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Ulam Stability of Quadratic Mapping Connected With Homomorphisms and Derivations in Non-Archimedean Banach Algebras

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Abstract. This study presents a novel quadratic functional equation. The primary objective of this study is to examine the stability of a quadratic functional equation associated with homomorphisms and derivations (briefly, hom-der) in non-Archimedean Banach algebras through direct and fixed point methodologies. Furthermore, we offer examples wherein the stability of this quadratic functional equation can be regulated by the summation and multiplication of powers of norms.

1. Introduction and Preliminaries

The theory of functional equations involves identifying functions that fulfil a specified equation. A functional equation resembles a standard algebraic equation; however, rather than seeking

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unknown elements within a set, the focus is on identifying a function that fulfils the equation's requirements.

The concept of functional equations raises a crucial question: Does a function that roughly fulfills a functional equation have to be near to the problem's exact solutions? If the equation has a unique solution, it is deemed stable.Ulam [31] developed a stability problem for the Cauchy functional equation, $\phi(x_1+x_2)=\phi(x_1)+\phi(x_2)$, as follows:

Consider a group $(A,*)$, a metric group $(A',\cdot)$ equipped with d, and a function $\phi: A\to A'$. Is there a $\delta>0$ such that

$$d(\phi(x_1 * x_2), \phi(x_1)\cdot\phi(x_2)) \leq \delta$$

holds for every $x_1, x_2 \in A$? If such a mapping ϕ exists, can we find a homomorphism $g: A\to A'$ fulfilling

$$d(\phi(x_1), g(x_1)) \leq \epsilon$$

for every $x_1\in A$?

In 1941, Hyers [11] offered a favorable resolution to the issue. Aoki [1] subsequently expanded Hyers' findings to additive mappings, while Rassias [24] further generalized this for linear mappings by examining stability in the context of unbounded Cauchy differences. In 1994, Găvruţa [10] expanded Rassias' theory by substituting the constraint $\epsilon(\|x_1\|^b+\|x_2\|^b)$ with a more adaptable general control function $\varphi(x_1,x_2)$. Furthermore, Rassias [23] proposed a less stringent condition that incorporates a mixture of powers of norms, thereby further generalizing the Hyers stability theorem.

In the domain of functional equation stability, the quadratic functional equation is expressed as follows:

$$\phi(x_1+x_2)+\phi(x_1-x_2)=2\phi(x_1)+2\phi(x_2), \tag{1.1}$$

represents the most renowned functional equation. The function $\phi(x_1)=x_1^2$ fulfills the functional equation (1.1). Arunkumar [2], Bae and Jun [5], Czerwik [6], Hyers [13,14], Chang and Kim [15], Pl. Kannappan [16], Kim [17] and Tamilvanan [29] have investigated the stability problem of several quadratic functional equations in extensive detail, and there have been a number of interesting results on this problem.

Eshaghi and Khodaei [8] recently introduced the quadratic functional equation and investigated the Ulam stability in Banach spaces. The homomorphism and derivations of non-Archimedean algebras were studied by Eshaghi Gordji in [9] and numerous functional equations were investigated by Semrl [26] regarding this problem. The authors established the Hyers-Ulam stability of hom-derivations in complex Banach algebras that are related to an additive functional equation in the paper referred to as [18]. Similar to the previous example, the notion of hyper homomorphisms and hyper derivations in Banach algebras was presented in [25]. Additionally, the stability of these structures was established for three-additive functional equations. The quadratic functional equations are studied further in [7,19–22].

The purpose of this study is to present a novel quadratic functional equation, which is denoted as

$$\phi(ut+vs)+\phi(ts-vu)=[\phi(v)+\phi(t)][\phi(u)+\phi(s)], \tag{1.2}$$

and to derive its universal solution. The major objective is to investigate the stability of this quadratic functional equation within the framework of homomorphisms and derivations performed within non-Archimedean Banach algebras. This will be accomplished through the use of both direct and fixed-point approaches. In addition, applications are described in which the stability of the equation is determined by ways in which sums and products of powers of norms are combined. The reader can get more details about notions of non-Archimedean Banach algebras from [27,28,30].

Ring homomorphism stability was introduced by Badora [3] in two unital Banach algebras. The stability results for operator algebras derivations was made for the first time by Semrl [26]. The stability of the equation $\phi(x_1x_2)=x_1\phi(x_2)+\phi(x_1)_2$, where ϕ is a function on a unitary normed algebra A, was demonstrated by Badora in [4].

Definition 1.1. *[2]*

Consider two algebras, A_1, A_2. If ϕ is a quadratic mapping that fulfills $\phi(x_1x_2)=\phi(x_2)\phi(x_1)$ for every $x_1, x_2 \in A_1$, then the function $\phi : A_1 \to A_2$ is referred to as a quadratic homomorphism.

For example, if A_1 is commutative, then $\phi(x_1)=x_1^2(x_1 \in A_1)$, which defines the function $\phi : A_1 \to A_2$, is a quadratic homomorphism.

Definition 1.2. *[2]*

If $\phi : A_1 \to A_2$ is a quadratic mapping that fulfills $\phi(x_1x_2)=x_1^2\phi(x_2)+\phi(x_1)x_2^2$ for every $x_1, x_2 \in A_1$, then ϕ is referred to as a quadratic derivation.

Quadratic derivations and ring derivations are distinct from one another.

To simplify notation, we adopt the following abbreviation for a mapping $\phi : M \to N$:

$$F_Q(s,t,u,v)=[\phi(v)+\phi(t)][\phi(u)+\phi(s)]-[\phi(sv+tu)+\phi(st-uv)],$$

for all $s,t,u,v \in M$.

