

SIMULATION OF NON-STATIONARY WIND SPEED AND DIRECTION TIME SERIES FOR COASTAL APPLICATIONS

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Accurate modeling of wind directions is as important as accurate modeling of wind speeds when it comes to studying locally generated waves, particularly in areas of limited fetch and intermediate water depths like estuaries, as well as when analyzing the reliability and operability of coastal and offshore structures. In this paper we propose a new methodology for the simulation of non-stationary auto- and cross-correlated time series of wind speed and direction, which takes into account the circular nature of both the wind direction variable and its probability distribution function.

Keywords: Wind time series; Monte Carlo; wind direction

INTRODUCTION

For probabilistic Level III verification of coastal and harbor systems that are not only affected by extreme wind conditions, but also by intermediate or moderate wind conditions, it is required to have time series simulation tools capable of adequately reproduce the whole range of values of the variable.

However, accurate modeling of wind direction is as important as accurate modeling of wind speed when it comes to studying locally generated waves, particularly in areas of limited fetch and intermediate water depth like estuaries, as well as when analyzing the reliability and operability of coastal and offshore structures (e.g. ships, mooring systems or offshore platforms), where the direction of attack of the wind on the structure determines the pressures and the wind-induced vibrations.

Thus, univariate time series simulation methodology, as presented in Solari and Losada (2011), useful for modeling linear variables (e.g. wind speed), will be of limited applicability, being necessary to develop simulation methodologies that address the cross-correlation between wind speed and direction, as well as the circular behavior of wind directions.

In this paper we propose a methodology for the simulation of non-stationary auto- and cross-correlated time series of wind speed and direction, which takes into account the circular nature of both the wind direction variable and its probability distribution function, and apply the methodology to a case study in the Rio de la Plata estuary.

PREVIOUS WORKS

Previous works deal with the simulation of univariate (e.g. Solari and Losada 2011) and multivariate (Solari and van Gelder 2012, Mendonca et al. 2012) time series. These methodologies are able to cover the entire range of values of the involved variables (i.e. wave and wind state variables), however the methodologies were developed for linear variables, not considering circular behavior of directional variables. For example, in Mendonca et al. (2012) the wind time series simulation methodology is based on the use of copulas and autoregressive models. Drawbacks of the methodology are that the models used to characterize the auto- and cross-correlation of the wind direction do not take into account the circular nature of the probability distribution function of the directions, thus introducing errors in the auto- and cross-correlation of the simulated series.

OBJECTIVES

The objective of this work is to development a methodology for the simulation of non-stationary auto- and cross-correlated time series of wind speed and direction, which takes into account the circular nature of both the wind direction variable and its probability distribution function, subject to the following constrains:

1. Marginal probability distribution functions of both wind speed and wind direction must reproduce the non-stationarity of the measured series.
2. Probability distribution function of the wind directions must be circular.
3. Dependence function between wind speed and wind direction must be circular on directions.
4. Dependence function should also be non-stationarity if required.

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METHODOLOGY

A methodology is proposed based on the use of mixture non-stationary probability distributions and multivariate copulas. This methodology is an extension of the methodology proposed by Solari and Losada (2011) for the simulation of univariate time series of linear variables (e.g. significant wave height, wind speed, etc.).

Non-stationary mixture distributions are used for modeling the marginal distribution of each variable, and include the long-term evolution of the variables. This distributions are able to accommodate the mean annual cycle of the variables as well as their inter-annual variability, for example by including the effect of covariables like El Niño/Southern Oscillation Index or the Antarctic Index (see e.g. Solari and Losada 2012b).

Multivariate copulas are used to model the short-term evolution of the variables. These copulas reproduce the dependence between one given variable (speed or direction) at different time steps and the dependence between both variables (speed and direction) at a given time step, i.e. the copulas are used to model the auto- and cross- correlation of the wind speed and wind direction series.

