ALGORITHM BASED ON PARTICLE SWARM OPTIMIZATION FOR HANDLING INCOMPLETE PAIRWISE COMPARISON SITUATIONS IN AHP

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ABSTRACT

The job of the AHP expert is to input his/her perceptions into a pairwise comparison matrix; however, there are times when the expert is unable to provide his/her opinions. This can be for many reasons such as there are numerous pairwise comparisons to be completed or the expert is unsure about the values that must be included in the pairwise comparison matrix. The AHP method cannot be performed without a complete comparison matrix; therefore, the aim of this article is to provide a novel approach based on the Particle Swarm Optimization (PSO) technique to estimate the missing values in pairwise comparison matrices. According to the findings of this study, the proposed algorithm can offer a suggested value for the pairwise comparison matrix with an acceptable Consistency Ratio (CR). Furthermore, the time required to find the suggested value is quite short.

Keywords: Analytic Hierarchy Process; incomplete pairwise matrices; Particle Swarm Optimization; suggested value

1. Introduction

Decision making is an integral activity in any company. Some examples of possible decisions are 1) determining order priority which can minimize delays in order completion; 2) determining the optimal delivery route for goods, 3) selecting suppliers, or 4) determining the right marketing strategy. Some of these decision-making problems can be solved using hard operation research (OR) methodology, especially when the problem involves quantitative criteria. In hard OR methodology, the problem can be solved by finding the value of the objective function using an analytical, numerical, metaheuristics or simulation technique. In addition to the complexity of the problem encountered, when the uncontrollable input of a system is a random variable, the mathematical models that are built become increasingly complex. This often results in a situation where an analytical approach cannot be used to obtain a solution; therefore, a metaheuristics technique is needed.

Existing metaheuristics techniques include genetic algorithms, particle swarm optimization, ant colonies and many more. Metaheuristics techniques are widely used for optimization problems that need to be solved with the help of a computer when an analytical solution is not possible. The application of metaheuristics techniques can be found in several problems both in manufacturing and service companies (Astanti et al., 2018; Özgün- Kibiroğlu et al., 2019; Li et al., 2024).

In real life, situations exist where the problem involves quantitative criteria as well as qualitative criteria. Therefore, a decision-making method to solve problems that consider both quantitative and qualitative aspects is needed (Benítez et al., 2015). One of the methods for solving decision making problems with multiple criteria is the Analytic Hierarchy Process (AHP). The AHP is applied in situations where there are no dependencies among criteria, and it allows both quantitative and qualitative criteria to be considered.

Emotion is one of the factors that can sometimes be useful in decision making (Lerner et al., 2015). Several scholars have conducted research to identify the influence of emotions on judgment and decision making (Solomon, 1993; Phelps et al., 2014; Lerner et al., 2015). Emotions can be triggered by existing and formed judgments or decisions (Damasio, 1994; Greene & Haidt, 2002). Emotions, according to Lerner et al. (2015), can have a positive or negative impact on decision making. The negative impact of emotions on decision making can be mitigated by, among other things, (a) reducing the intensity of emotions through delay, reappraisal, or inducing counteractive emotions, and (b) isolating the decision-making process from emotions by raising awareness of misattribution, changing the architecture, or crowding out emotions (Lerner et al., 2015).

The AHP as a decision-making technique that involves multiple criteria (Multi Criteria Decision Making) uses expert judgments to carry out pairwise comparisons both between criteria and alternatives (Saaty, 1980). A simple decision-making structure hierarchy with the AHP consists of three levels, namely, objective, criteria, and alternatives. This structure affects the number of pairwise comparisons that must be completed by the expert. When the AHP is used there is a consistency index the shows that the more objects that must be compared in pairs, the more the consistency of the expert as a decision maker will decrease (Saaty, 1980).

With the AHP, negative impact conditions on decision making processes, for example related to inconsistency in giving judgments in the pairwise comparison process, can be reduced by restructuring the decision hierarchy (Benítez et al., 2015). Therefore, at one level, the number of objects being compared is not large. However, this condition causes the hierarchical structure to become more complex, which involves criteria, sub-criteria, alternatives, and sub-alternatives. Hence, even though the objects being compared per level can be reduced, the addition of levels results in an increase in the number of pairwise comparisons that must be filled in by the expert. For this situation, according to Chen & Lin (2003), the decision makers need more time to compare all the paired elements. According to Harker (1987), due to this condition the decision maker may not be willing to express opinions on all the paired elements.