In the following sections, let us consider M to be a Banach module with norm $\|\cdot\|$, and N to be a non-Archimedean Banach module with norm $\|\cdot\|$.

2. Ulam Stability of (1.2): Direct Approach

Theorem 2.1. *Let a function $\vartheta : M^4 \to [0,\infty)$ such that the series*

$$\sum_{\zeta=0}^{\infty}\frac{\vartheta\left(2^{\zeta}s,2^{\zeta}t,2^{\zeta}u,2^{\zeta}v\right)}{2^{2\zeta}}$$

converges in $\mathbb{R}$ and

$$\lim_{\zeta\to\infty}\frac{\vartheta\left(2^{\zeta}s,2^{\zeta}t,2^{\zeta}u,2^{\zeta}v\right)}{2^{2\zeta}}<\infty,$$

for every $s,t,u,v \in M$ and a function $\phi : M \to N$ such that

$$\|F_Q(s,t,u,v)\| \leq \vartheta(s,t,u,v), \tag{2.1}$$

for every $s,t,u,v \in M$, and

$$\|\phi(st) - \phi(s)\phi(t)\| \leq \vartheta(s,t,0,0), \tag{2.2}$$

for every $s,t \in M$. Then there is only one quadratic homomorphism $H_1 : M \to N$ fulfilling (1.2) and

$$\|\phi(s) - H_1(s)\| \leq \frac{1}{2^2}\sum_{p=0}^{\infty}\frac{\vartheta\left(\left(\sqrt{2}\right)^p\sqrt{s},\left(\sqrt{2}\right)^p\sqrt{s},\left(\sqrt{2}\right)^p\sqrt{s},\left(\sqrt{2}\right)^p\sqrt{s}\right)}{2^{2p}}, \tag{2.3}$$

for all $s \in M$. The mapping $H_1(s)$ is defined by

$$H_1(s) = \lim_{\zeta\to\infty}\frac{\phi\left(2^{\zeta}s\right)}{2^{2\zeta}},$$

for all $s \in M$.

Proof. Replacing (s,t,u,v) by $(\sqrt{s},\sqrt{s},\sqrt{s},\sqrt{s})$ in (2.1) and dividing by 2^2, gives

$$\left\|\frac{\phi(2s)}{2^2} - \phi(s)\right\| \leq \frac{1}{2^2}\vartheta\left(\sqrt{s},\sqrt{s},\sqrt{s},\sqrt{s}\right), \tag{2.4}$$

for every $s \in M$. Setting $s = 2s$ in (2.4) and dividing by 2^2, we get

$$\left\|\frac{\phi\left(2^2 s\right)}{2^4} - \frac{\phi(2s)}{2^2}\right\| \leq \frac{1}{2^4}\vartheta\left(\sqrt{2s},\sqrt{2s},\sqrt{2s},\sqrt{2s}\right),$$

for every $s \in M$. In general, for any integer $\zeta > 0$, we obtain

$$\begin{aligned}\left\|\frac{\phi\left(2^{\zeta}s\right)}{2^{2\zeta}} - \phi(s)\right\| &\leq \frac{1}{2^2}\sum_{p=0}^{\zeta-1}\frac{1}{2^{2p}}\vartheta\left(\sqrt{2^p s},\sqrt{2^p s},\sqrt{2^p s},\sqrt{2^p s}\right)\\ &\leq \frac{1}{2^2}\sum_{p=0}^{\infty}\frac{1}{2^{2p}}\vartheta\left(\sqrt{2^p s},\sqrt{2^p s},\sqrt{2^p s},\sqrt{2^p s}\right),\end{aligned} \tag{2.5}$$

for all $s \in M$. To show that the sequence $\left\{\frac{\phi\left(2^{\zeta}s\right)}{2^{2\zeta}}\right\}$ is convergent, replace s by $2^q s$ in (2.5) and divide by 2^{2q}, for any $q,\zeta > 0$, we get

$$\begin{aligned}\left\|\frac{\phi\left(2^{\zeta+q}s\right)}{2^{2(\zeta+q)}} - \frac{\phi(2^q s)}{2^{2q}}\right\| &= \frac{1}{2^{2q}}\left\|\frac{\phi\left(2^{\zeta+q}s\right)}{2^{2\zeta}} - \phi(2^q s)\right\|\\ &\leq \frac{1}{2^2}\sum_{p=0}^{\zeta-1}\frac{1}{2^{2(q+p)}}\vartheta\left(\sqrt{2^{q+p}s},\sqrt{2^{q+p}s},\sqrt{2^{q+p}s},\sqrt{2^{q+p}s}\right)\\ &\leq \frac{1}{2^2}\sum_{p=0}^{\infty}\frac{1}{2^{2(q+p)}}\vartheta\left(\sqrt{2^{q+p}s},\sqrt{2^{q+p}s},\sqrt{2^{q+p}s},\sqrt{2^{q+p}s}\right)\\ &\longrightarrow 0 \quad \text{as } q\to\infty,\end{aligned}$$

for all $s \in M$. Thus, the sequence $\left\{\frac{\phi(2^{\zeta}s)}{2^{2\zeta}}\right\}$ is a Cauchy sequence. By the result N is complete, there exists a function $H_1 : M \to N$ such that

$$H_1(s) = \lim_{\zeta\to\infty} \frac{\phi\left(2^{\zeta}s\right)}{2^{2\zeta}},$$

for all $s \in M$. We obtain (2.3) by taking the limit as $\zeta \to \infty$ in (2.5). Now, we prove that H_1 fulfills (1.2). Replacing (s,t,u,v) by $(2^{\zeta}s, 2^{\zeta}t, 2^{\zeta}u, 2^{\zeta}v)$ and dividing by $2^{2\zeta}$ in (2.1), we get