Marginal distribution of each variable

For the marginal probability distribution function of the wind speed a mixture of a non-stationary truncated Weibull distribution (central part) and a non-stationary Generalized Pareto distribution (upper tail) is used (see e.g. Solari and Losada 2012a). The mixture is given by Eq. 1, where the threshold u is one of the parameters of the distribution. The non-stationarity is imposed by modeling the parameters of the two distributions by means of Fourier series.

$$f_{Speed}(x) = \begin{cases} f_{Weibull}(x) & x \leq u \\ (1 - F_{Weibull}(u))f_{GPD}(x) & x > u \end{cases} \quad (1)$$

For the marginal distribution of the wind direction a mixture of non-stationary Wrapped-Normal distributions is used, as given by Eq. 2 (this distribution is discussed on Solari and Losada 2012b). For details on the Wrapped Normal distribution (Eq. 3) the reader is referred to Fisher (1993).

$$f_{Direction}(\theta) = \sum_{i=1}^N \alpha_i WN(\theta | \mu_i, \rho_i) \quad \sum \alpha_i = 1; 0 \leq \alpha_i \leq 1 \quad (2)$$

$$WN(\theta | \mu, \rho) = \frac{1}{2\pi} \left(1 + 2 \sum_{p=1}^{\infty} \rho^{p^2} \cos p(\theta - \mu) \right) \quad 0 \leq \theta \leq 2\pi; 0 \leq \rho \leq 1 \quad (3)$$

Multivariate copula based on several bivariate copulas

For a formal definition of a copula the reader is referred to Joe (1997) and Salvadori et al. (2007). Here it is enough to bear in mind that a bivariate copula as a bivariate probability distribution function with uniform $U(0,1)$ univariate margins. The use of bivariate copulas to model the short-term auto- and cross-correlation of the wind speed and wind directions is described next.

First, non-stationary original variables (i.e. wind speed and direction) are transformed into stationary uniform $U(0,1)$ variables using the non-stationary marginal distributions proposed above. This is sketched on figure 1, where it is shown how the original variables at time steps t and $t-1$, (V_t, θ_t) and (V_{t-1}, θ_{t-1}) respectively, are transformed into the new variables pairs (F_1, F_A) and (F_2, F_B) by means of Eq. 4

$$\begin{aligned} F_1 &= F_{Speed}(V_t | t) & F_A &= F_{Direction}(\theta_t | t) \\ F_2 &= F_{Speed}(V_{t-1} | t-1) & F_B &= F_{Direction}(\theta_{t-1} | t-1) \end{aligned} \quad (4)$$

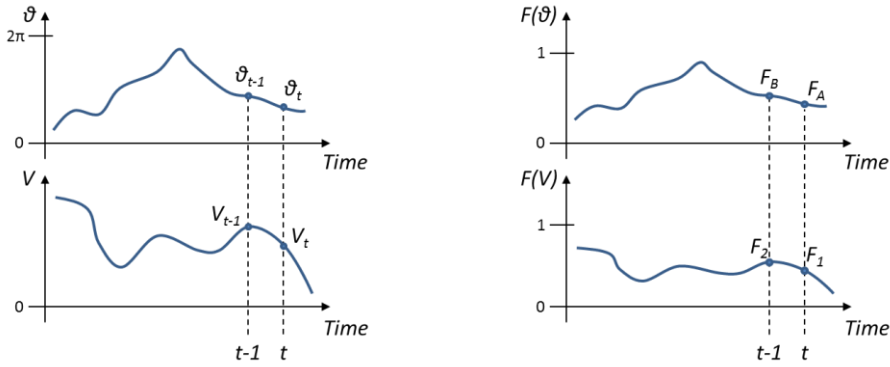


Figure 1. Original variables (left) and new variables (right) at time steps $t-1$ and t .

Secondly three bivariate copulas are estimated: one for the dependence between consecutive wind speeds (C_{12}), i.e. auto-correlation of the wind speed, one for the dependence between consecutive wind directions (C_{AB}), i.e. auto-correlation of the wind direction, and one for the dependence between simultaneous wind speed and direction (C_{B2}), i.e. cross-correlation between wind speed and direction.

Then, the bivariate copulas are used for the construction of a trivariate copula that gives the joint distribution of (F_1, F_2, F_B) . To this end the procedure described in Joe (1997), sketched on figure 2, is used. The procedure is as follows.