In research conducted by Pujiastuti et al. (2016), a situation was found where the decision maker was unable to provide the judgments on several pairwise comparison matrices because the hierarchical structure of the problem had many levels and the decision maker was not willing to make judgements for all of the paired elements. Under these conditions, the pairwise comparison matrix was left incomplete and the synthesis steps in the AHP method could not be carried out.

Incomplete pairwise comparison describes the existence of unknown values in the pairwise comparison. When this situation exists, the AHP method cannot be completed (Ichihashi & Turksen, 1993; Nishizawa, 1997; Shiraishi et al., 1998; Hu & Tsai, 2006; Gomez-Ruiz et al., 2010; Bernroider et al., 2010; Chen et al., 2015). Previous research on incomplete pairwise comparisons with the AHP focuses on two situations, namely 1) how to simplify the questions needed to fill in pairwise comparison (Harker, 1987; Shen et al., 1992; Fedrizzi & Giove, 2013; Shen et al., 1992) and 2) how to complete pairwise comparisons when the value of pairwise comparison is not known (Ichihashi & Turksen, 1993; Nishizawa, 1997; Shiraishi et al.,1998; Hu &Tsai, 2006; Gomez-Ruiz et al., 2010; Bernroider et al., 2010; Srdjevic et al., 2014; Pan et al., 2014; Chen et al., 2015; Jandova, 2016; Zhou et al., 2018; Maharani et al., 2019; Maleki, 2020; Tekile et al., 2021; Sharabiani, 2023).

Particle Swarm Optimization (PSO) is an evolutionary computation proposed by Kennedy & Ebenhart (1995). PSO is a method that exhibits speed and efficiency in determining optimal values (Marini & Walczak, 2015). At present, PSO has been used in many applications, including as an artificial neural network approach, multi-objectives optimization, classification, pattern recognition, decision making and simulation (Eberhart & Shi, 2001). The reason PSO is interesting to apply is because very few parameters must be adjusted (Eberhart & Shi, 2001). According to Eberhart & Shi (2001) the application of PSO is considered simple and efficient. The use of PSO to solve supply chain problems using fuzzy AHP was conducted by Che (2019). However, to the best of the author’s knowledge, there has been no research discussing how PSO can be used for incomplete pairwise comparison problems.

The goal of the research presented in this article is to provide a suggested value to fill the incomplete pairwise comparison matrix using a novel approach based on Particle Swarm Optimization (PSO). It is not required to use the value obtained from the proposed approach but rather, it is a suggestion.

The subsequent sections of this article provide an explanation of the proposed PSO-based algorithm for providing a suggested value to complete the pairwise comparison matrix, discusses how the proposed algorithm can be applied in a real case study and discusses the results.

2. Literature review

2.1 Review of incomplete pairwise comparison in Analytic Hierarchy Process

A literature review was carried out with the Scopus database using the keyword "incomplete pairwise comparison" which resulted in 199 documents. Next, another screening was carried out with a more detailed search, namely, "incomplete pairwise comparison AHP" and 45 documents were obtained. The process of searching the literature is shown in Figure 1.

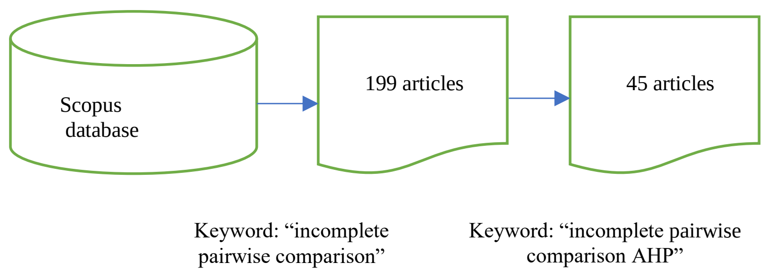

Figure 1 Literature review search

As previously mentioned, incomplete pairwise comparison needs to be overcome so that the AHP method can be performed (Harker, 1987; Shen et al., 1992; Fedrizzi & Giove, 2013; Shen et al., 1992; Ichihashi & Turksen, 1993; Nishizawa, 1997; Shiraishi et al., 1998; Hu & Tsai, 2006; Gomez-Ruiz et al., 2010; Bernroider et al., 2010; Srdjevic et al., 2014; Pan et al., 2014; Chen et al., 2015; Jandova, 2016; Zhou et al., 2018; Maharani et al., 2019; Maleki, 2020; Tekile et al., 2021; Sharabiani, 2023). One of the ways to overcome this situation is simplifying the pairwise comparisons (Harker, 1987; Shen et al., 1992; Fedrizzi & Giove, 2013). Research on how to simplify questions to fill in pairwise comparison was initiated by Harker (1987) who proposed reducing the number of questions based on criteria in the AHP hierarchy. Another study was conducted by Shen et al. (1992) who proposed reducing the number of questions by dividing the objects from a level into several subsets so that each subset has one common object that can be used as a standard. Fedrizzi & Giove (2013) proposed using the sequencing method to select questions.

