International Electronic Journal of Elementary Education
January 2022, Volume 14, Issue 3, 387-403

An Instructional Design for The Improvement of Counting Skills in 3-Year-Old Children

Aida Carbonell-Jornet[a], Pascual D. Diago[b,*], David Arnau[c], Miguel A. García-Moreno[d]

Received : 14 December 2021
Revised : 29 December 2021
Accepted : 28 January 2022
DOI : 10.26822/iejee.2022.251

[a]Aida Carbonell-Jornet, Facultat de Magisteri, Universitat de València, Spain.
E-mail: aidacarbonell37@gmail.com
ORCID: https://orcid.org/0000-0002-2004-2481

[b,*]**Corresponding Author:** Pascual D. Diago, Departament de Didàctica de la Matemàtica, Universitat de València Spain.
E-mail: pascual.diago@uv.es
ORCID: https://orcid.org/0000-0002-0900-8098

[c]David Arnau, Departament de Didàctica de la Matemàtica, Universitat de València, Spain.
E-mail: david.arnau@uv.es
ORCID: https://orcid.org/0000-0002-2849-8248

[d]Miguel A. García-Moreno, Departament de Didàctica de la Matemàtica, Universitat de València, Spain.
E-mail: miguel.garcia-moreno@uv.es
ORCID: https://orcid.org/0000-0001-6182-4335

Copyright ©
www.iejee.com
ISSN: 1307-9298

Abstract

The aim of this pre-experimental study is to evaluate the acquisition level of counting skills of a 3-year-old classroom made up of 14 children through a specific instructional design. To this end, an instructional proposal to improve these mathematical skills was designed. Before and after the intervention, we measured the students' level regarding counting skills through an evaluation of their counting abilities. The results indicate that the designed intervention increased the acquisition level of skills related to counting principles, constituting an effective instrument to enhance counting skills for 3-year-old children. In particular, after the intervention children improved significantly in skills related to the one-to-one correspondence principle and the order-irrelevance principle, both showing a large effect size in their observed differences. The cardinality principle, stable-order principle and abstraction principle also showed gains, but the differences were found to be statistically non-significant. Finally, the role of the age of the participants was also analyzed in relation to their acquired counting skills, indicating that children in the older age range improved their counting skills more than children in the younger group.

Keywords:

Early Childhood, Mathematics Education, Counting Skills, Counting Principles, Instructional Design

Introduction

In recent decades, mathematical learning has been a subject of study at both social and educational levels. The case of early counting skills, and its projection in the educational field, is notable since numerous studies have highlighted the relevant role that counting abilities play in typical and atypical cognitive development of early-childhood students (Baroody, 1992; Dowker, 2005; Gelman & Gallistel, 1978; Gelman & Meck, 1983; Hwang, 2020; Johnson et al., 2019; Rittle-Johnson & Siegler, 1998; Sarnecka & Carey, 2008; Wynn, 1992; Yilmaz, 2017). Mathematical educational research in pre-school stages reveals the importance of the acquisition of the necessary principles for counting (Baroody, 1992; Gelman & Gallistel, 1978). These principles have been identified as a base tool to start the learning of basic operations, as well as to establish a clear relationship between number and quantity. Despite the fact that the correct acquisition of these principles is necessary to properly perform counting-related tasks, from an adult point of view they may be considered extremely simple.

It is assumed that counting is a universal ability. All children over the age of 6 or 7 will count equally well, unless those with severe learning difficulties (Dowker, 2005). Research differs in how counting skills are acquired. Some researchers claim that counting principles master children's learning to count (Gelman & Gallistel, 1978; Gelman & Meck, 1983). Others state that the counting experience offers the child knowledge of counting skills (Fuson, 1988; Siegler, 1991). Yet others propose a mutual development whereby counting principles and counting procedures develop together during the learning of counting abilities (Baroody, 1992; Rittle-Johnson & Siegler, 1998).

Research has shown that students with learning mathematics disabilities and reading disabilities tend to show limited math abilities (Salihu & Räsänen, 2018). In particular, those children tend to show problems in understanding counting principles and detecting counting errors (Dowker, 2005;). For example, children with dyscalculia often demonstrate an incomplete understanding of some counting principles (Geary et al., 2000). Some other studies had shown that 6-years-old children who had difficulties with both mathematics and reading understand most of the counting principles, but consistently fail on tasks that assess order-irrelevance or consecutive count of contiguous objects (Geary et al., 1992; Geary et al., 2000). Studies centered on arithmetic abilities also showed that children with arithmetic disabilities in Grade 1 and 2 already had encountered problems on counting in kindergarten (Desoete & Grégoire, 2007; Stock et al., 2010).

Some studies have suggested that giving unselected children individual or small-group sessions of training in specific mathematical content and procedures, such as counting principles, comparison of quantities or quantity estimation can lead to significant improvement in typical and atypical children's number development (Ansari et al., 2003; Geary, 2011; Kaufmann et al., 2003; LeFevre et al.,2006; Stock et al., 2010). Instruction sequences based on counting tasks have also shown effects on individual differences in motor coordination, specifically related to the motor skills involved in using the fingers for counting (Dowker, 2005). In particular, in this study we present an instructional design aimed at favoring the typical development of counting skills for 3-year-old children.

The Acquisition of the Five Counting Principles for the Execution of Early Mathematical Action

The existence of different capacities or abilities that are key to learning mathematics from an early age have been a recurring subject of study for different relevant authors within the area of mathematics didactics and developmental psychology (Fuson, 1988; Sarama & Clements, 2009). One of these skills, which has been studied in depth from different perspectives, is counting. Counting is considered to be decisive for the progress of the cognitive and mathematical development of children (Fuson, 1988).

The work 'The child's understanding of number' published in 1978 by Gelman and Gallistel (1978) evidenced the existence of five counting principles that guide the acquisition of the ability to count, and allow the realization of a correct-counting process. According to this model, counting is made up of five principles namely: the one-to-one correspondence principle, the stable-order principle, the cardinality principle, the abstraction principle, and order-irrelevance principle.

The one-to-one correspondence principle. The one-to-one correspondence principle is defined as the assignment of a single number-word (Fuson, 1988) to each object in a collection. This skill involves the coordination of two processes: partitioning and labeling. On the one hand, as Lagos (1992) mentioned, the partition process is identified as the ability to divide the collection into two sub-collections: the elements counted and the elements that have not yet been counted. On the other hand, the labeling process refers to the ability of children to assign a numerical label to each of the objects that has been counted. Thus, Gelman and Gallistel (1978) consider that a child acquires the one-to-one correspondence principle when he/she is able to point to each element once, while assigning it a specific number-word. In addition, in relation to the principle of one-to-one correspondence, Briars and Secada (1988) identify three types of errors that occur in children's verbal productions when they count the elements of a collection. These errors are the omission of an object, the assignment of more than one number-word to the same object and the non-assignment of a number-word to an omitted object, even though this has been pointed out during the counting process. Regarding the age at which the one-to-one correspondence principle is acquired, Potter and Levy (1968) confirm that this skill is acquired at the age of two. However, many authors consider that this principle is not mastered before the age of 4 (Chamorro et al., 2005), and state that it is acquired by the age of 5 years (Briars & Siegler, 1984).

