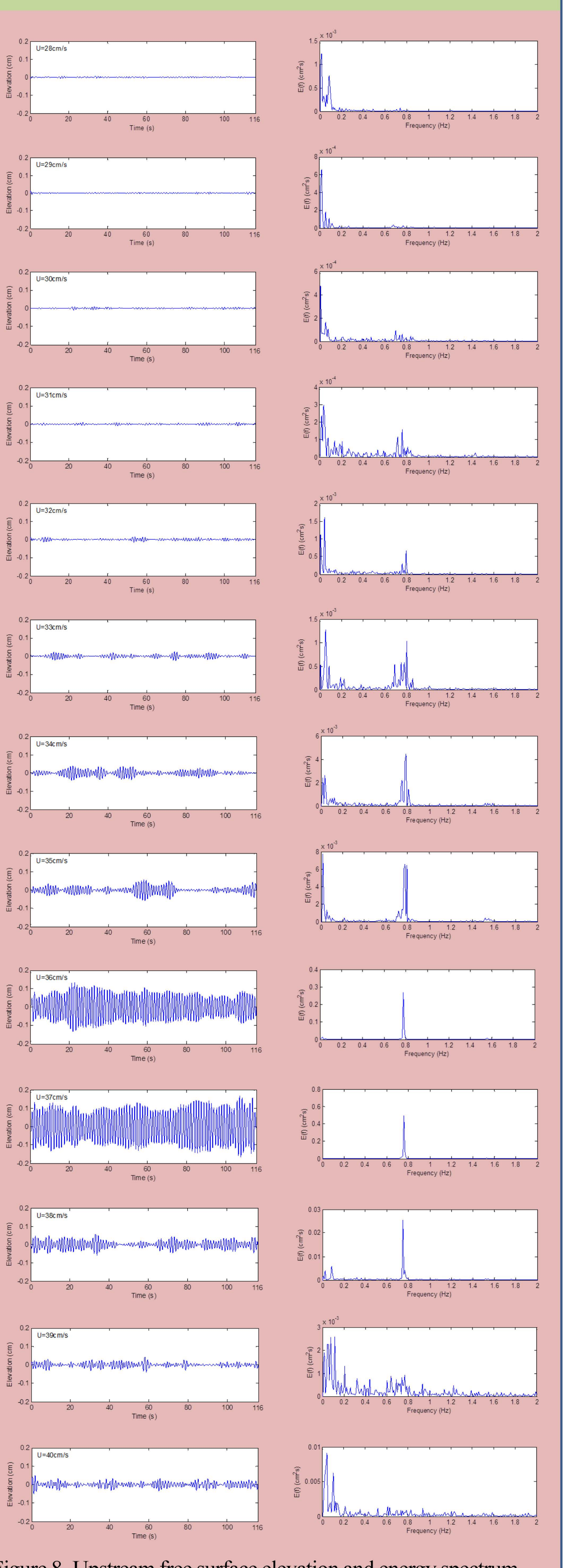


Figure 8–Upstream free surface elevation and energy spectrum with the increment of flow velocity from wave gauge 10#, Group 5

II. The influence of the variation of water depth

- The upstream-advancing waves will not be generated when the ratio of water depth to bottom wavelength is larger than 1.4 .
- The maximum amplitudes from each group rise to a peak value and drop down with the increase of the water depth. Besides, the current velocities which excite the maximum upstream-advancing waves in each group shift towards larger magnitudes as the water depth increases.

III. The period and existence of upstream-advancing waves

- The existence of upstream-advancing waves is smaller than the region from Kyotoh (Fig.11).
- The periods of the waves are concentrated between 1.2s-1.5s (Fig.12).