First, conditional probability of F_1 given F_2 and of F_B given F_2 are estimated by means of copulas C_{12} and C_{B2} (Eq. 5).

$$F_{1|2} = \Pr[x \leq F_1 | y = F_2] = C_{1|2}(F_1, F_2) = \frac{\partial C_{12}}{\partial y}(F_1, F_2)$$

$$F_{B|2} = \Pr[x \leq F_B | y = F_2] = C_{B|2}(F_B, F_2) = \frac{\partial C_{B2}}{\partial y}(F_B, F_2) \quad (5)$$

Then, a new bivariate copula $C_{1B|2}$ is fitted to reproduce the dependence relationship between $F_{1|2}$ and $F_{B|2}$. This bivariate copula $C_{1B|2}$ could be used for constructing the trivariate copula C_{12B} by integration. However, for simulation purposes it is not required to have an explicit expression for C_{12B} , being enough to have the expression for $C_{1B|2}$ (see Joe 1997 chap 4.5 for more details or Solari and Losada 2011 for an application of this methodology to significant wave height time series).

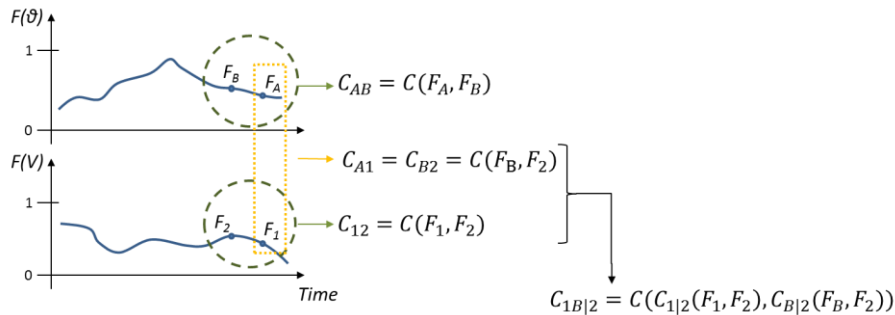


Figure 2. Outline of the procedure followed for the construction of the trivariate copula.

Simulation procedure

Once the previously described distributions and copulas are fitted to the data, the simulation of new time series is accomplished as described below.

First, two random independent, identically distributed time series $U_V(t)$ and $U_D(t)$ are simulated from a uniform distribution.

Secondly, using C_{AB} and the series $U_D(t)$ the time series of wind direction probabilities $PD(t)$ is obtained by solving Eq. 6 for every time step t . The series $PD(t)$ obtained from Eq. 6 is stationary, uniformly distributed and auto-correlated.

$$U_D(t) = C_{A/B}(P_D(t), P_D(t-1)) \quad (6)$$

Then, using the copulas C_{12} , C_{2B} and $C_{1B|2}$ and the series $U_V(t)$, the time series of wind speed probabilities $P_V(t)$ is obtained by solving Eq. 7 to 9 below. The obtained series is stationary, uniformly distributed, auto-correlated and cross-correlated with $P_D(t)$.

$$F_{B|2}(t) = C_{B|2}(P_D(t-1), P_V(t-1)) \quad (7)$$

$$U_P(t) = C_{1B|2}(F_{1|2}(t), F_{B|2}(t)) \quad (8)$$

$$F_{1|2}(t) = C_{1|2}(P_V(t), P_V(t-1)) \quad (9)$$

Lastly, using the non-stationary mixtures distributions, the simulated time series of probabilities $P_V(t)$ and $P_D(t)$ are transformed to time series of wind speed and direction as $V(t)=F_V^{-1}(P_V(t)/t)$ and $D(t)=F_D^{-1}(P_D(t)/t)$.

CASE STUDY

The proposed methodology was applied to a series of reanalysis wind taken at the mouth of the Rio de la Plata estuary, serving as a weather generator for a management and optimization tool that is under development for the entrance channel of Montevideo harbor following the methodology presented in Solari et al. (2010). The series is 34 years long (1979-2012) with one data every 6 hours (ERA-Interim reanalysis at node 35,5°S 56°W; see figure 3(a)).

Figure 3 shows the mean annual non-stationary distributions of wind speed (b) and wind direction (c), along with the bivariate distribution of the two variables (d). It is seen a clear annual cycle in the directions, with more E-SE data in summer and more W data in winter. With regards to speeds, a mean annual cycle can also be identify, although not as pronounced as in the case of the directions.