The stable-order principle. The acquisition of this principle is identified with the ability to count a collection repeatedly and assign the correct and conventional number-word to each item. For this principle to be acquired, the counting sequence has to be repeatable, this means that the number-words need to follow a stable and conventional order: a numerical sequence (Fuson, 1988). To be able to affirm that a child has assimilated the stable-order principle, it is necessary for the total number of elements of the collection to coincide with the stable and conventional part of the numerical sequence that the child masters. Fuson (1988), in his work, states that the average number of elements of the numerical sequence that 3-year-olds can recite adequately is five. Also, children around the age of four and a half are beginning to be able to recite a sequence of between 10 and 20 elements. Moreover, Chamorro et al. (2005) establish that learning the number sequence up to 10 corresponds to children aged four and a half years old, although this age is approximate because each child has individual characteristics and different learning rates. Finally, errors that are observed during the counting process in reference to the stable-order principle are called labeling errors. These are identified as errors derived from the action of labeling and are relative to the numerical sequence (Fuson, 1988). Geary et al. (1992) have shown that first-grade children with mathematics and reading disabilities understand stable-order principle.

The cardinality principle. The cardinality principle refers to the ability to designate the total number of elements in a set. In this way, the last number-word emitted when counting a collection has two different functions: to designate the last element of the collection and to determine the cardinality of the set. Gelman and Gallistel (1978) affirm that this counting principle is acquired at the moment in which the child repeats the last number of the counting sequence, or shows a special emphasis when pronouncing it aloud during the counting process. The acquisition of this counting principle occurs between four and five years old, considering that the acquisition of the principle implies having the ability to give cardinal meaning to the different numerical symbols (Chamorro et al., 2005). However, it should be mentioned that in order to acquire the cardinality principle it is necessary to firstly acquire the principles of one-to-one correspondence and stable-order (Gelman & Gallistel, 1978). As stated by Ansari et al (2003), the understanding of the cardinality principle is a good assessment on a proficient development of exact number representation and whether this follows a typical developmental trajectory.

The abstraction principle. This counting principle is defined as the ability to count a collection regardless of its qualitative aspect. In other words, it implies counting the elements of a collection without paying attention to concrete or abstract changes, such as alterations in the properties of objects, for example color or shape. In this way, children progressively understand that the properties of objects have no relevance during the counting process and, therefore, any object can be counted without influencing its qualitative characteristics. Regarding the age of acquisition of this counting principle, Gast (1957; cited in Gelman & Galistell, 1978) specifies that the age of acquisition of the abstraction principle in its entirety is identified at the age of seven.

The order-irrelevance principle. This last principle refers to the ability to count the elements of a collection without following any specific order. In other words, it is identified with the ability to understand that the total quantity of the set does not change regardless of the order in which its elements are counted. Chamorro et al. (2005) establish that the acquisition of the four previous counting principles is necessary to develop a numbered count. However, to affirm that a child is already capable of carrying out a total and correct counting process, it is necessary for the child to internalize the principle of order-irrelevance which allows the counting process to be given understanding and significance. Gelman and Gallistel (1978) affirm that the principle of irrelevance of order implies that the child is aware that the counted elements are independent of the label (number-word) assigned to them. This means that the number-word labels assigned during the counting process to each of the elements are assigned in a temporary and arbitrary way, and that the same cardinal of the collection is always obtained regardless of the order that is followed during the counting process. The principle of irrelevance of order, in the case of students with learning disabilities, is usually not fully understood (Geary et al., 2000).

Relevant Studies Regarding the Counting Principles

There are abundant studies regarding the counting principles in early school ages and, according to empirical evidence, there is a correlation between the acquisition of the counting principles and the success in which counting tasks are developed by typical children (Gelman & Gallistel, 1978; Gelman & Meck, 1983; Wynn, 1990; Lagos, 1992; Sarnecka & Carey, 2008; Johnson et al., 2019). Frequently, these investigations have tried to evaluate and analyze the performances of students based on different variables related to these tasks.

In the first place, with regard to the one-to-one correspondence principle, studies such as those by Gelman and Meck (1983) or Sarnecka and Carey (2008) evaluated via The one-one study and Correspondence task, respectively, the degree of acquisition of the one-to-one correspondence principle in pre-school children. Gelman and Meck (1983) presented a row of objects to be counted by children between three and four years old. After, the children had to detect any type of error in the counting carried out. The conclusion, and subsequent discussion of these tests, showed that children of both ages were able to identify the counting sequences where no type of error occurred and those in which errors were detected, although the percentage of children who detected errors was between 67% and 83%, versus 100% identifying the correct sequences. Likewise, in parallel to this study, Sarnecka and Carey (2008) evaluated the level of acquisition of the one-to-one correspondence principle in children from 2 to 4 years old. Within the analysis of the results, they highlighted that most of the children pointed to each of the objects and only assigned them a number-word. However, a previous study carried out by Gelman and Gallistel (1978) evaluated the one-to-one correspondence principle with the Videotape Counting Study task and concluded that two- and three-year-old children had almost no errors in the correspondence principle as long as the collections were made up of a reduced set of elements (between three and five). Thus, as the number of items increased, children began to show errors in this principle. Following the same line of research, Johnson et al. (2019), evaluated the level of acquisition of the one-to-one correspondence principle in pre-school children, through the tasks Count eight bears and Count 31 pennies. In their sample, 16% of the children were capable of developing this skill while 57.1% could only demonstrate the principle at certain times and, 26.9% had not yet acquired this counting principle.

Regarding the stable-order principle, Gelman and Gallistel (1978) developed the task The Magic Experiment. This task aimed to assess if young children differentiate between two categories of transformations performed on a collection of items. This study revealed that the majority of two-year-old children use a stable order sequence in spontaneous counting, in some cases with particularities. Three- and four-year-old children also use stable order sequences and made errors only in collections of larger sizes. Gelman and Meck (1983) concluded that the reason why the children presented problems with the stable-order principle was mainly due to the fact that they had not acquired the numerical sequence, in its entirety, following a stable and conventional order and, therefore, if they were asked to count a collection greater than the known numerical sequence, significant errors were found. In contrast, the study by Johnson et al. (2019), developed the Count 31 pennies and Count out loud tasks, the latter aimed to detect the highest number reached by a student using the standard sequence. A comparison between both tasks shows that slightly more children counted up to higher number when counting the pennies than when asked to count out loud without any objects (40.5% versus 35.5%).

Concerning the cardinal principle, the study by Gelman and Gallistel (1978) reported that the majority of four-year-old children were able to obtain, in most of the tests, the cardinal of the represented set. Gelman and Meck (1983), with The Cardinal Study, found that between 85% and 96% of three-year-old children already had an implicit knowledge of the cardinality principle. Wynn (1992) found that the mean age of the youngest children tested in their experiments was over 3-and-a-half, consistent with previous studies. A more recent work by Sarnecka and Carey (2008), reported that in 83% of the trials, the children, aged between two and four, answered the total amount of the collection adequately. In contrast, in 13% of the trials, the children answered incorrectly, and in the remaining 4% of the trials, they counted out loud instead of determining an exact final amount. The study by Johnson et al. (2019) provided a different analysis concerning the cardinal principle, as they studied the knowledge of the principle when engaging in more challenging tasks (counting 8 bears versus counting 31 pennies). Their results showed that, out of 317 children who provided a cardinal response to the bears task, 83 (26%) did not do so when counting the larger collection.

There are few references in relation to the acquisition of the abstraction principle and the order-irrelevance principle in early childhood education. Concerning abstraction, Markman (1979) and Fuson (1988) carried out counting experiments involving heterogeneous and homogeneous objects, indicating that by age 3 most children seemed to be able to take many different kinds of entities as separate equivalent "countable" units. Over the different studies reported in Fuson (1988), multiple count errors increased with object heterogeneity; however, children made more skim errors when the items counted were homogeneous (all the objects in a row were identical) than when they were heterogeneous. Concerning order-irrelevance, the dissertation by Lagos (1992) reported that the counting performance of 3 and 4-year-old children was significantly higher when few elements were presented in the counting collections and they were row-distributed instead of being arranged randomly.