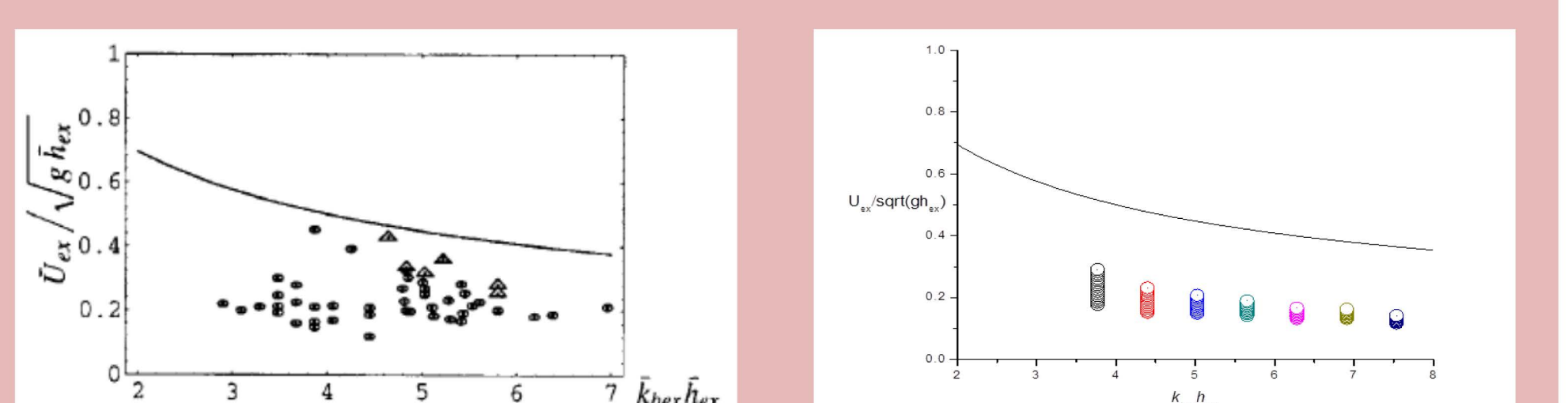


Figure 11– Comparison of the existence (Left: Kyotoh ; Right: present experiment)

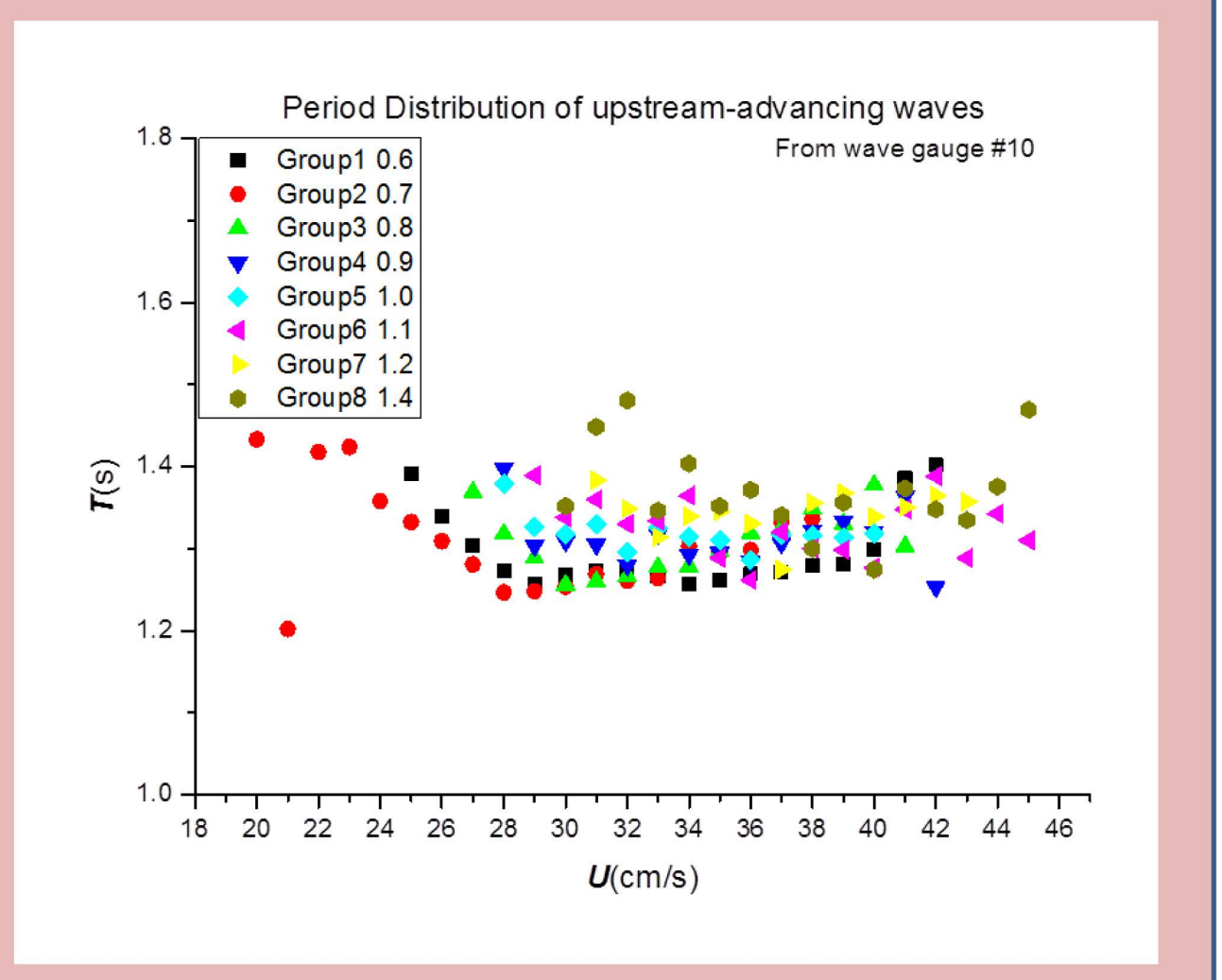


Figure 12– Period distribution of upstream-advancing waves

In summary, the generation of upstream-advancing waves reflects the resonant interaction between the primary stationary waves and free surface disturbance wave components. The detailed features of this resonant type as well as the mechanism of this instability process are worthy of further study.

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