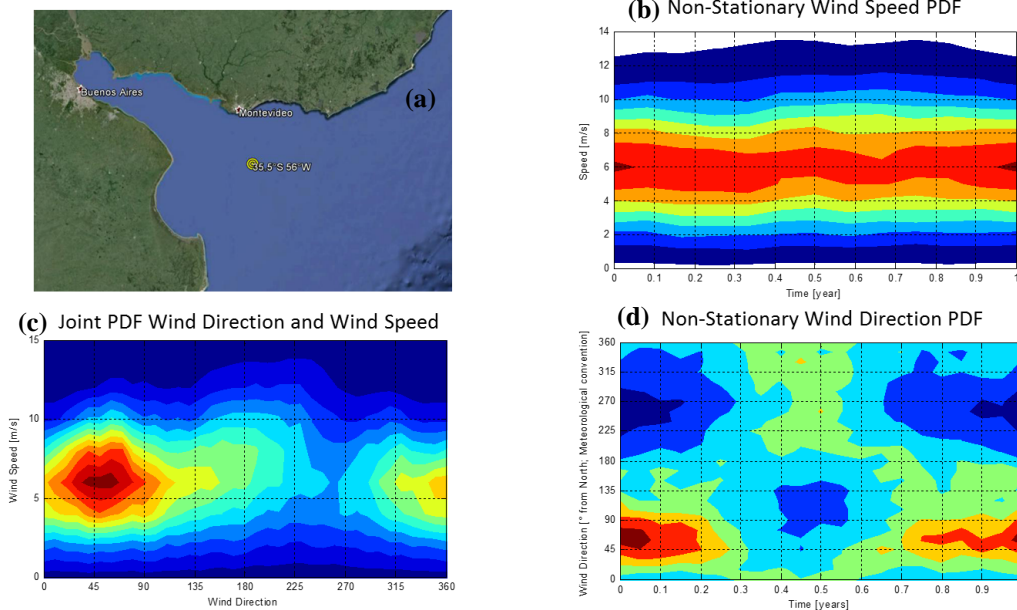


Figure 3. Location of the data point (a), empirical non-stationary distribution of wind speeds (b), empirical joint distribution of wind speed and direction (c) and empirical non-stationary distribution of wind directions.

The non-stationary mixture distributions proposed above are fitted to the data. Figure 4 shows the obtained fit. For the wind direction a mixture of two non-stationary Wrapped Normals was used. It is seen that the obtained fit is good, and that not only the mean distribution is properly reproduced but its

mean annual cycle as well. In particular, the proposed distributions are able to adequately reproduce the shift of the mode that occurs between summer and winter and the more severe wind speeds that are registered on winter.

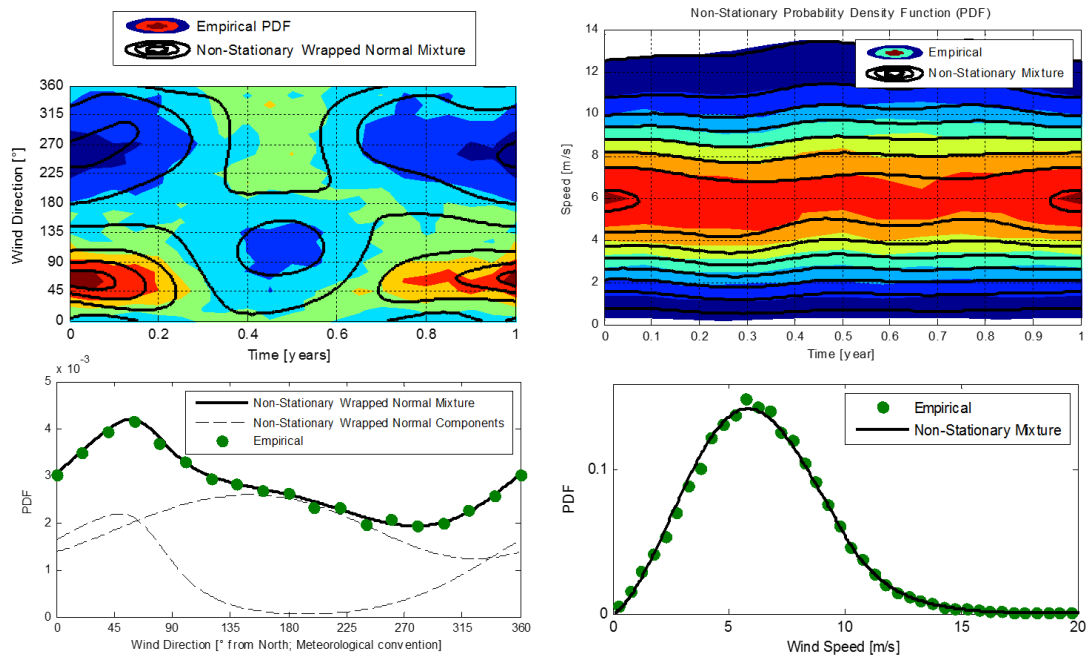


Figure 4. Non-stationary mixtures distributions fitted to the wind directions (left) and to the wind speeds (right).