Research Questions

The aim of this pre-experimental study is to evaluate the acquisition level of counting skills for 3-year-old children through a specific instructional design. Within this context, the research questions are as follows:

- RQ1: Is it possible to significantly increase the acquisition level of skills related to counting principles in 3-year-old students with an ad hoc designed intervention?

- RQ2: With this particular instructional design, are there significant improvements in any of the counting principles?
- RQ3: Is the difference in age a key factor for the improvement in counting principles in a 3-year-old Early Childhood Education classroom?

Method

In this study we follow a pre-experimental design with an ad hoc instructional sequence. We employed a pre-test/post-test design with the aim of assessing the children's improvement in counting skills before and after the intervention. The decision to not include a control group in this experiment was based on professional ethics, as we preferred an all-class intervention in order to maintain equity in the children's learning processes, which are highly influenced by what the teacher does in the classroom.

Participants

The study sample consists of a natural 3-year-old Early Childhood Education classroom in a public school in Valencia (Spain). A total of 14 children (8 boys and 6 girls) aged between 3 years 5 months and 4 years 4 months (M = 3.97) participated in the study. None of the children has special needs or a diagnosed learning disability.

Instrument

The measurement instrument was used to collect the data in both the pre-test and post-test with the aim of determining improvements in the acquisition of the students' counting principles after the intervention. To this end, the same evaluation method (rubrics) was applied to compare both measurements with a critical and objective character. In addition, the rubrics have served to determine the strengths and weaknesses of the design and the implementation of the instructional intervention. The rubrics were developed based on the studies of Gelman and Gallistel (1978) and Fuson (1988). Different rubrics were developed to measure the degree of acquisition of each of the counting principles.

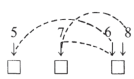
The results obtained by each student were coded by assigning specific scores. For each of the counting principles, the student's ability was evaluated over five consecutive attempts, each scored with 0.2. Thus, a total of five successful attempts results in the highest score (1) in that specific counting principle. In addition, since the five counting principles were evaluated and each one of them was assigned up to a score of 1, the sum of the test scores of each child was valued over 5 points.

The one-to-one correspondence principle was evaluated through the rubric presented in Table 1. The successful and failed counting attempts were defined, scoring the correct answers with 0.2 each. In addition, the type of error that the student made was determined according to the work of Briars and Secada (1988).

The evaluation of the stable-order principle was carried out in the same way, by counting the number of successful counting attempts. Following the study by Fuson (1988) regarding the learning of the numerical sequence during the acquisition phase, three significant parts in the structure of the counting productions of children were differentiated: the stable and conventional portion (an accurate number-word sequence), the stable but non-conventional portion (incorrect number-word sequence consistently produced) and the non-stable and non-conventional portion (incorrect number-word sequence that varied over trials). The rubric is shown in Table 2.

In the rubric referring to the cardinality principle (Table 3), three different categories were distinguished among the students' productions during the repetitions: i) cardinal not indicated; ii) cardinal indicated, but incorrect; and iii) correct cardinal indicated. Gelman and Gallistel (1978) consider that students have understood the cardinality principle as

Table 1
One-to-one correspondence principle rubric (add rows for each attempt)

ONE-TO-ONE CORRESPONDENCE PRINCIPLE					
Trial number	Correct trials	Trials with errors	Type of one-one error		
			Item omission	Assignment of more than one number-word	Non assignment of number-word
					

long as they show behavioral manifestations, such as repeating the last element of the counting sequence, placing special emphasis on the last element of the counting sequence, or spontaneously repeating the last number-word once they finish counting. In addition, in the study by Wynn (1992) the responses of those students who are able to identify the last emitted number-word as the cardinal of the set are considered correct. Thus, the attempts in which the students indicate the cardinal are considered valid, regardless of whether the numerical result is correct or incorrect.

The abstraction principle rubric shown in Table 4 presents two variations in the counting items. Two changes have been evaluated in the properties of the collection objects: the color and the shape. Thus, five attempts were addressed varying the color, and five more attempts varying the shape. Regarding the scores, in both cases the trials were evaluated with 1, followed by an average of the results of both variations.

Finally, in Table 5 we show the rubric corresponding to the order-irrelevance principle. In each of the five counting attempts, the order of the elements was altered with random positions. To each of the trials, a score of 0.2 was assigned, giving a total score of 1.

Instructional Design

In order to answer the research questions, an intervention proposal was designed with a total of eight tasks devoted to the improvement in children' skills related to counting. The design of the tasks was based on manipulative materials as during the stage of Early Childhood Education, children begin to learn intuitive and informal mathematical knowledge based on exploration, experimentation, manipulation and situations involving play (Baroody, 1987). Moreover, all the tasks are based on common activities in school settings which they are likely to be familiar with. These tasks made up an instructional sequence to be implemented to the natural group of students. The details of the intervention are given in the following subsection. Below we give a brief description of each task:

Task 1: Put each cube in a recipient. To complete this task, the children must place a cube inside each container while they count out loud (Figure 1). Although different counting principles can be worked on with this activity, it is mainly oriented to improve the one-to-one correspondence principle, since the objective is to relate one cube with its container.

Table 2
Stable-order principle rubric (add rows for each attempt)

STABLE-ORDER PRINCIPLE					
Trial number	Correct trials	Trials with errors	Label errors		
			Stable and conventional portion	Stable but non-conventional portion	Non-stable and non-conventional portion
5					

Table 3
Cardinal principle rubric (add rows for each attempt)

CARDINALITY PRINCIPLE			
Trial number	Cardinal indicated, but incorrect	Correct cardinal	Cardinal not indicated

Table 4
Abstraction principle rubric (add rows for each attempt)

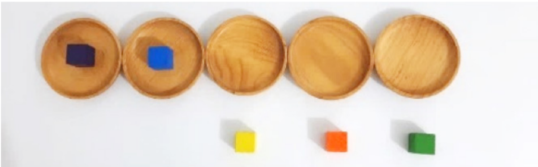
ABSTRACTION PRINCIPLE		
Trial number	Color variation	Shape variation
	YES / NO	YES / NO

Table 5
Order-irrelevance principle rubric (add rows for each attempt)

ORDER-IRRELEVANCE PRINCIPLE	
Trial number	Order alteration
	YES / NO

Figure 1

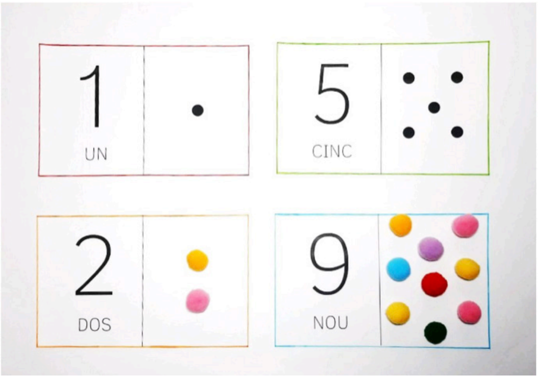
Put each cube in a recipient.



Task 2: Determine the quantity with modelling clay balls. A picture with a dual representation of a quantity is given on a sheet of paper, showing the number-word and the quantity in a black dotted-constellation format (see Figure 2, upper panels). The children have to assign a modelling clay ball to each of the depicted dots, while reciting the number sequence and determining the exact number of modelling clay balls they have to place, i.e., the cardinal of the set. This task is aimed at improving the one-to-one correspondence principle, the stable-order principle and the cardinality principle.

Figure 2

Determine the quantity with modelling clay balls



Task 3: How many are there? A series of cards with different numbers is distributed to a group of students. Then, the teacher sets up a collection of objects (feathers in the example shown in Figure 3). The children are asked to count the items out loud in order to determine the quantity. Once the total quantity has been indicated, the student who has the card with the correct number places it next to the collection. This task is mainly oriented to improve the stable-order principle, the cardinality principle and also the recognition of numerical symbols.