Maharani et al. (2019) used fuzzy AHP to overcome the incomplete pairwise comparison problem, especially when the decision maker still has doubts about giving his/her perceptions for the pairwise comparison. Besides reducing the number of questions, another approach to deal with incomplete pairwise comparison is done by complete pairwise comparison when the value of pairwise comparison is not known. Ichihashi & Turksen (1993) introduced a neuro-fuzzy approach for calculating pairwise comparisons even when the value was unknown. Research has also focused on how to estimate unknown values in pairwise comparisons, including Nishizawa (1997), Shiraishi et al. (1998), Hu and Tsai (2006), Gomez-Ruiz et al. (2010), Bernroider et al. (2010), Srdjevic et al. (2014), Pan et al. (2014), Chen et al. (2015), Jandova (2016), Zhou et al. (2018), Maharani et al. (2019), Maleki (2020), Tekile et al. (2021), and Sharabiani (2023).

Nishizawa (1997) introduced a syllogistic approach to estimate unknown values in pairwise comparison matrices. Shiraishi et al. (1998) proposed the Characteristic Polynomial-Based Method (CPB), while Hu & Tsai (2006) suggested the Back-

ISSN 1936-6744
https://doi.org/10.13033/ijahp.v16i1.1136

Propagation Multi-Layer Perceptron. Gomez-Ruiz et al. (2010) utilized the Multi-Layer Perceptron (MLP) neural network method, followed by Bernroider et al. (2010), who employed graph theory to address incomplete pairwise comparison matrices. Other researchers, such as Srdjevic (2014), put forward the first-level transitivity rule model. Pan et al. (2014) proposed the use of information entropy and Dempster–Shafer Evidence Theory, and Chen et al. (2015) developed the Connecting Path Method (CPM) for dealing with incomplete pairwise comparison matrices. Jandova (2016) proposed an interactive algorithm based on sequential optimal choice and the concept of weak consistency, while Zhou (2018) introduced the Dematel-based completion method. Maharani et al. (2019) introduced unsymmetrical triangular fuzzy numbers, Maleki (2020) suggested the use of the Choquet integral, and Tekile (2021) proposed the application of the Nelder-Mead Algorithm. In recent research, Sharabiani (2023) presented a goal programming and similarity function approach to overcome incomplete pairwise comparisons. Details are shown in Table 1.

Table 1
Previous research on incomplete pairwise comparison in the AHP

Authors	**Problem**	**Method**
Harker (1987)	Reduction of the number of questions in AHP	Reducing the number of questions based on criteria in the AHP hierarchy
Shen et al. (1992)	Reduction of the number of questions in the AHP using common objects from existing subsets	Dividing the objects from a level into several subsets
Ichihashi & Turksen (1993)	The value of the pairwise comparison of the neuro-fuzzy approach is not known	Neuro-Fuzzy Approach
Nishizawa (1997)	The value of the pairwise comparison with the syllogistic approach is not known	Syllogistic approach
Shiraishi et al. (1998)	The value of the pairwise comparison Characteristic Polynomial-Based Method (CPB) is unknown	Characteristic Polynomial-Based Method (CPB)
Hu & Tsai (2006)	The value of the pairwise comparison Back-Propagation Multi-Layer Perceptron is not known	Back-Propagation Multi-Layer Perceptron
Gomez-Ruiz et al. (2010)	The value of the pairwise comparison Multi-Layer Perceptron (MLP) neural network is not known	Multi-Layer Perceptron (MLP) neural network
Bernroider et al. (2010)	The value of pairwise comparison graph theory is not known	Graph Theory
Fedrizzi & Giove	Reduction of the number of	Sequencing method

ISSN 1936-6744
https://doi.org/10.13033/ijahp.v16i1.1136

Authors	**Problem**	**Method**
(2013)	questions in AHP	
Srdjevic (2014)	Estimate unknown values in pairwise comparisons	First level transitivity rule model
Pan et al. (2014)	Estimate unknown values in pairwise comparisons	Information entropy & Dempster – Shafer Evidence Theory
Chen et al. (2015)	The value of the pairwise comparison Connecting Path Method (CPM) is not known	Connecting Path Method (CPM)
Jandova (2016)	Estimate unknown values in pairwise comparisons	Interactive algorithm based on sequential optimal choice and concept of weak consistency
Zhou (2018)	Estimate unknown values in pairwise comparisons	Dematel – based completion method
Maharani et al.(2019)	Estimate unknown values in pairwise comparisons	Unsymmetrical triangular fuzzy number
Maleki (2020)	Estimate unknown values in pairwise comparisons	Choquet integral
Tekile (2021)	Estimate unknown values in pairwise comparisons	Nelder-Mead Algorithm
Sharabiani (2023)	Estimate unknown values in pairwise comparisons	Goal programming & similarity function

Based on Table 1, no research has been found related to the use of Particle Swarm Optimization (PSO) for dealing with incomplete pairwise comparisons.