$$\frac{1}{2^{2\zeta}}\left\|F_Q\left(2^{\zeta}s, 2^{\zeta}t, 2^{\zeta}u, 2^{\zeta}v\right)\right\| \leq \frac{\vartheta\left(2^{\zeta}s, 2^{\zeta}t, 2^{\zeta}u, 2^{\zeta}v\right)}{2^{2\zeta}},$$

for all $s,t,u,v \in M$. Taking the limit as $\zeta \to \infty$ in the above inequalities, we get $H_1 = 0$. Thus, H_1 fulfills the equation (1.2) for every $s,t,u,v \in M$. This proves that the function H_1 is quadratic. Then,

$$\begin{aligned}\left\|H_1(st) - H_1(s)H_1(t)\right\| &= \lim_{\zeta\to\infty} \frac{1}{2^{4\zeta}}\|\phi\left(2^{2\zeta}st\right) - \phi\left(2^{\zeta}s\right)\phi\left(2^{\zeta}t\right)\| \\ &\leq \lim_{\zeta\to\infty} \frac{1}{2^{4\zeta}}\vartheta\left(2^{\zeta}s, 2^{\zeta}t, 0, 0\right) = 0\end{aligned}$$

for all $s,t \in M$. Therefore, H_1 is quadratic homomorphism. To show that H_1 is the only one solution, let us consider another quadratic homomorphism $H_2 : M \to N$ fulfilling (1.2) and (2.3). Then

$$\begin{aligned}\left\|H_1(s) - H_2(s)\right\| &= \frac{1}{2^{2\zeta}}\left\|H_1\left(2^{\zeta}s\right) - H_2\left(2^{\zeta}s\right)\right\| \\ &\leq \frac{1}{2^{2\zeta}}\left\{\left\|H_1\left(2^{\zeta}s\right) - \phi\left(2^{\zeta}s\right)\right\| + \left\|\phi\left(2^{\zeta}s\right) - H_2\left(2^{\zeta}s\right)\right\|\right\} \\ &\leq \frac{2}{2^2}\sum_{p=0}^{\infty} \frac{1}{2^{2(p+\zeta)}}\vartheta\left(\sqrt{2^{p+\zeta}s}, \sqrt{2^{p+\zeta}s}, \sqrt{2^{p+\zeta}s}, \sqrt{2^{p+\zeta}s}\right) \\ &\to 0 \quad as \quad \zeta \to \infty\end{aligned}$$

for all $s \in M$. Thus, the quadratic function H_1 is unique. Therefore, there is a unique quadratic homomorphism $H_1 : M \to N$ satisfying (2.3). □

The following theorem gives an alternative stability of Theorem 2.1.

Theorem 2.2. *Let* $\vartheta : M^4 \to [0,\infty)$ *be a mapping such that*

$$\sum_{\zeta=0}^{\infty} 2^{2\zeta}\vartheta\left(\frac{s}{2^{\zeta}}, \frac{t}{2^{\zeta}}, \frac{u}{2^{\zeta}}, \frac{v}{2^{\zeta}}\right)$$

converges in $\mathbb{R}$ *and*

$$\lim_{\zeta\to\infty} 2^{2\zeta}\vartheta\left(\frac{s}{2^{\zeta}}, \frac{t}{2^{\zeta}}, \frac{u}{2^{\zeta}}, \frac{v}{2^{\zeta}}\right) < \infty,$$

for every $s,t,u,v \in M$ and $\phi : M \to N$ be a function fulfills (2.1) *and* (2.2). *Then is only one quadratic homomorphism $H_1 : M \to N$ fulfilling (1.2) and*

$$\|\phi(s) - H_1(s)\| \leq \sum_{b=0}^{\infty} 2^{2b}\vartheta\left(\frac{\sqrt{s}}{\left(\sqrt{2}\right)^{1+b}}, \frac{\sqrt{s}}{\left(\sqrt{2}\right)^{1+b}}, \frac{\sqrt{s}}{\left(\sqrt{2}\right)^{1+b}}, \frac{\sqrt{s}}{\left(\sqrt{2}\right)^{1+b}}\right), \tag{2.6}$$

for every $s \in M$. The mapping $H_1 : M \to N$ is defined by

$$H_1(s) = \lim_{\zeta\to\infty} 2^{2\zeta}\phi\left(\frac{s}{2^{\zeta}}\right),$$

for every $s \in M$.

Proof. Replacing (s,t,u,v) by $(\sqrt{s}, \sqrt{s}, \sqrt{s}, \sqrt{s})$ in (2.1), we get

$$\left\|\phi(2s) - 2^2\phi(s)\right\| \leq \vartheta\left(\sqrt{s}, \sqrt{s}, \sqrt{s}, \sqrt{s}\right), \tag{2.7}$$

for every $s \in M$. Setting $s = \frac{s}{2}$ in (2.7), we have

$$\left\|\phi(s) - 2^2\phi\left(\frac{s}{2}\right)\right\| \leq \vartheta\left(\sqrt{\frac{s}{2}}, \sqrt{\frac{s}{2}}, \sqrt{\frac{s}{2}}, \sqrt{\frac{s}{2}}\right),$$

for every $s \in M$. For any positive integer $\zeta > 0$, we arrive

$$\begin{aligned}\left\|\phi(s) - 2^{2\zeta}\phi\left(\frac{s}{2^{\zeta}}\right)\right\| &\leq \sum_{b=0}^{\zeta-1} 2^{2b}\vartheta\left(\sqrt{2^{-(1+b)}s}, \sqrt{2^{-(1+b)}s}, \sqrt{2^{-(1+b)}s}, \sqrt{2^{-(1+b)}s}\right)\\ &\leq \sum_{b=0}^{\infty} 2^{2b}\vartheta\left(\sqrt{2^{-(1+b)}s}, \sqrt{2^{-(1+b)}s}, \sqrt{2^{-(1+b)}s}, \sqrt{2^{-(1+b)}s}\right),\end{aligned} \tag{2.8}$$