For fitting the dependence between consecutive wind speed probabilities (i.e. copula C_{12}) an asymmetric Gumbel copula is used (see e.g. Salvadori et al. XXX). This copula has already proved to be adequate to fit the dependence structure of other met-ocean variables, like significant wave heights (see Solari and Losada 2011). For the dependence between consecutive wind direction probabilities (copula C_{AB}) a circular-circular parametric “pseudo-copula” is developed, based on the non-stationary truncated t-Student distribution (see Appendix). Lastly, for the dependence between simultaneous wind speed and wind direction probabilities (copula $C_{1A}=C_{2B}$), the linear-circular non-parametric Bernstein copula is used (Carnicero et al. 2013). This last copula is taken piecewise stationary, fitting 12 copulas, one per month, since the dependence between wind speed and direction probabilities was found to be non-stationary. For fitting the copula $C_{12/B}$ circular-linear non-parametric Bernstein copula is used (Carniero et al. 2013).

Figure 5 shows the fit obtained to the copulas C_{12} and C_{AB} , while Figure 6 shows the fit of the copula C_{2B} for January and July.

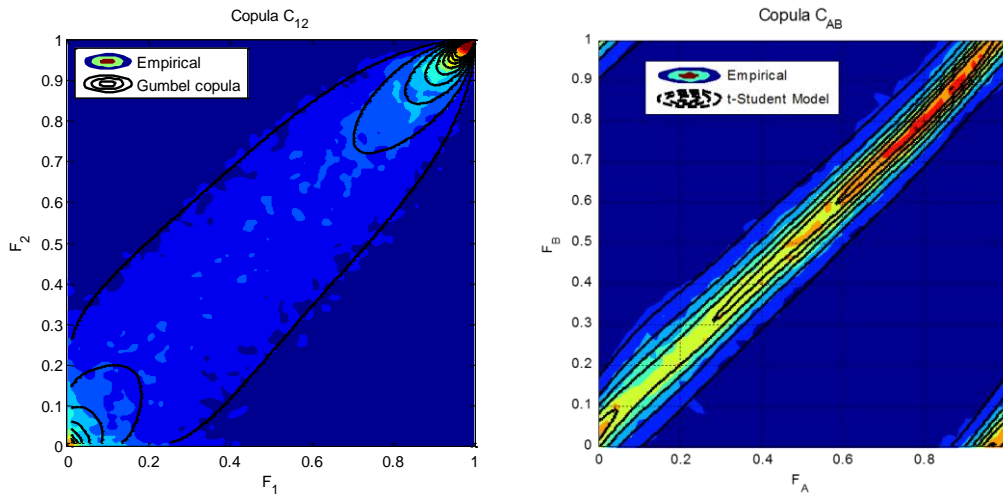


Figure 5. Gumbel copula fitted to (F_1, F_2) (left) and t-Student “pseudo-copula” fitted to (F_A, F_B) (right)