In general, the common method for solving incomplete pairwise comparison problems is to provide suggestions regarding the recommended judgment values to fill in the gaps in the pairwise comparison. Several studies have provided consistency indicators to illustrate that if the proposed judgment value is applied by decision makers, it will provide good a consistency ratio (i.e., CR $\leq$ 0.1) (Harker, 1987; Shen et al., 1992; Fedrizzi & Giove, 2013, Shen et al., 1992; Ichihashi & Turksen, 1993; Nishizawa, 1997; Shiraishi et al., 1998; Hu & Tsai, 2006; Gomez-Ruiz et al, 2010; Bernroider et al, 2010; Srdjevic et al., 2014; Pan et al., 2014; Chen et al., 2015; Jandova, 2016; Zhou et al., 2018; Maharani et al., 2019; Maleki, 2020; Tekile et al., 2021; Sharabiani, 2023). According to Hu and Tsai (2006), the more efficient the estimation method used, the lower the consistency ratio (CR) value.

2.2 Review of Particle Swarm Optimization (PSO)

The concept of Swarm Intelligence (SI) was introduced in 1989 as an optimization method inspired by the social behavior of animals, such as birds, fish, ants, bees, and termites (Marini & Walczak, 2015). One Swarm Intelligence (SI) method is Particle Swarm Optimization (PSO) proposed by Russell C. Eberhart and James Kennedy in 1995 (Eberhart & Shi, 2001). PSO is inspired by information circulation and social behavior of flocks of birds and schools of fish (Marini & Walczak, 2015).

ISSN 1936-6744
https://doi.org/10.13033/ijahp.v16i1.1136

PSO has been used often because its application requires very few parameters to be adjusted (Eberhart & Shi, 2001). According to Eberhart & Shi (2001), the application of PSO is considered simple and efficient. PSO focuses on the formation of cooperation and competition between individuals (particles) to obtain the best position close to fitness by sharing information both in the local environment (partial PSO) and the global environment (global PSO) (Marini & Walczak, 2015). The position achieved by each particle is called the personal best (*pbest*) and the best position achieved by all particles is called the global best (*gbest*). From sharing the *pbest* and *gbest* information, each particle determines a new position and velocity to find the most optimal position that can be used as a solution (Eberhart & Kennedy, 1995).

The following are the steps for the PSO algorithm described by Marini & Walczak (2015):

a. Initial population initialization
   The population referred to in PSO is the alternative answers that are evaluated during each iteration. Meanwhile, the initial population is an alternative answer when the first iteration is carried out. During the first iteration, the following steps are conducted:
   i. Initialize the initial velocity of each particle $\boldsymbol{V}_i(0)\ as\ 0$.
   ii. Initialize the initial position of each particle $\boldsymbol{X}_i(0)$ randomly.
   iii. Calculate the fitness of each particle.
   iv. Initialize *pbest* for each particle using the initial position, and *gbest* using the best *pbest.*

b. Velocity update
   During the iteration, velocity updates are performed using Equation (1) as follows:

$$\boldsymbol{V}_i(t+1) = w.\boldsymbol{V}_i(t) + C_1.\left(\boldsymbol{p}_i - \boldsymbol{x}_i(t)\right).\boldsymbol{R}_1 + C_2.\left(\boldsymbol{g} - \boldsymbol{x}_i(t)\right).\boldsymbol{R}_2 \quad (1)$$

   where:
   $\boldsymbol{V}$: Particle velocity
   $\boldsymbol{x}$: Particle position
   $\boldsymbol{p}$: *pbest* value
   $\boldsymbol{g}$: *gbest* value
   w : inertia weight
   C_1: coefficient of cognitive particle
   C_2: coefficient of social particle

   Previous research discussed the suggested value of parameters w, C_1 and C_2 (Clerc & Kennedy, 2002; Zhang et al., 2005). According to Clerc and Kennedy (2002) and Zhang et al. (2005), the selection of the values of parameters C_1 and C_2 is such that the sum C_1+C_2=4. Shi and Eberhart (1998a, 1998b) and Shi and Eberhart (1999) suggested the value of parameter w, where the value of w is linear decreasing from 0.9 to 0.4. This step is very important because the algorithm performance may be sensitive to initialization of position and velocity. However, with proper selection of parameters C_1 and C_2 as suggested by Clerc and Kennedy (2002) and Zhang et al. (2005), the rapid convergence tendency can be controlled.