for every $s \in M$. To show that the sequence $\left\{2^{2\zeta}\phi\left(\frac{s}{2^{\zeta}}\right)\right\}$ is convergent, replace s by $\frac{s}{2^q}$ in (2.8) and multiply by 2^{2q}, for every $q, \zeta > 0$, we get

$$\begin{aligned}\left\|2^{2(\zeta+q)}\phi\left(\frac{s}{2^{\zeta+q}}\right) - 2^{2q}\phi\left(\frac{s}{2^q}\right)\right\| &= 2^{2q}\left\|2^{2\zeta}\phi\left(\frac{s}{2^{\zeta+q}}\right) - \phi\left(\frac{s}{2^q}\right)\right\|\\ &\leq \sum_{b=0}^{\zeta-1} 2^{2(q+b)}\vartheta\left(\sqrt{2^{-(1+q+b)}s}, \sqrt{2^{-(1+q+b)}s}, \sqrt{2^{-(1+q+b)}s}, \sqrt{2^{-(1+q+b)}s}\right)\\ &\leq \sum_{b=0}^{\infty} 2^{2(q+b)}\vartheta\left(\sqrt{2^{-(1+q+b)}s}, \sqrt{2^{-(1+q+b)}s}, \sqrt{2^{-(1+q+b)}s}, \sqrt{2^{-(1+q+b)}s}\right)\\ &\longrightarrow 0 \quad \text{as } q \to \infty.\end{aligned}$$

Hence $\left\{2^{2\zeta}\phi\left(\frac{s}{2^{\zeta}}\right)\right\}$ is a Cauchy sequence. As N is complete, there is a function $H_1 : M \to N$ satisfies

$$H_1(s) = \lim_{\zeta\to\infty} 2^{2\zeta}\phi\left(\frac{s}{2^{\zeta}}\right),$$

for every $s \in M$. We obtain (2.6) by taking the limit as $\zeta \to \infty$ in (2.8). Next, we need to show that H_1 fulfills (1.2). Switching (s,t,u,v) by $\left(\frac{s}{2^{\zeta}}, \frac{t}{2^{\zeta}}, \frac{u}{2^{\zeta}}, \frac{v}{2^{\zeta}}\right)$ and multiplying by $2^{2\zeta}$ in (2.1), we get

$$2^{2\zeta}\left\|F_Q\left(\frac{s}{2^{\zeta}}, \frac{t}{2^{\zeta}}, \frac{u}{2^{\zeta}}, \frac{v}{2^{\zeta}}\right)\right\| \leq 2^{2\zeta}\vartheta\left(\frac{s}{2^{\zeta}}, \frac{t}{2^{\zeta}}, \frac{u}{2^{\zeta}}, \frac{v}{2^{\zeta}}\right),$$

for every $s, t, u, v \in M$. Taking the limit as $\zeta \to \infty$ in the above inequalities, we get $H_1 = 0$. Hence, H_1 fulfills the equation (1.2) for every $s, t, u, v \in M$. This proves that the function H_1 is quadratic. Then,

$$\begin{aligned}\left\|H_1(st) - H_1(s)H_1(t)\right\| &= \lim_{\zeta\to\infty} 2^{4\zeta}\left\|\phi\left(\frac{st}{2^{2\zeta}}\right) - \phi\left(\frac{s}{2^{\zeta}}\right)\phi\left(\frac{t}{2^{\zeta}}\right)\right\| \\ &\leq \lim_{\zeta\to\infty} 2^{4\zeta}\vartheta\left(\frac{s}{2^{\zeta}}, \frac{t}{2^{\zeta}}, 0, 0\right) = 0,\end{aligned}$$

for all $s, t \in M$. Therefore, the function H_1 is a quadratic homomorphism. Finally, to show that H_1 is unique, let us consider another quadratic homomorphism $H_2 : M \to N$ satisfying the functional equation (1.2) and (2.6). Then

$$\begin{aligned}\left\|H_1(s) - H_2(s)\right\| &= 2^{2\zeta}\left\|H_1\left(\frac{s}{2^{\zeta}}\right) - H_2\left(\frac{s}{2^{\zeta}}\right)\right\| \\ &\leq 2^{2\zeta}\left\{\left\|H_1\left(\frac{s}{2^{\zeta}}\right) - \phi\left(\frac{s}{2^{\zeta}}\right)\right\| + \left\|\phi\left(\frac{s}{2^{\zeta}}\right) - H_2\left(\frac{s}{2^{\zeta}}\right)\right\|\right\} \\ &\leq 2\sum_{b=0}^{\infty} 2^{2(b+\zeta)}\vartheta\left(\sqrt{2^{-(1+b+\zeta)}s}, \sqrt{2^{-(1+b+\zeta)}s}, \sqrt{2^{-(1+b+\zeta)}s}, \sqrt{2^{-(1+b+\zeta)}s}\right) \\ &\to 0 \quad as \quad \zeta \to \infty\end{aligned}$$

for every $s \in M$. Thus, the quadratic solution H_1 is unique.