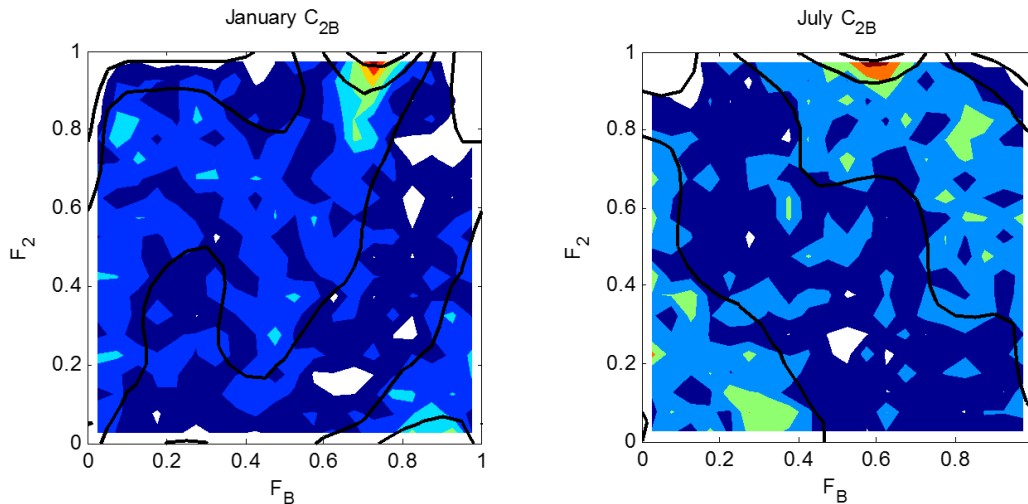


Figure 6. Circular-linear Bernstein fitted to (F_B, F_2) for Januaries (left) and for Julies (right).

RESULTS

Using the fitted models new time series are simulated and compared with the original ones.

It is found that the simulated series obtained with the proposed methodology reproduce well the auto-correlation of both variables (Figure 7) up to lag 12, even though the proposed methodology only models the dependence structure between consecutive time steps (i.e. only up to lag 1). For the wind direction, circular correlation was used for the calculation of the auto-correlation (see Fisher 1993).

With regards to the non-stationary probability distribution function of the simulated series, a very good agreement between the original and the simulated series is achieved (figure 8 a), since the simulated series are obtained by inverting the non-stationary mixture models that were fitted to the original data. However, when it comes to the bivariate distribution of wind speed and direction, that is not explicitly modeled but obtained indirectly by the dependence structure modeled with copula $C_{1B|2}$, it is found that although the main characteristics of the distribution are reproduced, the proposed methodology is not able to reproduce the high probability of strong winds observed for S-SW directions, between 180° and 225° (see figure 8 b). This results in a not so good agreement between the annual maxima distribution of the original and simulated data (see figure 9).

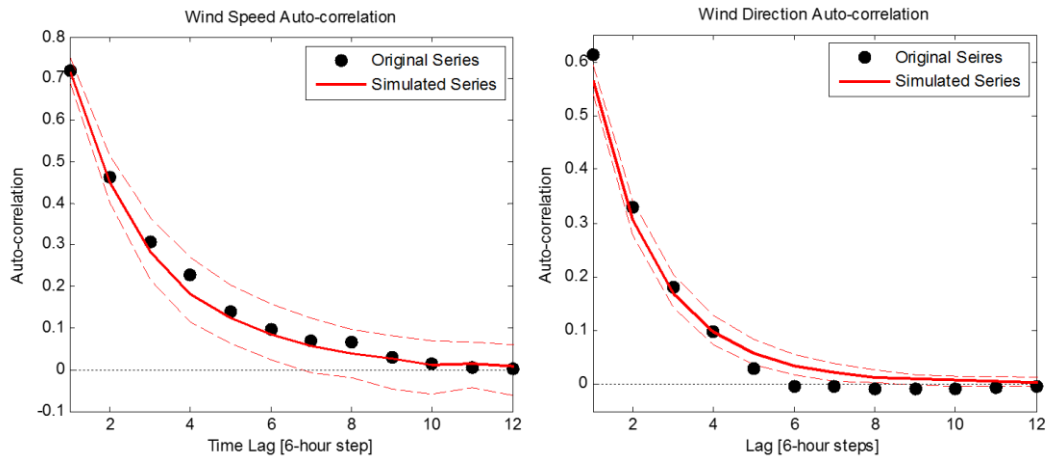


Figure 7. Auto-correlation of the original (black dots) and simulated (red lines) data. Left: wind speeds. Right: wind directions.