ISSN 1936-6744
https://doi.org/10.13033/ijahp.v16i1.1136

c. Update position and calculate fitness value
   Perform velocity updates using Equation (2):

$$\boldsymbol{x}_i(t+1) = \boldsymbol{x}_i(t) + \boldsymbol{V}_i(t+1) \tag{2}$$

   where
   $\boldsymbol{V}$ = Particle velocity
   $\boldsymbol{x}$ = Particle position

   Recalculate the fitness for each particle.

d. Update *pbest* and *gbest*

e. Compare the fitness value with the *pbest* in the previous iteration for each particle and take the best value as the current *pbest*. Next, compare the *gbest* in the previous iteration with the current *pbest*, and take the best value as the current *gbest*.

f. Perform iteration by repeating the velocity update in step b.

3. Proposed algorithm based on PSO for providing the suggested value to complete the pairwise comparison matrices

In this section, the proposed algorithm based on PSO is explained to obtain the proposed preference values in a pairwise comparison matrix. PSO can be used because it is a technique that can find the optimal value of a target value. In the AHP, the objective function is a Consistency Ratio (CR) ≤ 0.1 after the expert fills in the pairwise comparison. Meanwhile, the optimal value is the pairwise comparison value such that the CR is ≤ 0.1. The optimal value referred to by the pairwise comparison value is the recommended value for the decision maker in conditions where due to doubts the decision maker chooses not to provide his/her perception on the pairwise comparison matrix, resulting in an incomplete pairwise comparison matrix condition.

The unique feature of the PSO application in the proposed algorithm is the definition of particles and particle dimensions as follows:

a) One pairwise comparison matrix is represented by one particle

b) Every incomplete comparison is considered as one particle dimension

c) The particle has a position where the value of the particle position in a certain dimension represents a pairwise comparison value.

   - The PSO method utilizes the particles to obtain particle positions in such a way that it can suggest the preferred value in the pairwise comparison matrix. The suggested value is designed so that it can result in a minimum Consistency Ratio (CR) value. This can be achieved because each particle informs the other particles of its position and value. Therefore, each particle can update its position to obtain a better value with the help of other particles. The value of input parameters refers to the work of Clerc and Kennedy (2002), Zhang et al. (2005),

ISSN 1936-6744
https://doi.org/10.13033/ijahp.v16i1.1136

Shi and Eberhart (1998a, 1998b), and Shi and Eberhart (1999). This selection of parameters can avoid the rapid convergence tendency.

- When the program detects an empty value in the input of the pairwise comparisons, the program generates dimensions. The amount of data that is not known in a pairwise comparison generates dimensions with the same number. Each particle in the dimension randomly has a position that describes the empty pairwise comparison value according to the change in the random value to the AHP value. It is noted that the proposed particle position is unique for every problem solved using the PSO approach. Therefore, to solve the incomplete pairwise comparison problem, this research proposes the particle position, which is a real number with a lower bound of 0 and an upper bound of 1. Those position values can be converted to the AHP pairwise comparison value, which vary from 1/9 to 9. The conversion of PSO positions to AHP pairwise comparison values is shown in Table 2.

Table 2
Conversion of PSO position to AHP pairwise comparison value

PSO position	**AHP pairwise comparison value**
0<= x <=1/17	1/9
1/17< x <=2/17	1/8
2/17< x <=3/17	1/7
3/17< x <=4/17	1/6
4/17< x <=5/17	1/5
5/17< x <=6/17	1/4
6/17< x <=7/17	1/3
7/17< x <=8/17	1/2
8/17< x <=9/17	1
9/17< x <=10/17	2
10/17< x <=11/17	3
11/17< x <=12/17	4
12/17< x <=13/17	5
13/17< x <=14/17	6
14/17< x <=15/17	7
15/17< x <=16/17	8
16/17< x <=1	9

d) Each particle calculates the CR value from the filled pairwise comparison after the filled pairwise value is converted from its corresponding PSO position value. The CR value of each particle is compared. The best CR value is selected as the best result and its position as the best global position. Its position value is made the best particle position. The particle with the best CR is called the *gbest*. In every iteration each particle changes its position following the movement equation; the particle moves toward *gbest* and *pbest*. *pbest* is the best position that each particle ever experiences according to Equation (1).