□

Theorem 2.3. *Let $\epsilon \in \{-1, 1\}$ and $\vartheta : M^4 \to [0, \infty)$ be a mapping such that*

$$\sum_{\zeta=0}^{\infty} \frac{\vartheta(2^{\zeta\epsilon}s, 2^{\zeta\epsilon}t, 2^{\zeta\epsilon}u, 2^{\zeta\epsilon}v)}{2^{2\zeta\epsilon}} \text{ converges in } \mathbb{R}$$

and

$$\lim_{\zeta\to\infty} \frac{\vartheta(2^{\zeta\epsilon}s, 2^{\zeta\epsilon}t, 2^{\zeta\epsilon}u, 2^{\zeta\epsilon}v)}{2^{2\zeta\epsilon}} < \infty,$$

for all $s, t, u, v \in M$. Let $\phi : M \to N$ be a function satisfies the inequality (2.1) *and*

$$\left\|\phi(st) - s^2\phi(t) - \phi(s)t^2\right\| \leq \vartheta(s, t, 0, 0),$$

for every $s, t \in M$, then there is only one quadratic derivation $D : M \to M$ fulfilling (1.2) and

$$\| \phi(s) - D(s) \| \leq \frac{1}{2^2} \sum_{b=\frac{1-\epsilon}{2}}^{\infty} \frac{\vartheta((\sqrt{2})^{b\epsilon}\sqrt{s}, (\sqrt{2})^{b\epsilon}\sqrt{s}, (\sqrt{2})^{b\epsilon}\sqrt{s}, (\sqrt{2})^{b\epsilon}\sqrt{s})}{2^{2b\epsilon}}, \tag{2.9}$$

for every $s \in M$. The function $D : M \to M$ is defined by

$$D(s) = \lim_{\zeta\to\infty} \frac{\phi(2^{\zeta\epsilon}s)}{2^{2\zeta\epsilon}},$$

for every $s \in M$.

Proof. By using the same argument used to prove the Theorem 2.1, there is only one quadratic function $D : M \to M$ fulfilling (2.9). The function $D : M \to M$ is defined as

$$D(s) = \lim_{\zeta\to\infty} \frac{\phi(2^{\zeta\epsilon}s)}{2^{2\zeta\epsilon}}.$$

The inequality (2.1) implies

$$\begin{aligned} \left\|D(st) - s^2D(t) - D(s)t^2\right\| &= \lim_{\zeta\to\infty} \frac{1}{2^{4\zeta\epsilon}} \left\|\phi(2^{2\zeta}st) - (2^{\zeta}s)^2\phi(2^{\zeta}t) - \phi(2^{\zeta}s)(2^{\zeta}t)^2\right\| \\ &\leq \lim_{\zeta\to\infty} \frac{1}{2^{4\zeta\epsilon}} \vartheta(2^{\zeta}s, 2^{\zeta}t, 0, 0) = 0 \end{aligned}$$

for every $s, t \in M$. Thus, there is a quadratic derivation function $D : M \to M$ satisfying (2.9). □

3. Ulam Stability of (1.2): Fixed Point Method

Theorem 3.1. *[2] Let (∂, d) be a complete generalized metric space and $T : \partial \to \partial$ be a strictly contractive function with $0 < L < 1$. Then, for every $u \in \partial$, either*
$(B1)$ $d(T^n u, T^{n+1}u) = \infty$ for all $n \leq 0$,

or

$(B2)$ there is an integer $n_0 > 0$ fulfills

- (i) *$d(T^n u, T^{n+1}u) < \infty$ for every $n \geq n_0$;*
- (ii) *the sequence $\{T^n u\}$ is convergent to a fixed point v^* of T.*
- (iii) *v^* is a unique fixed point of T in the set*
  $\Delta = \{v \in \partial : d(T^{n_0}u, v) < \infty\}$
- (iv) *$d(v^*, v) \leq \frac{1}{1-L} d(v, T_v)$ for every $v \in \Delta$.*

Theorem 3.2. *Let $\phi : M \to N$ be a mapping and there is a mapping $\beta : M^4 \to [0, \infty)$ with*

$$\lim_{\zeta\to\infty} \frac{1}{\tau_p^{2\zeta}} \beta(\tau_p^{\zeta}s, \tau_p^{\zeta}t, \tau_p^{\zeta}u, \tau_p^{\zeta}v) = 0,$$

where $\tau_p = 2$ if $p = 0$ and $\tau_p = \frac{1}{2}$ if $p = 1$,

$$\| F_Q(s, t, u, v) \| \leq \beta(s, t, u, v), \tag{3.1}$$

for every $s, t, u, v \in M$, and

$$\| \phi(st) - \phi(s)\phi(t) \| \leq \beta(s, t, 0, 0),$$

for every $s, t \in M$. If there is $L_c = L_c(p) < 1$ satisfies

$$s \to \Phi(s) = \beta\Big(\sqrt{\frac{s}{2}}, \sqrt{\frac{s}{2}}, \sqrt{\frac{s}{2}}, \sqrt{\frac{s}{2}}\Big), \tag{3.2}$$

has the property

$$\Phi(s) = \frac{L_c}{\tau_p^2} \Phi(\tau_p s), \tag{3.3}$$

for all $s \in M$. Then there is only one quadratic homomorphism $H_1 : M \to N$ fulfilling (1.2) and

$$\| \phi(s) - H_1(s) \| \leq \frac{L_c^{1-p}}{1 - L_c}\Phi(s), \tag{3.4}$$

for every $s \in M$.

Proof. Let us take $\omega = \{r | r : M \to N, r(0) = 0\}$.