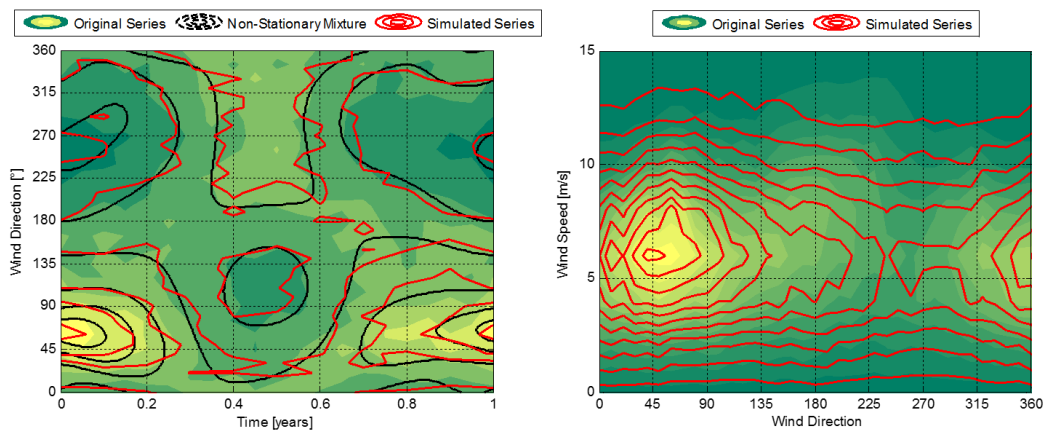


Figure 8. Non-stationary annual PDF of the wind directions (left) and joint distribution of wind speed and wind direction (right). Color filled contours: empirical PDF of the original data. Black contour lines: non-stationary mixture distribution fitted to the data. Red contour lines: empirical PDF from simulated data.

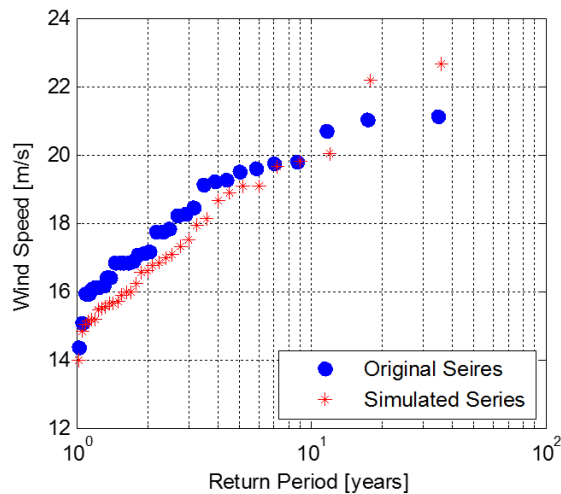


Figure 9. Comparison of the extremes of the original and a simulated series.

DISCUSSION

In general terms the proposed methodology was able to reproduce the behavior of the bivariate time series of wind speed and direction at the mouth of the Rio de la Plata estuary, except for some limitation to reproduce the extreme wind speed associated with the S-SW wind directions.

It is believed that this inability to reproduce southern direction severe conditions may be related to an excessive smoothing of the copula C_{2B} introduced by the non-parametric Bernstein copulas. As shown in figure 6 (a), corresponding to the dependence structure between simultaneous wind direction and wind speed during Januaries, the empirical copula has a maximum around $F_B=0,7$ and $F_2=1$ that is clearly smoothed by the Bernstein copula. Something similar, although less pronounced, is shown in figure 6 (b) for Julies.

CONCLUSIONS

A new methodology was introduced for the simulation of bivariate time series of wind speed and wind direction that takes account of: (a) the non-stationarity of the series, (b) the circular behavior of the directions.

The methodology was applied to a case study, showing its ability to reproduce most of the characteristics observed on the original series. However, it is required to explore the use of alternatives to the non-parametric Bernstein copulas in order to check if severe and extreme conditions could be better reproduced by the methodology.

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APPENDIX

A bivariate distribution with uniform margins is constructed by means of a truncated t-Student distribution. Although this is not strictly a copula, since one of its margins is not uniform for the whole parameter space of the distribution, it is shown that when properly fitted to a bivariate sample with uniform marginal distributions, it behaves as a proper copula.

For defining this “pseudo-copula” the parameters of the t-Student distribution are taken as Fourier series whose main period is one, and the distribution is truncated at $\mu-0,5$ and $\mu+0,5$, where μ is the mean value. As an example, Figure 10 shows how this model is able to reproduce the conditional probability of F_B given a value of F_A .

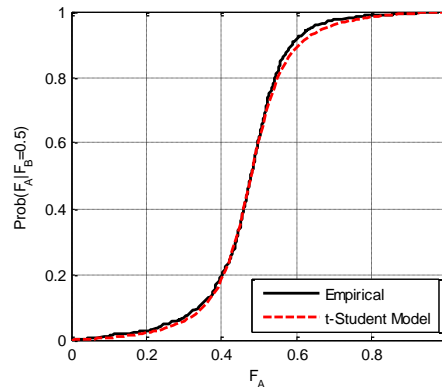


Figure 10. Example of the fit obtained with the truncated t-Student distribution

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