ISSN 1936-6744
https://doi.org/10.13033/ijahp.v16i1.1136

Each particle looks for a velocity value that is influenced by the best position of all particles (*gbest*) and the particle itself (*pbest*). This velocity is used to find a new position for each particle. The position in the PSO which is less than or equal to 0 will be changed by Equation (3):

$$\text{Position} = 0 \tag{3}$$

While those greater than 1 are changed with Equation (4):

$$\text{Position} = 1 \tag{4}$$

In addition, for both cases, the velocity is also updated following Equation (5):

$$\text{Velocity} = 0 \tag{5}$$

This change is made to avoid values that exceed the range which causes the value to not exceed the upper bound value, which is 1, and not be less than the lower bound value, which is 0. With Equations (3) – (5), position values that exceed the range can be reflected.

e) The activity of each particle is repeated as many times as iterations are used as follows:
  - p*best* and g*best* will always be updated.
    If the current position of FITNESS is better than the previous *pbest* FITNESS, then the *pbest* is updated with the current position.
    If the current position of FITNESS is better than the previous *gbest* of FITNESS, then the *gbest* is updated with current position.

  In each replication, the best position of each particle and the best position of all particles can be obtained. Then, the resulting value can be used as a suggested value to fill in the incomplete pairwise comparisons.

f) The decision maker looks at the results of the suggested value and then compares it with his/ her perception. If the decision maker feels that the suggested value from the algorithm is appropriate and can increase his/her confidence regarding the perception about a certain question in the pairwise comparison matrix, then the decision maker can use that value. If not, the decision maker can adjust, using the suggested value as the initial basis for the preference value. In the case of multiple decision makers, if not all of the decision makers agree with the suggested value then they can come to a consensus to discuss the value to be input into the matrix.

4. Case study to illustrate the application of the proposed algorithm

Here we explain how the proposed algorithm can be applied to provide preference value suggestions for incomplete pairwise comparison matrices through a case study of a decision-making problem. The case study was conducted in a garment manufacturer located in Indonesia. This company has a decision-making problem related to supplier selection. The expert for this decision-making problem is the owner of the company. The steps taken to solve the supplier selection problem follow the steps in the AHP method (Saaty, 2008).

4.1 Modeling hierarchical structure of supplier selection

A hierarchical structure is needed in the AHP technique to simplify the complexity of the problem to be solved. In this study, a hierarchical structure was formed with the aim of selecting a fabric supplier for a garment manufacturer. The hierarchical structure was obtained from the results of interviews with the owner. Based on the interviews, the factors that have been considered by this company in selecting fabric suppliers are price, quality, and variety of materials.

Price as one of the criteria to select supplier has previously been studied by Kumar Kar & Pani (2014); Jayaraman et al. (1999); Kannan et al. (2013); Asamoah et al. (2012); Weber and Elram (1993); Thakkar et al. (2012); Li et al. (2013) and Nazari-Shirkouhi et al. (2013). The quality criterion in the selection of a supplier has been studied by Mirabi et al. (2010), Mendoza (2007), and Li et al. (2013). The variety of material provided by suppliers criterion can be categorized as the capability of the supplier. This criterion to select suppliers has been previously studied by Çebi and Bayraktar (2003), Gnanasekaran et al. (2006), Vonderembse and Tracey (1999), Gonzales et al. (2004), Mirabi et al. (2010), Thakkar et al. (2012), Ruiz-Torres et al. (2013), Songhori et al. (2011), Li et al. (2013) and Kannan et al.(2013). These three criteria are in a hierarchical structure to evaluate five alternative suppliers, namely, supplier A, supplier B, supplier C, supplier D and supplier E. The hierarchical structure of the supplier selection problem can be seen in Figure 2.