Now, we can introduce the generalized metric d on ω,

$$d(p, b) = \inf\{r \in (0, \infty) : \|p(s) - b(s)\| \leq r\Phi(s),\ \forall s \in M\}.$$

Clearly, (ω, d) is complete. Now, we may define a function $\Psi : \omega \to \omega$ by

$$\Psi a(s) = \frac{1}{\tau_p^2} a(\tau_p s),$$

for all $s \in M$. One can show that $(\Psi a, \Psi b) \leq L_c\ d(a, b)$, for all $a, b \in \omega$. That is, the function Ψ is strictly contractive on ω with L_c. Replacing (s, t, u, v) by $(\sqrt{s}, \sqrt{s}, \sqrt{s}, \sqrt{s})$ in (1.2), we obtain

$$\| 2^2\phi(s) - \phi(2s) \| \leq \beta(\sqrt{s}, \sqrt{s}, \sqrt{s}, \sqrt{s}), \tag{3.5}$$

for all $s \in M$. From the above inequality, we obtain

$$\left\|\phi(s) - \frac{\phi(2s)}{2^2}\right\| \leq \frac{1}{2^2}\beta(\sqrt{s}, \sqrt{s}, \sqrt{s}, \sqrt{s}), \tag{3.6}$$

for all $s \in M$. Using the inequality (3.2) and (3.3), for the case $p = 0$, the inequality (3.6) becomes

$$\left\|\phi(s) - \frac{\phi(2s)}{2^2}\right\| \leq \frac{1}{2^2}\Phi(s),$$

for all $s \in M$. That is,

$$d(\phi, \Psi\phi) \leq \frac{1}{2^2} \leq L_c < \infty.$$

Again, replacing s by $\frac{s}{2}$ in (3.5), we get

$$\left\|2^2\phi\left(\frac{s}{2}\right) - \phi(s)\right\| \leq \beta\left(\sqrt{\frac{s}{2}}, \sqrt{\frac{s}{2}}, \sqrt{\frac{s}{2}}, \sqrt{\frac{s}{2}}\right), \tag{3.7}$$

for all $s \in M$. Using the inequality (3.2) and (3.3), for the case $p = 1$, the inequality (3.7) becomes

$$\left\|\phi(s) - 2^2\phi\left(\frac{s}{2}\right)\right\| \leq \Phi(s),$$

for all $s \in M$. That is,

$$d(\phi, \Psi\phi) \leq 1 \leq L_c^0 < \infty.$$

In the above two cases, we conclude that

$$d(\phi, \Psi\phi) \leq L_c^{1-p}.$$

Therefore, the condition (i) holds. By condition (ii), it follows that there is a fixed point H_1 of Ψ in ω fulfills

$$H_1(s) = \lim_{\zeta\to\infty} \frac{1}{\tau_p^{2\zeta}}\phi(\tau_p^{\zeta}s),$$

for every $s \in M$. Next, we need to prove that the mapping $H_1 : M \to N$ is quadratic. Replacing (s,t,u,v) by $(\tau_p^{\zeta}s, \tau_p^{\zeta}t, \tau_p^{\zeta}u, \tau_p^{\zeta}v)$ in (3.1) and dividing by τ_p^{ζ}, we get

$$\begin{aligned} \| H_1(s,t,u,v) \| &= \lim_{\zeta\to\infty} \frac{\| F_Q(\tau_p^{\zeta}s, \tau_p^{\zeta}t, \tau_p^{\zeta}u, \tau_p^{\zeta}v) \|}{\tau_p^{2\zeta}} \\ &\le \lim_{\zeta\to\infty} \frac{\beta(\tau_p^{\zeta}s, \tau_p^{\zeta}t, \tau_p^{\zeta}u, \tau_p^{\zeta}v)}{\tau_p^{2\zeta}} = 0 \end{aligned}$$

for every $s,t,u,v \in M$. That is, H_1 fulfills (1.2). Hence,

$$\begin{aligned} \| H_1(st) - H_1(s)H_1(t) \| &= \lim_{\zeta\to\infty} \frac{1}{\tau_p^{4\zeta}} \| \phi(\tau_p^{2\zeta}st) - \phi(\tau_p^{\zeta}s)\phi(\tau_p^{\zeta}t) \| \\ &\le \lim_{\zeta\to\infty} \frac{1}{\tau_p^{4\zeta}}\beta(\tau_p^{\zeta}s, \tau_p^{\zeta}t, 0, 0) = 0 \end{aligned}$$

for all $s,t \in M$. Therefore, the function H_1 is a quadratic homomorphism. By condition (iii), H_1 is a unique fixed point of Ψ in $\Theta = \{H_1 \in \omega : d(\phi, H_1) < \infty\}$, H_1 is the only one function which satisfies

$$\| \phi(s) - H_1(s) \| \le r\Phi(s),$$

for all $s \in M$ and $r > 0$. Condition (iv) implies

$$d(\phi, H_1) \le \frac{1}{1-L_c} d(\phi, \Psi\phi).$$

This implies

$$\begin{aligned} d(\phi, H_1) &\le \frac{L_c^{1-p}}{1-L_c} \\ \Rightarrow \| \phi(s) - H_1(s) \| &\le \frac{L_c^{1-p}}{1-L_c}\Phi(s), \end{aligned}$$

which ends the proof. □

The stability results of the quadratic functional equation (1.2), applicable to non-Archimedean Banach algebra derivations by the fixed point technique, are given by the following theorem.

Theorem 3.3. *Let $\phi : M \to N$ be a function for which there is $\beta : M^4 \to [0,\infty)$ satisfies*

$$\lim_{\zeta\to\infty} \frac{1}{\tau_p^{2\zeta}}\beta(\tau_p^{\zeta}s, \tau_p^{\zeta}t, \tau_p^{\zeta}u, \tau_p^{\zeta}v) = 0,$$

where $\tau_p = 2$ if $p = 0$ and $\tau_p = \frac{1}{2}$ if $p = 1$ such that (3.1) *and*

$$\| \phi(st) - \phi(s)\phi(t) \| \le \beta(s,t,0,0),$$

for all $s, t \in M$. If there is $L_c = L_c(i) < 1$ fulfills

$$s \to \Phi(s) = \beta\Big(\sqrt{\frac{s}{2}}, \sqrt{\frac{s}{2}}, \sqrt{\frac{s}{2}}, \sqrt{\frac{s}{2}}\Big),$$

has the property

$$\Phi(s) = L_c \frac{1}{\tau_p^2} \Phi(\tau_p s),$$

for all $s \in M$, then there is only one quadratic derivation $D : M \to M$ fulfilling (1.2) and

$$\| \phi(s) - D(s) \| \leq \frac{L_c^{1-p}}{1 - L_c} \Phi(s), \tag{3.8}$$

for all $s \in M$.