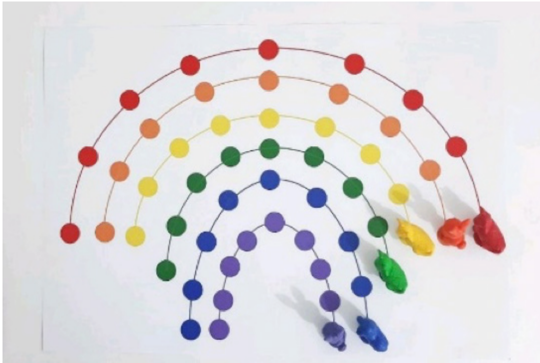
Figure 3

How many are there?

Task 4. Jump over the rainbow. The children are provided with the game board shown in Figure 4. The game shows a rainbow with different colored figures assigned to dotted paths over each rainbow color. The children are asked to move each figure from the starting point to the other rainbow side counting the jumps over the dots. After the counting process, the children are asked to state the total number of jumps that determine the cardinal of the collection for each color. Finally, the children are asked to start counting from the other side of the rainbow to move the figure to the starting position. The activity is focused on the one-to-one correspondence principle, the stable-order principle, the cardinality principle and the order-irrelevance principle.

Figure 4

Jump over the rainbow



Task 5. Roll, count and jump. The classroom floor is prepared with colored sheets of paper as depicted in Figure 5. The task consists of rolling the dice and counting the total number of dots on it. Once the children have determined the cardinal, they have to jump over the squares, while counting aloud, until they reach the corresponding number of colored sheets. The purpose of this task is to improve the one-to-one correspondence principle, the stable-order principle and the cardinality principle.

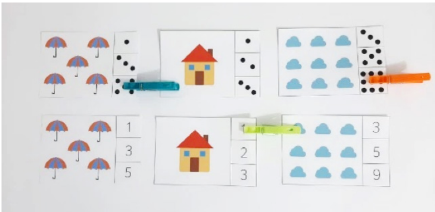
Figure 5

Roll, count and jump

Task 6. Count and identify the cardinal. The task consists of counting the total number of elements and identifying them with the correct cardinal through different paper cards (Figure 6). As some students have difficulties with the recognition of numerical symbols, two models of cards are presented: those with numerical symbols (Figure 6, upper cards) and those with a dot-constellation representation (Figure 6, lower cards). This task aims to improve the one-to-one correspondence principle, the stable-order principle and, especially, the cardinality and order-irrelevance principles.

Figure 6

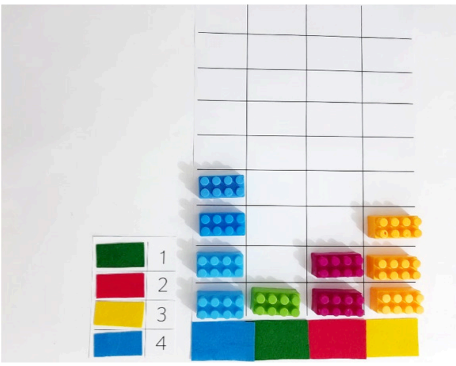
Count and identify the cardinal



Task 7. Count building blocks. The aim of this task is for the children to place building blocks on a grid (Figure 7, right), one piece in each square, as indicated by a card which has been previously given to each child (Figure 7, left), while counting out loud. This task is especially oriented to the one-to-one correspondence principle, the order-stable principle and the cardinality principle.

Figure 7

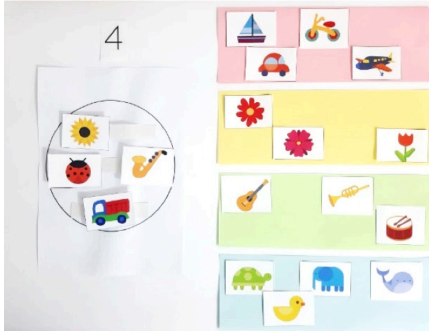
Count building blocks



Task 8. Create new collections. In this activity, the children are given different sets of items, classified into four categories: vehicles, flowers, instruments and animals (as depicted in the right panel of Figure 8). The children are asked to form a new collection made up of items from the given collections. The new collection has to be formed in the given circle (left panel of Figure 8) and, once the collection has been completed, the children have to count the total number of objects out loud and determine the final amount. Although all five principles are present in this activity, it is specifically focused on improving the abstraction principle.

Figure 8

Create new collections



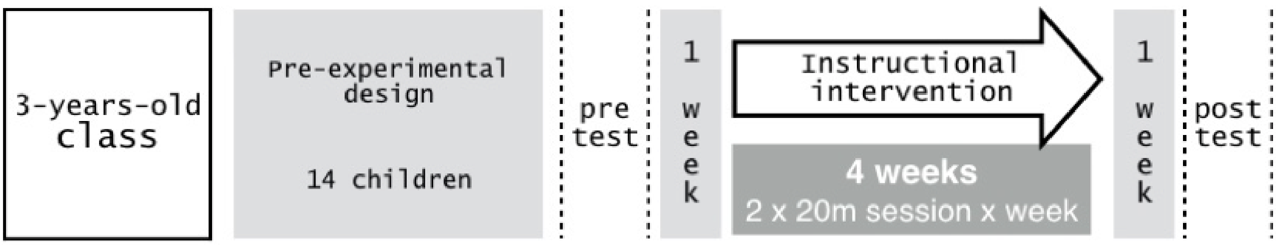
Procedure

In this pre-experimental design, all the children in the 3-year-old classroom were assessed with a pre-test for their counting-principle skills. The pre-test took place one week prior to the instructional intervention. After, the instructional sequence took place. One week after the end of the intervention the children were again assessed on their counting-principle skills through a post-test. Figure 9 gives a schematic view of the pre-experimental design. In what follows, we delve into the methodology carried out both in the pre/post-test and in the intervention.

Pre-test evaluation. The pre-test data collection was done by means of the instrument described above. An individual test, lasting approximately between ten and fifteen minutes, was completed by each child. To avoid unnecessary distractions, for each child the rubric assessment took place on a table away from the rest of the class. For each child, we first assessed, at the same time, the one-to-one correspondence, the stable-order and the cardinality principles with a total of five counting attempts, described as follows. A collection of nine blue rubber animal figures was presented with the intention of carrying out the counting sequences (Figure 10). The nine elements were provided in a linear arrangement in a horizontal format, and there was a space between each of the figures of approximately one centimeter. Each child was asked to count the entire collection out loud, pointing at each item as it was counted. Then, at the

Figure 9

Pre-experimental scheme



end of the counting process, the children were asked for the final quantity of the set by asking the question "How many animals are there?"

Figure 10

Collection for the pre-tests initial counting tasks

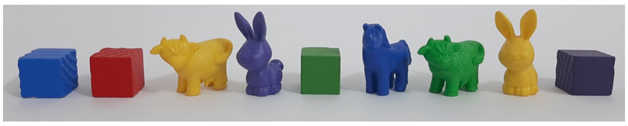
Secondly, the abstraction principle was evaluated with the two mentioned variants of the items: the alteration of color and shape. To get the children more involved, they were asked to choose the colors of the animals they wanted to count. So, the students lined up animals of different colors until they were told to stop, as shown in Figure 11. Afterwards, they were asked to count the collection in the same way as before (five attempts), out loud and pointing to the objects as they were counting them. Once the five counting attempts had been performed, four of the animals were exchanged for four different-colored cubes, as shown in Figure 12. The cubes were placed randomly within the horizontal objects' row and, therefore, sometimes there were cubes together or, on the contrary, they were interspersed forming a series with the rest of the animals.