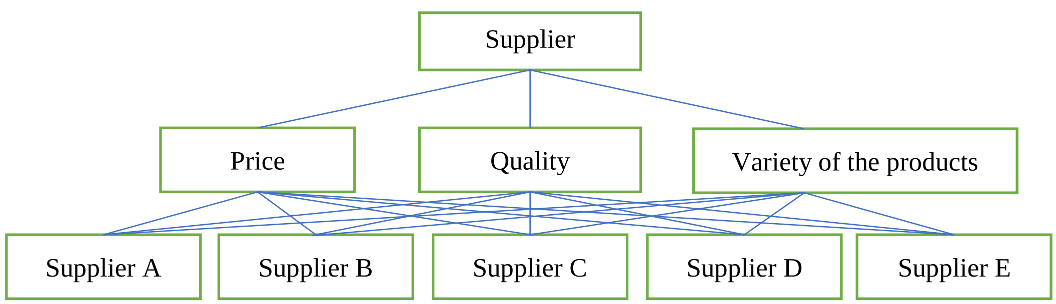

Figure 2 Hierarchical structure of supplier selection problem

4.2 Pairwise comparison for criteria level and alternative

At this stage, pairwise comparisons are carried out at the criterion level and at the alternative level. Unfortunately, in some comparisons, the owner as a decision maker experiences doubts when assessing which alternatives are better than others being compared according to certain criteria. This is because the decision maker does not have complete information about the performance of a supplier based on certain criteria. The owner is not sure which supplier is better in terms of price, Supplier C or Supplier D. He is uncertain because even though the prices offered by Supplier C are higher, sometimes they offer discounts. The results of the pairwise comparison are shown in Table 3.

ISSN 1936-6744
https://doi.org/10.13033/ijahp.v16i1.1136

Table 3
Incomplete pairwise comparisons of alternatives based on Price criteria

	Supplier A	Supplier B	Supplier C	Supplier D	Supplier E
Supplier A	1	1/5	1/3	1/3	1/3
Supplier B	5	1	3	3	3
Supplier C	3	1/3	1	-	1
Supplier D	3	1/3	-	1	1
Supplier E	3	1/3	1	1	1

4.3 Use of the proposed algorithm to complete the incomplete pairwise comparison matrix

The algorithm created is then written in the C# programming language. The proposed algorithm can provide a preference value suggestion for the incomplete pairwise comparison matrix where the proposed value ensures that a good CR is achieved. The result can be seen in Table 4 where the proposed values are shown in red.

Table 4
Complete pairwise comparisons with suggested preference value based on the proposed algorithm for alternatives based on Price criteria

	Supplier A	Supplier B	Supplier C	Supplier D	Supplier E
Supplier A	1	1/5	1/3	1/3	1/3
Supplier B	5	1	3	3	3
Supplier C	3	1/3	1	**1**	1
Supplier D	3	1/3	**1**	1	1
Supplier E	3	1/3	1	1	1

4.4 Feedback mechanism for the decision maker

Based on Table 4, the suggested value resulting from the algorithm is 1. The decision maker then checks the suggested value by comparing it with his/her perception. If the decision maker thinks that the suggested value is acceptable then it can be applied. For the case study, the decision maker agreed with the suggested value; therefore, it is applied in the matrix. Once the matrix becomes complete, then the next step of the AHP methodology which is synthesis and checking the CR can be performed.

4.5 Synthesis

The complete matrix is now synthesized according to the AHP stages presented by Saaty (2008). The results can be seen in Table 5.

Table 5
Result of the synthesis process using the AHP method

Supplier	Weight	Rank
Supplier A	21.4833	2
Supplier B	29.6901	1
Supplier C	16.1714	4
Supplier D	15.2651	5
Supplier E	17.3900	3

The results in Table 5 can be used by the company owner to determine the priority of the supplier to be selected. Suppliers with the lowest ranking are prioritized and ranked first.

4.6 Consistency Ratio (CR) checking

The complete matrix is checked for consistency and the CR value for this pairwise comparison is 0.0094. Since this CR <0.1, hence it can be concluded that the pairwise comparison is consistent.

4.7 Computational Time

The computational time to perform the AHP steps starting from finding the suggested value to completing the incomplete pairwise comparison matrices for CR checking is only 0.13 seconds. This computational time was obtained when the proposed method was implemented on a computer with Intel i-3 1.8GHz processor and 8GB RAM.

5. Numerical illustration of the proposed method to solve a larger problem

The first numerical illustration that shows the applicability of the proposed method occurs when the number of incomplete values in the pairwise comparison matrix from the previous case study is increased. Instead of one missing value in the incomplete pairwise comparison matrix, in this numerical illustration, two missing values are considered. The incomplete pairwise comparison is presented in Table 6, in which the first missing value is denoted as I_1 and the second missing value is denoted as I_2. It should be noted that the pairwise values in Table 6 are almost entirely the same as Table 3 for the case study discussed earlier. In addition, the second missing value ($\boldsymbol{I_2}$) is the same as the missing value presented in Table 3. The only value that is different is the first missing value ($\boldsymbol{I_1}$) in Table 6, which is actually already known in the case study above.