Proof. By using the same argument used to prove the Theorem 3.2, there is only one quadratic solution $D : M \to M$ fulfilling (3.8). Let us define the function $D : M \to M$ by

$$D(s) = \lim_{\zeta \to \infty} \frac{\phi(\tau_p^{\zeta} s)}{\tau_p^{2\zeta}},$$

for every $s \in M$. It follows from (3.1) that

$$\begin{aligned} \left\| D(st) - s^2 D(t) - D(s) t^2 \right\| &= \lim_{\zeta \to \infty} \frac{1}{\tau_p^{4\zeta}} \left\| \phi(\tau_p^{2\zeta} st) - (\tau_p^{\zeta} s)^2 \phi(\tau_p^{\zeta} t) - \phi(\tau_p^{\zeta} s)(\tau_p^{\zeta} t)^2 \right\| \\ &\leq \lim_{\zeta \to \infty} \frac{1}{\tau_p^{4\zeta}} \beta(\tau_p^{\zeta} s, \tau_p^{\zeta} t, 0, 0) = 0 \end{aligned}$$

for every $s, t \in M$. Hence, the function $D : M \to M$ is a quadratic derivation satisfying (3.8). □

Corollary 3.1. *Let $\phi : M \to N$ be a function such that*

$$\| F_Q(s,t,u,v) \| \leq \begin{cases} \xi \\ \xi\Big\{ \| s \|^i + \| t \|^i + \| u \|^i + \| v \|^i \Big\} \\ \xi\Big\{ \| s \|^i \| t \|^i \| u \|^i \| v \|^i \Big\} \\ \xi\Big\{ \| s \|^i \| t \|^i \| u \|^i \| v \|^i \\ \quad +(\| s \|^{4i} + \| t \|^{4i} + \| u \|^{4i} + \| v \|^{4i})\Big\}, \end{cases}$$

for every $s, t, u, v \in M$. Then there is only one quadratic homomorphism $H_1 : M \to N$ fulfilling

$$\| \phi(s) - H_1(s) \| \leq \begin{cases} \frac{\xi}{|3|} \\ \frac{4\xi \|s\|^{\frac{i}{2}}}{|2^2 - 2^{\frac{s}{2}}|}; \quad i \neq 4 \\ \frac{\xi \|s\|^{2i}}{|2^2 - 2^{2i}|}; \quad i \neq 1 \\ \frac{5\xi \|s\|^{2i}}{|2^2 - 2^{2i}|}; \quad i \neq 1, \end{cases}$$

for all $s \in M$, where ξ and i are real numbers.

Proof. Setting

$$\beta(s,t,u,v) \leq \begin{cases} \xi \\ \xi\Big\{ \parallel s \parallel^i + \parallel t \parallel^i + \parallel u \parallel^i + \parallel v \parallel^i \Big\} \\ \xi\Big\{ \parallel s \parallel^i \parallel t \parallel^i \parallel u \parallel^i \parallel v \parallel^i \Big\} \\ \xi\Big\{ \parallel s \parallel^i \parallel t \parallel^i \parallel u \parallel^i \parallel v \parallel^i \\ \quad +(\parallel s \parallel^{4i} + \parallel t \parallel^{4i} + \parallel u \parallel^{4i} + \parallel v \parallel^{4i})\Big\}, \end{cases}$$

for all $s,t,u,v \in M$. Now

$$\begin{aligned} \frac{\beta(\tau_p^{\zeta}s, \tau_p^{\zeta}t, \tau_p^{\zeta}u, \tau_p^{\zeta}v)}{\tau_p^{2\zeta}} &= \begin{cases} \xi\tau_p^{-2\zeta} \\ \xi\tau_p^{(i-2)\zeta}\Big\{ \parallel s \parallel^i + \parallel t \parallel^i + \parallel u \parallel^i + \parallel v \parallel^i \Big\} \\ \xi\tau_p^{(4i-2)\zeta}\Big\{ \parallel s \parallel^i \parallel t \parallel^i \parallel u \parallel^i \parallel v \parallel^i \Big\} \\ \xi\tau_p^{(4i-2)\zeta}\Big\{ \parallel s \parallel^i \parallel t \parallel^i \parallel u \parallel^i \parallel v \parallel^i \\ \qquad\qquad +(\parallel s \parallel^{4i} + \parallel t \parallel^{4i} + \parallel u \parallel^{4i} + \parallel v \parallel^{4i})\Big\} \end{cases} \\ &= \begin{cases} \rightarrow \ 0 \ as \ \zeta \rightarrow \infty \\ \rightarrow \ 0 \ as \ \zeta \rightarrow \infty \\ \rightarrow \ 0 \ as \ \zeta \rightarrow \infty \\ \rightarrow \ 0 \ as \ \zeta \rightarrow \infty. \end{cases} \end{aligned}$$