Figure 11

Collection for the pre-test second counting task

Figure 12

Variant collection for the third counting task in the pre-test



Finally, to evaluate the order- irrelevance principle, a collection of nine blue animals was used again. Over five different attempts, the animals were randomly placed on the table in no particular order as shown in Figure 13. The children were asked to count the total number of animals out loud and point at them as they were counted.

Figure 13

Randomly placed figures for the last counting task of the pre-test

Instructional intervention. The instructional sequence was aimed to work with the children through the specially designed tasks. The methodology during the instructional intervention was based on learning corners, in order to favor the acquisition of mathematical knowledge (Clements et al., 2002; Sarama & Clements, 2003). In this way, the children were given more personalized attention and the opportunity to learn at their own pace. To this end, the classroom was divided into four groups (G1 to G4). Although this methodology is used for our specific classroom reality, the materials used in this proposal can also be adapted to other school realities where teaching resources do not allow the learning corners methodology.

Each day of the week (except Friday) two small groups carried out a 20-minute session devoted to completing one of the eight designed tasks. Hence, every two days a specific task was completed by all the children. Thus, it took four weeks to complete the instructional designed sequence, as shown in Table 6. The organization of the tasks in 20-minute sessions responds to the necessity of children's sustained attention during a short period of time. At age 4, it has been observed that children's attention span-persistence significantly predicted math achievement (McClelland et al., 2013).

Table 6

Task organization for implementing the instructional intervention

Week	Monday	Tuesday	Wednesday	Thursday
1	Task 1 G1 and G2	Task 1 G3 and G4	Task 2 G1 and G2	Task 2 G3 and G4
2	Task 3 G1 and G2	Task 3 G3 and G4	Task 4 G1 and G2	Task 4 G3 and G4
3	Task 5 G1 and G2	Task 5 G3 and G4	Task 6 G1 and G2	Task 6 G3 and G4
4	Task 7 G1 and G2	Task 7 G3 and G4	Task 8 G1 and G2	Task 8 G3 and G4

Post-test evaluation. One week after the instructional intervention finished, we collected the post-test data using the same measuring instrument used in the pre-test. Thus, the data collection was once again carried out individually, presenting the same counting activities to the children as described above.

Data Analysis

To address the research questions, we studied the differences obtained in the scores on the pre-test and post-test. Although the participating population is made up of only 14 students, the significance of the differences between the initial and final tests was determined using paired t-tests. To this end, the normality of the datasets was checked previously using the Saphiro-Wilk test (Saphiro & Wilk, 1965). The comparison of means was done using the standard t-student test for dependent samples in the case of normal distributed datasets. We used the Wilcoxon signed-ranked test (Wilcoxon, 1945), a non-parametric equivalent of the *t*-test, for non-normal data (Field et al., 2012). The analysis showed a significance level of 0.05. In the case where significant differences were found, the effect size r was calculated. A value of r = .10 means a small effect size, r = .30 can be classified as a medium effect size and r > .50 means a large effect size (Field et al., 2012).

Together with the mean score analysis we performed a detailed study on the success- per-counting attempt for each principle. These detailed studies were carried out in terms of the assessment-rubric attempts performed by the children both in the pre-test and post-test, as described in the following section.

Results

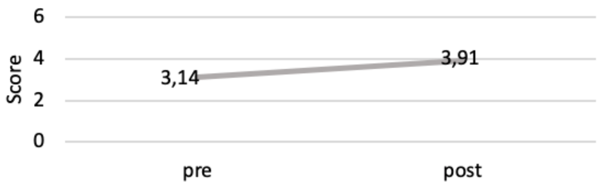
The results section followed a quantitative analysis on the gains in the children's counting skills, assessed from the pre-test and post-test. To explain the results in more detail, three subsections have been organized corresponding to the research questions initially posed. Thus, we will make the following distinctions: i) a global analysis regarding all five counting principles; ii) a specific and independent analysis on the evolution of each of the counting principles; and iii) an analysis concerning the improvement in the counting principles related to the ages of the participants.

Global Analysis

The obtained results in the pre-test ranged between 0.7 and 4.4, with 5 being the maximum possible score. The mean value of the pre-test scores was M = 3.14 (N = 14). As regards the post-test, the lowest score was 1.3 and the maximum score was 5 out of 5. The mean value for the post-test was M = 3.91 (N = 14). As shown in Figure 14 comparing both means, in general terms, a gain of 0.77 was achieved between the pre-test and post-test scores. Moreover, this difference between pre- and post-test scores is statistically significant with p = .0092, and r = .70, indicating a large effect size improvement.

Figure 14

Evolution of global counting skills assessed from the pre-test and post-test



Analysis of the evolution of each of the counting principles

Regarding each individual counting principle, the difference between the pre and post interventions shows gains in each of the counting principles, as shown in Table 7. In particular, the gain in the order-irrelevance principle (.23) was particularly noteworthy. The gain in the one-to-one correspondence principle (.21) and the gain in the cardinality principle (.20) are also remarkable. The rest of the counting principles also evolved, although with lower gains. Below, we discuss the significant differences encountered for each of the counting principles.

Regarding the one-to-one correspondence principle, the data from the pre-test and post-test was non-normal distributed, hence, the non-parametric Wilcoxon signed-ranked test was used for the contrast of means. The analysis revealed that the mean score obtained in the post-test (M = .90) was significantly higher than the mean score obtained in the pre-test (M = .69), p = .0069, with a large effect size r = .72.

In the case of the order-stable principle, both the pre-test and post-test scores followed a normal distribution. The dependent t-test reveals that the difference between the pre-test mean (M = .77) and the post-test mean (M = .80) was non-significant (p > .05).

The data obtained concerning the cardinality principle, the abstraction principle and the order-irrelevance principle in both the pre- and post-test assessments were non-normal for the three principles, thus the comparison of means was carried out again using the non-parametric test. As for the cardinality principle data, although finding differences between

Table 7.

Gains and scores obtained in the pre-test and post-test for each counting principle

	Counting principle										TOTAL	
	One-to-one correspondence		Stable-order		Cardinality		Abstraction		Order-irrelevance			
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Mean	.69	.90	.77	.80	.66	.86	.80	.89	.23	.46	3.14	3.91
Gain	.21		.03		.20		.09		.23		.77	

the pre-test score (*M* = .66) and the post-test score (*M* = .86), this was non-significant (*p* > .05). For the abstraction principle the results showed that the difference between the obtained mean in the pre-test (*M* = .80) and in the post-test (*M* = .89) was also non-significant (p > 0.05). Finally, for the order-irrelevance principle the Wilcoxon test showed a significant difference between the score obtained in the pre-test (*M* = .23) and the value obtained in the post-test (*M* = .46), *p* = .0042, *r* = .76, which represented a large effect size.

In the following, we report the results concerning the analysis based on the rate of -success attempts for each counting principle.

Detailed results on the one-to-one correspondence principle. Table 8 describes the evolution of the success rate per counting attempt for the one-to-one correspondence principle during the pre-test and post-test.

Table 8

Success rate per counting attempt on the one-to-one correspondence principle

Correct attempts	Pre-test	Post-test
5	28.57%	64.28%
4	14.29%	21.43%
3	35.71%	14.29%
2	14.29%	0.00%
1	7.14%	0.00%
0	0.00%	0.00%

A clear evolution and improvement can be seen in reference to the one-to-one correspondence principle after the intervention, since the total number of successful attempts was not less than 3 out of 5. In addition, during the post-test 64.28% of the children from the sample, equivalent to 9 students, managed to establish a correct one-to-one correspondence principle in each of the 5 attempts, compared to 28.57% of the children, 4 students, in relation to the pre-test.

Regarding the type of errors, three types of errors were evaluated during the counting attempts: item omission, assignment of more than one number-word and non-assignment of a number-word. As seen in Table 9, all three types of errors occurred on various occasions in both tests. Even so, there was a difference between the mean errors committed during the initial test (*M* = 1.57) and the final test (*M* = 0.5).