Thus, (3.1) holds. But, we obtain

$$\Phi(s) = \beta\left(\sqrt{\frac{s}{2}}, \sqrt{\frac{s}{2}}, \sqrt{\frac{s}{2}}, \sqrt{\frac{s}{2}}\right),$$

has the property

$$\Phi(s) = L_c \frac{1}{\tau_p^2}\Phi(\tau_p s),$$

for all $s \in M$. Hence

$$\begin{aligned} \Phi(s) &= \beta\left(\sqrt{\frac{s}{2}}, \sqrt{\frac{s}{2}}, \sqrt{\frac{s}{2}}, \sqrt{\frac{s}{2}}\right) \\ &= \begin{cases} \xi \\ \frac{4\xi\|s\|^{\frac{i}{2}}}{2^{\frac{i}{2}}} \\ \frac{\xi\|s\|^{2i}}{2^{2i}} \\ \frac{5\xi\|s\|^{2i}}{2^{2i}}. \end{cases} \end{aligned}$$

Now,

$$\frac{1}{\tau_p^2}\Phi(\tau_p s) = \begin{cases} \frac{\xi}{\tau_p^2} \\ \frac{4\xi\|s\|^{\frac{i}{2}}}{\tau_p^2 2^{\frac{i}{2}}} \\ \frac{\xi\|s\|^{2i}}{\tau_p^2 2^{2i}} \\ \frac{5\xi\|s\|^{2i}}{\tau_p^2 2^{2i}} \end{cases}$$

$$= \begin{cases} \tau_p^{-2}\Phi(s) \\ \tau_p^{\frac{i-4}{2}}\Phi(s) \\ \tau_p^{2i-2}\Phi(s) \\ \tau_p^{2i-2}\Phi(s), \end{cases}$$

for all $s \in M$. From (3.4), we verify the below cases:

Case 1: If $p = 0$ then $L_c = 2^{-2}$;

$$\| \phi(s) - H_1(s) \| \leq \frac{L_c^{1-p}}{1 - L_c}\Phi(s) = \frac{\xi}{3}.$$

Case 2: If $p = 1$ then $L_c = 2^2$;

$$\| \phi(s) - H_1(s) \| \leq \frac{L_c^{1-p}}{1 - L_c}\Phi(s) = \frac{\xi}{-3}.$$

Case 3: $L_c = 2^{\frac{i-4}{2}}$ for $i < 4$ if $p = 0$;

$$\| \phi(s) - H_1(s) \| \leq \frac{L_c^{1-p}}{1 - L_c}\Phi(s) = \frac{4\xi \| s \|^{\frac{i}{2}}}{\left(2^2 - 2^{\frac{i}{2}}\right)}.$$

Case 4: $L_c = 2^{\frac{4-i}{2}}$ for $i > 4$ if $p = 1$;

$$\| \phi(s) - H_1(s) \| \leq \frac{L_c^{1-p}}{1 - L_c}\Phi(s) = \frac{4\xi \| s \|^{\frac{i}{2}}}{\left(2^{\frac{i}{2}} - 2^2\right)}.$$

Case 5: $L_c = 2^{2i-2}$ for $i < 1$ if $p = 0$;

$$\| \phi(s) - H_1(s) \| \leq \frac{L_c^{1-p}}{1 - L_c}\Phi(s) = \frac{\xi \| s \|^{2i}}{\left(2^2 - 2^{2i}\right)}.$$

Case 6: $L_c = 2^{2-2i}$ for $i > 1$ if $p = 1$;

$$\| \phi(s) - H_1(s) \| \leq \frac{L_c^{1-p}}{1 - L_c}\Phi(s) = \frac{\xi \| s \|^{2i}}{\left(2^{2i} - 2^2\right)}.$$

Case 7: $L_c = 2^{2i-2}$ for $i < 1$ if $p = 0$;

$$\| \phi(s) - H_1(s) \| \leq \frac{L_c^{1-p}}{1 - L_c} \Phi(s) = \frac{5\xi \| s \|^{2i}}{\left(2^2 - 2^{2i}\right)}.$$

Case 8: $L_c = 2^{2-2i}$ for $i > 1$ if $p = 1$;

$$\| \phi(s) - H_1(s) \| \leq \frac{L_c^{1-p}}{1 - L_c} \Phi(s) = \frac{5\xi \| s \|^{2i}}{\left(2^{2i} - 2^2\right)}.$$

□

Corollary 3.2. *Let $\phi : M \to N$ be a function satisfies*

$$\| F_Q(s,t,u,v) \| \leq \begin{cases} \xi \\ \xi\Big\{ \| s \|^i + \| t \|^i + \| u \|^i + \| v \|^i \Big\} \\ \xi\Big\{ \| s \|^i \| t \|^i \| u \|^i \| v \|^i \Big\} \\ \xi\Big\{ \| s \|^i \| t \|^i \| u \|^i \| v \|^i \\ \quad +(\| s \|^{4i} + \| t \|^{4i} + \| u \|^{4i} + \| v \|^{4i})\Big\}, \end{cases}$$

for every $s, t, u, v \in M$. Then there is only one quadratic derivation $D : M \to M$ fulfilling

$$\| \phi(s) - H_1(s) \| \leq \begin{cases} \frac{\xi}{|3|} \\ \frac{4\xi \|s\|^{\frac{i}{2}}}{|2^2 - 2^{\frac{i}{2}}|}; \quad i \neq 4 \\ \frac{\xi \|s\|^{2i}}{|2^2 - 2^{2i}|}; \quad i \neq 1 \\ \frac{5\xi \|s\|^{2i}}{|2^2 - 2^{2i}|}; \quad i \neq 1, \end{cases}$$

for all $s \in M$, where ξ and i are real numbers.

4. Conclusion

We found the general solution of a new kind of quadratic functional problem and presented it in this paper. We primarily used the direct and fixed point methods to study the stability of a quadratic functional equation relating to homomorphisms and derivations in non-Archimedean Banach algebra. We also give examples of situations where sums and products of powers of norms control the stability of this quadratic functional equation.

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