Table 9

Types of detected errors during the pre-test and the post-test

Student	Pre-test	Post-test
1	1 assignment error	0 errors
2	2 assignment errors 1 non-assignment error	1 assignment error 1 non-assignment error
3	1 item omission error 1 assignment error	0 errors
4	1 item omission error 1 assignment error	2 assignment errors
5	0 errors	0 errors
6	0 errors	0 errors
7	0 errors	0 errors
8	1 assignment error	0 errors
9	0 errors	0 errors
10	2 item omission errors 1 non-assignment error	1 item omission error
11	1 item omission error 1 assignment error	0 errors
12	1 item omission error 1 assignment error	1 item omission error
13	2 item omission errors 1 assignment error 1 non-assignment error	1 assignment error
14	2 assignment errors	0 errors
Mean error	1.57	.50

As can be seen in Table 10, during the pre-test the assignment of more than one number-word and item omission errors were more common. Regarding the post-test, although the number of errors was smaller, the error commission was still maintained. The distribution of error types was similar in both assessments: more assignment of more than number-word errors, followed by errors of item omission and, finally, errors of non-assignment of a number-word during the counting process.

Table 10

Absolute frequency of error types related to the one-to-one correspondence principle

Error type	Pre-test	Post-test
Item omission	8	2
Assignment of more than one number-word	11	4
Non-assignment of number-word	3	1
TOTAL	22	7

Detailed results on the stable-order principle. As shown in Table 11, there were slight differences between the success rates in the pre-test and post-test assessments. The results showed that 64.28% of the children achieved the maximum number of correct answers in both the pre-test and post-test. However, an improvement can be observed from the pre-test to the post-test, as the percentage of students who managed 4 correct attempts increased to 14.49% in the final test, compared to the initial test which was 7.14%. Even so, the rate of children who managed 2 correct attempts has reduced to 0.00% in the post-test, compared to 7.14% in the pre-test.

Table 11

Success rate per counting attempt on the stable-order principle

Correct attempts	Pre-test	Post-test
5	64.28%	64.28%
4	7.14%	14.29%
3	7.14%	7.14%
2	7.14%	0.00%
1	0.00%	0.00%
0	14.29%	14.29%

Regarding the type of errors detected during the evaluation of the order-stable principle (label errors), in general terms we found that, for our sample, in the stable and conventional counting sequence portion the children were able to count up to the number-word "five". Counting sequences with a stable, although non-conventional portion, were observed. Other cases with non-stable and unconventional portions were also detected. It was decided not to include any more information regarding this topic as it is out of the scope of the research questions posed.

Detailed results on the cardinality principle. During the pre-test, as specified in Table 12, the results showed that 64.28% of the children managed to establish the cardinality of the posed set in the five counting attempts. In addition, 7.14% made only one correct cardinality assignment and 28.58% were unsuccessful at all of the attempts. Regarding the post-test, the rate of children who acquired the cardinality principle in all five attempts increased to 85.71%, while 14.29% of the children were unable to correctly answer in any of the five attempts. Certainly, a clear evolution and improvement can be seen in reference to the cardinality principle after the intervention.

Table 12

Success rate per counting attempt on the cardinality principle

Correct attempts	Pre-test	Post-test
5	64.28%	85.71%
4	0.00%	0.00%
3	0.00%	0.00%
2	0.00%	0.00%
1	7.14%	0.00%
0	28.58%	14.29%

As mentioned in the instrument description, following the study by Wynn (1992), the attempts in which children identified the last emitted number-word as the cardinal of the set were considered correct. But in order to delve in the correct acquisition of counting skills, we analyzed the attempts in which the children indicate the correct or incorrect number-word. As can be seen in Table 13, the mean score of correct or incorrect last number-word emissions among all the students has been analyzed. Note that in Table 13 scores showing 0.00% in both correct and incorrect last number-word columns mean that the specific student failed in all counting attempts with regard to the cardinality principle. The results obtained show that children who emitted the correct last number-word in their counting attempts increased in the post-intervention assessment, as the mean score has increased from .43 to .76, showing an improvement in the general counting process.

Table 13

Correct and incorrect number-word emissions among the successful counting attempts performed by the children during the cardinality principle assessment

		Pre-test		Post-test
Child	Last number-word correct	Last number-word incorrect	Last number-word correct	Last number-word incorrect
1	.80	.20	1.00	.00
2	.40	.60	.60	.40
3	.60	.40	1.00	.00
4	.60	.40	.60	.40
5	.00	.00	1.00	.00
6	.60	.40	1.00	.00
7	1.00	.00	.80	.20
8	.80	.20	1.00	.00
9	1.00	.00	1.00	.00
10	.00	.00	.80	.20
11	.60	.40	1.00	.00
12	.00	.20	.00	.00
13	.00	.00	.80	.20
14	.00	.00	.00	.00
Mean value	.43	.20	.76	.10

Detailed results on the abstraction principle. The results obtained from the alteration of color in the counting tasks in the pre-test and post-test showed no differences among counting attempts. Thus, in this subsection the abstraction principle will be specifically addressed concerning the shape variation in the collection to be counted over the attempts. As can be seen in Table 14, the success rates among attempts showed a slight variation from pre-test to post-test. This fact could be an indication that the abstraction principle is the one that is acquired later, as we will discuss.

Table 14

Success rate per counting attempt on the abstraction principle regarding shape-variations

Correct attempts	Pre-test	Post-test
5	57.15%	71.43%
4	0.00%	0.00%
3	0.00%	7.14%
2	7.14%	7.14%
1	0.00%	0.00%
0	35.71%	14.29%

Detailed results concerning the order-irrelevance principle. To conclude, in Table 15 we report the results concerning the success rate for the attempts on the order-irrelevance principle. As can be seen in Table 15, the children seemed to improve in correct attempts after the intervention. This is so because prior to the intervention the success rates were distributed between 0 to 3 correct attempts; after the intervention the percentages were more distributed, as 7.14% of the children did not succeed in any attempt, 35.71% succeeded in one attempt, and 14.29% succeeded in two and three attempts. Furthermore, unlike the pre-test, 21.43% of the children have four correct answers and 7.14% have five correct answers in four and five attempts, respectively.

Table 15

Success rate per counting attempt on the order-irrelevance principle

Correct attempts	Pre-test	Post-test
5	0.00%	7.14%
4	0.00%	21.43%
3	21.43%	14.29%
2	14.29%	14.29%
1	21.43%	35.71%
0	42.85%	7.14%

Analysis of the Influence of the Age Factor on the Scores Obtained

Finally, we report the results taking into account age as an analysis factor. To this end, the children were divided into two differentiated groups: Group 1 included children from 3.5 to 3.9 years-old; and Group 2 included children from 4 to 4.4 years-old. The mean scores obtained in the pre-test and post-test were again compared and analyzed based on the age factor. As can be seen in Table 16, the pre-test and post-test score comparison shows that the performance of the children in group 1 (3.5 to 3.9 years-old) was lower than the children in group 2 (4 to 4.4-old years), as the gain between the pre-test and post-test was 0.50 for group 1 and 1.03 for group 2. The mean scores were M = 3.1 in the pre-test and M = 3.6 in the post-test for the younger group, in contrast to the older group who scored M = 3.18 in the pre-test and M = 4.21 in the post-test. Statistical significance has not been determined for this age-separated sample, since the sample is too small and would lack statistical robustness, nevertheless, it seems that age is a key factor that affects the acquisition of counting skills.

Table 16

Mean scores and gain analyzed by age

	Age factor analysis			
	Group 1: 3.5 to 3.9 years-old		Group 2: 4 to 4.4 years-old	
	Pre-test	Post-test	Pre-test	Post-test
Mean score	3.10	3.60	3.18	4.21
Gain		.50		1.03

Discussion and Conclusions

This study has allowed us to explore the counting abilities of 3-year-old children to develop skills related to the counting principles, as well as to design an ad hoc intervention that improves these cognitive skills. This section discusses the results obtained in line with the bibliographic review proposed at the beginning of the paper in order to answer the research questions posed. We have arranged this section in terms of the research questions.

RQ1: Is it possible to significantly increase the acquisition level of skills related to counting principles in 3-year-old students with an ad hoc designed intervention?

To answer the first research question, an analysis of the mean scores obtained, both in pre-test and post-test, has been carried. Prior to the 4-week intervention, the children obtained an overall mean score lower than the average obtained after the intervention. This difference turns out to be statistically significant, indicating that the intervention was effective for the 3-year-old children, resulting in an improvement in the acquisition of counting principles. Also, the effect size of the intervention has been estimated as large.

RQ2: With this particular instructional design, are there significant improvements in any of the counting principles?

The results on the evolution of each of the counting principles, analyzed separately in the previous section, have shown that the five counting-principle skills improved after the designed classroom intervention.

In particular, concerning the one-to-one correspondence principle, the results showed that there is a statistically significant gain after the instruction. The performance of the children during the post-test showed that nearly 2/3 of the participants (64.28%) managed to establish the one-to-one correspondence when counting collections after the intervention, while less than 1/3 of the participants (28.57%) were able do this in the pre-test. Our findings are in line with Potter and Levy (1968), who affirm that the ability to establish one-to-one correspondence when counting collections is acquired at the age of two. In addition, our results align with the results obtained by other researchers (Gelman & Gallistel, 1978; Gelman & Meck, 1983) who state that children aged between three to four years-old are in the process of acquiring the one-to-one correspondence principle. In contrast, other authors report lower success rates in one-to-one correspondence counting skills. In Sarnecka and Carey (2008), children obtained an almost excellent global mean score in their one-to-one counting test. Similarly, Johnson et al. (2019) showed that less than 25% of their sample were able to successfully solve the one-to-one correspondence tasks. A detailed analysis concerning the errors related to the one-to-one correspondence principle has shown that all three types of errors were reduced after the intervention. As stated by Dowker (2005), children with low IQs are less good at number naming. They use to performed worse than their counterparts at detecting counting errors, especially with counting sequences beyond 5. From our study, although we have not carried out any IQ measurement, we concluded that the number of counting errors has been decreased after the intervention. In both assessments, prior to and after the intervention, the assignment of more than one number-word to an item when counting a collection was the most common error.

Skills related to the stable-order principle improved after the intervention, but with no statistical significance. Almost 2/3 of the participants managed to establish this principle over all the counting attempts in both tests. After the intervention, except for two children who had not yet acquired the stable-order principle, the rest of participants were able to establish the stable-order principle with a higher number of correct answers than errors. These results differ from those found by Johnson et al. (2019), who conclude that the percentage of students who could follow a stable and conventional order during counting tasks was less than 50%. Also, in contrast to the results obtained in our study, Sarnecka and Carey (2008) found that almost all the children in their study had already acquired the stable-order principle in its entirety. However, Fuson (1988) claims that 3-year-old students are already capable of using a stable and conventional sequence when counting up to five, while children of approximately four and a half years old are already beginning to be able to recite a stable and conventional sequence when counting between 10 and 20 elements. Chamorro et al. (2005) state that children are able to successfully count up to 10 with a stable and conventional number sequence at the age of four and a half years old. Thus, the results obtained in our sample, with children between 3.5 and 4.4 years old, align with Fuson (1988) and Chamorro et al. (2005).

Regarding the cardinality principle, the difference between the pre-test and the post-test scores was not significant. However, the percentage of children who improved their skills related to the cardinality principle increased after the intervention. In particular, 85.71% of the children established the cardinality of the set in all five counting attempts after the post-test. This finding aligns with the results of several authors (Gelman & Gallistel, 1978; Gelman & Meck, 1983; Wynn, 1992; Sarnecka & Carey, 2008), who state that children between 3 and 4 years old have already, broadly, acquired the cardinality principle. Moreover, the designed intervention has shown a positive effect on the rate of correct last number-words emitted during the counting attempts, as the rate of correct cardinality number-words increased after the intervention from .43 to .76.

The abstraction principle assessment showed a non-significant improvement after the instruction. The rate of children who correctly carried out the five counting attempts with varying shapes increased from 57.15% in the pre-test, to 71.43% in the post-test. Gast (1957) found the age of full acquisition of the abstraction principle to be seven years old. According to this claim, our post-test results report that 28.57% of the children have not yet fully acquired this counting principle.

The differences in the assessment of the order-irrelevance principle were statistically significant. Prior to the intervention, almost half of the children (42.85%) were unable to succeed in any of the five attempts. After the intervention, the rate of children who were unable to apply the order-irrelevance principle in any of the attempts decreased to 7.14%. Taking into account the success on the five performed counting attempts, this principle seems to be the one that was least strengthened by our designed intervention. This issue could be due to the fact that the order-irrelevance principle can only be fully acquired if the other four counting principles have been previously acquired, as stated by Chamorro et al. (2005).

RQ3: Is the difference in age a factor key for the improvement in counting skills in a 3-year-old Early Childhood Education classroom?

As observed, the mean scores after the intervention were influenced by the age of the children. Although both groups (3.5-3.9 years-old and 4-4.4 years-old) obtained very similar results in the pre-test, there was a considerable gain in the post-test scores for the oldest group. In fact, after the intervention the gain score was .50 for the younger group, meanwhile the older group obtained a gain score of 1.03. This result is aligned with the those of Gelman and Meck (1983), since, according to them, 3-year-old children show greater difficulty in identifying errors when performing a counting task compared to 4-year-old children. However, other studies, report that no significant differences are found between the counting skills of 3-year-old and 4-year-old children (Lagos, 1992).

Limitations and Final Remarks

Our study has shown the potential and the effect of an ad hoc intervention focused on improving counting abilities in 3-year-old children. Nevertheless, some limitations need to be underlined. First of all, the sample size is small as our study includes only 14 children, which could make the results not specifically representative. However, the diversity on the cognitive level observed among the participants, and the mathematics education literacy consulted, lead us to believe that a study with a larger sample will report similar results, although this claim should be confirmed by an experimental study. Moreover, this study was carried out in a real classroom scenario, in this way, the effects of these intervention could be exported to other school realities by other Early Childhood teachers. Another limitation is the pre-experimental design. The absence of a control group may pose problems regarding the intervention's effect validity on the level of acquisition of the counting principles. However, as has been argued in the methodology description, the pre-experimental design was intentionally chosen in order to offer the opportunity to the whole class-group to carry out the counting tasks and improve their cognitive abilities. The approach followed in this study was aimed at avoiding an imbalance in the class-group in relation to the counting process, as counting skills are a basic, essential part of elementary school practice.

Finally, despite that our intervention has been carried out with children without diagnosed learning difficulties, previous studies have shown the effectiveness of teaching sequences aimed at enhancing basic counting skills in both children who follow a typical or atypical developmental trajectory (Ansari et al., 2003; Kaufmann et al., 2003; Stock et al., 2010). With this in mind, the proposal that has been presented here and that has shown that it favors the acquisition of counting skills related to counting principles in typical 3-year-old students could also be useful as an effective instrument in students with learning difficulties.

Acknowledgements

Thanks to the tutor of the school where this research was carried out and to the children who have collaborated during the intervention. A. C.-J. thanks his family and Vicente Alfonso Benlliure for their support. This research was supported by Conselleria d'Innovació, Universitats, Ciència i Societat Digital de la Generalitat Valenciana through grant number GV/2019/146.

References

Ansari, D., Donlan, C., Thomas, M. S. C., Ewing, S. A., Peen, T., & Karmiloff-Smith, A. (2003). What makes counting count? Verbal and visuo-spatial contributions to typical and atypical number development. *Journal of Experimental Child Psychology*, *85*(1), 50–62. https://doi.org/10.1016/S0022-0965(03)00026-2

Baroody, A. J. (1987). The Development of Counting Strategies for Single-Digit Addition. *Journal for Research in Mathematics Education JRME*, *18*(2), 141–157. https://doi.org/10.5951/jresematheduc.18.2.0141

Baroody, A. J. (1992). The development of preschoolers' counting skills and principles. In J. Bideaud, C. Meljac, & J.-P. Fischer (Eds.), *Pathways to number: Children's developing numerical abilities.* (pp. 99–126). Lawrence Erlbaum Associates, Inc.

Briars, D. J., & Secada, W. G. (1988). Correspondence errors in children's counting. In *Children's counting and concepts of number* (pp. 63–91). Springer. https://doi.org/10.1007/978-1-4612-3754-9_3

Briars, D., & Siegler, R. S. (1984). A featural analysis of preschoolers' counting knowledge. *Developmental Psychology*, *20*(4), 607–618. https://doi.org/https://doi.org/10.1037/0012-1649.20.4.607

Chamorro, M. del C. (2005). *Didáctica de las Matemáticas para Educación Infantil*. Pearson Educación.

Clements, D. H., Sarama, J., & DiBiase, A.-M. (2002). Early Childhood Corner: Preschool and Kindergarten Mathematics: A National Conference. *Teaching Children Mathematics TCM*, *8*(9), 510–514. https://doi.org/10.5951/TCM.8.9.0510

Desoete, A., & Grégoire, J. (2006). Numerical competence in young children and in children with mathematics learning disabilities. *Learning and Individual Differences*, *16*(4), 351–367. https://doi.org/10.1016/j.lindif.2006.12.006

Dowker, A. (2005). *Individual differences in arithmetic: Implications for psychology, neuroscience and education*. Psychology Press. https://doi.org/10.4324/9780203324899

Field, A., Miles, J., & Field, Z. (2012). *Discovering Statistics Using R*. SAGE Publications

Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: Cognitive predictors of achievement growth in mathematics. *Developmental Psychology*, *47*(6), 1539–1552. https://doi.org/10.1037/a0025510.Cognitive

Geary, D. C., Bow-Thomas, C. C., & Yao, Y. (1992). Counting knowledge and skill in cognitive addition: A comparison of normal and mathematically disabled children. *Journal of Experimental Child Psychology*, *54*(3), 372–391. https://doi.org/10.1016/0022-0965(92)90026-3

Geary, D. C., Hamson, C. O., & Hoard, M. K. (2000). Numerical and Arithmetical Cognition: A Longitudinal Study of Process and Concept Deficits in Children with Learning Disability. *Journal of Experimental Child Psychology*, *77*(3), 236–263. https://doi.org/10.1006/jecp.2000.2561

Gelman, R., & Galistell, C. H. (1978). *The Child's Understanding of Number*. Harvard University Press.

Gelman, R., & Meck, E. (1983). Preschoolers' counting: Principles before skill. *Cognition*, *13*(3), 343–359. https://doi.org/https://doi.org/10.1016/0010-0277(83)90014-8

Hwang, S. (2020). Examining the effect of students' early numeracy activities at home on later mathematics achievement via early numeracy competencies and self-efficacy beliefs. *International Electronic Journal of Elementary Education*, *13*(1), 47–56. https://doi.org/10.26822/iejee.2020.172

Johnson, N. C., Turrou, A. C., McMillan, B. G., Raygoza, M. C., & Franke, M. L. (2019). "Can you help me count these pennies?": Surfacing preschoolers' understandings of counting. *Mathematical Thinking and Learning*, *21*(4), 237–264. https://doi.org/10.1080/10986065.2019.1588206

Kaufmann, L., Handl, P., & Thöny, B. (2003). Evaluation of a Numeracy Intervention Program Focusing on Basic Numerical Knowledge and Conceptual Knowledge: A Pilot Study. *Journal of Learning Disabilities*, *36*(6), 564–573.

Lagos, M. O. (1992). *Análisis estructural de la adquisición y desarrollo de la habilidad de contar* (PhD Thesis). Universidad Complutense de Madrid.

LeFevre, J. A., Smith-Chant, B. L., Fast, L., Skwarchuk, S. L., Sargla, E., Arnup, J. S., Penner-Wilger, M., Bisanz, J., & Kamawar, D. (2006). What counts as knowing? The development of conceptual and procedural knowledge of counting from kindergarten through Grade 2. *Journal of Experimental Child Psychology*, *93*(4), 285–303. https://doi.org/10.1016/j.jecp.2005.11.002

Fuson, K. C. (1988). *Children's counting and concepts of number*. Springer-Verlag.

Markman, E. M. (1979). Classes and collections: Conceptual organization and numerical abilities. *Cognitive Psychology*, *11*(4), 395–411. https://doi.org/https://doi.org/10.1016/0010-0285(79)90018-5

Potter, M. C., & Levy, E. I. (1968). Spatial enumeration without counting. *Child Development*, *39*, 265–273.

McClelland, M. M., Acock, A. C., Piccinin, A., Rhea, S. A., & Stallings, M. C. (2013). Relations between preschool attention span-persistence and age 25 educational outcomes. *Early Childhood Research Quarterly*, *28*(2), 314–324. https://doi.org/10.1016/j.ecresq.2012.07.008

Rittle-Johnson, B., & Siegler, R. (1998). The relation between conceptual and procedural knowledge in learning mathematics: A review. In C. Donlan (Ed.), *The development of mathematical skills* (pp. 75–110). Lawrence Erlbaum Associates, Inc.

Shapiro, S. S., & Wilk, M. B. (1965). An Analysis of Variance Test for Normality (Complete Samples). *Biometrika*, *52*(3/4), 591. https://doi.org/10.2307/2333709

Salihu, L., & Räsänen, P. (2018). Mathematics skills of Kosovar primary school children: A special view on children with mathematical learning difficulties. *International Electronic Journal of Elementary Education*, *10*(4), 421–430. https://doi.org/10.26822/iejee.2018438132

Sarama, J., & Clements, D. H. (2003). Early Childhood Corner: Building Blocks of Early Childhood Mathematics. *Teaching Children Mathematics TCM*, *9*(8), 480–484. https://doi.org/10.5951/TCM.9.8.0480

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. Routledge.

Sarnecka, B. W., & Carey, S. (2008). How counting represents number: What children must learn and when they learn it. *Cognition*, *108*(3), 662–674. https://doi.org/10.1016/j.cognition.2008.05.007

Siegler, R. S. (1991). In young children's counting, procedures precede principles. *Educational Psychology Review*, *3*(2), 127–135. https://doi.org/10.1007/BF01417924

Stock, P., Desoete, A., & Roeyers, H. (2010). Detecting children with arithmetic disabilities from kindergarten: Evidence from a 3-year longitudinal study on the role of preparatory arithmetic abilities. *Journal of Learning Disabilities*, *43*(3), 250–268. https://doi.org/10.1177/0022219409345011

Wilcoxon, F. (1945). Individual Comparisons by Ranking Methods. *Biometrics Bulletin*, *1*, 80–83.

Wynn, K. (1990). Children's understanding of counting. *Cognition*, *36*(2), 155–193. https://doi.org/https://doi.org/10.1016/0010-0277(90)90003-3

Yilmaz, Z. (2017). Young children's number sense development: Age related complexity across cases of three children. *International Electronic Journal of Elementary Education*, *9*(4), 891–902.