Table 6
Incomplete pairwise comparison matrix

	A	B	C	D	E
A	1	$\boldsymbol{I_1}$	1/3	1/3	1/3
B	$\boldsymbol{1/I_1}$	1	3	3	3
C	3	1/3	1	$\boldsymbol{I_2}$	1
D	3	1/3	$\boldsymbol{1/I_2}$	1	1
E	3	1/3	1	1	1

ISSN 1936-6744
https://doi.org/10.13033/ijahp.v16i1.1136

Using the proposed method, the best values of I_1 and I_2 are 1/7 and 1, respectively. Using these values, the CR is 0.0017. The proposed method, using the same computational device as in Section 4, was able to find this solution in 0.37 seconds.

These results must also be compared with previous case studies. The $\boldsymbol{I_2}$ value obtained with this calculation is 1, which turns out to be the same as the only missing value in the case study discussed in Section 4. Meanwhile the $\boldsymbol{I_1}$ value obtained in the results of this calculation is 7, while the actual value in Table 3 is 5. However, if we look at the CR together, both the preferences given by the decision maker and the recommended value of the proposed algorithm produced a CR $\leq$ 0.1.

To demonstrate the applicability of the proposed method, we also considered a larger pairwise comparison matrix that has a 7x7 dimension. There are four scenarios considered, in which one, two, three, and four missing values are present in the incomplete pairwise comparison matrix of each scenario, respectively.

The incomplete pairwise comparison matrix for the first scenario is presented in Table 7, which consists of only one missing value, denoted as I_1.

Table 7
First scenario of incomplete pairwise comparison matrix

	A	B	C	D	E	F	G
A	1	4	4	$\boldsymbol{I_1}$	4	3	2
B	1/4	1	2	1/2	1/2	1/3	1/2
C	1/4	1/2	1	1/3	1/2	1/2	1/2
D	$\boldsymbol{1/I_1}$	2	3	1	2	3	2
E	1/4	2	2	1/2	1	1/2	1/2
F	1/3	3	2	1/3	2	1	1/3
G	1/2	2	2	1/2	2	3	1

Using the proposed method, the best value of I_1 is 1. Using this value, the CR is 0.0499. The proposed method, using the same computational device, was able to find this solution within 0.27 seconds.

The incomplete pairwise comparison matrix for the second scenario is presented in Table 8, which consists of two missing values, denoted as I_1 and I_2.

Table 8
Second scenario of incomplete pairwise comparison matrix

	A	B	C	D	E	F	G
A	1	4	4	3	$\boldsymbol{I_1}$	3	2
B	1/4	1	2	1/2	1/2	1/3	1/2
C	1/4	½	1	1/3	$\boldsymbol{I_2}$	1/2	1/2
D	1/3	2	3	1	2	3	2
E	$\boldsymbol{1/I_1}$	2	$\boldsymbol{1/I_2}$	1/2	1	1/2	1/2
F	1/3	3	2	1/3	2	1	1/3
G	1/2	2	2	1/2	2	3	1

ISSN 1936-6744
https://doi.org/10.13033/ijahp.v16i1.1136

Using the proposed method, the best values of I_1 and I_2 are 5 and 1, respectively. Using this value, the CR is 0.0568. The proposed method, using the same computational device, was able to find this solution within 0.82 seconds.

The incomplete pairwise comparison matrix for the third scenario is presented in Table 9, which consists of three missing values, denoted as I_1, I_2, and I_3.

Table 9
Third scenario of incomplete pairwise comparison matrix

	A	B	C	D	E	F	G
A	1	4	4	3	4	3	2
B	1/4	1	2	1/2	1/2	$\boldsymbol{I_1}$	1/2
C	1/4	1/2	1	1/3	1/2	1/2	1/2
D	1/3	2	3	1	2	3	$\boldsymbol{I_2}$
E	1/4	2	2	1/2	1	$\boldsymbol{I_3}$	1/2
F	1/3	$\mathbf{1/}\boldsymbol{I_1}$	2	1/3	$\mathbf{1/}\boldsymbol{I_3}$	1	1/3
G	1/2	2	2	$\mathbf{1/}\boldsymbol{I_2}$	2	3	1

Using the proposed method, the best values of I_1, I_2, and I_3 are 1/2, 1/2, and 1, respectively. Using this value, the CR is 0.0390. The proposed method, using the same computational device is able to find this solution within 0.41 seconds.

The incomplete pairwise comparison matrix for the fourth scenario is presented in Table 10, which consists of four missing values, denoted as I_1, I_2, I_3 and I_4.

Table 10
Fourth scenario of incomplete pairwise comparison matrix

	A	B	C	D	E	F	G
A	1	$\boldsymbol{I_1}$	4	3	I_2	3	2
B	$\mathbf{1/}\boldsymbol{I_1}$	1	2	1/2	1/2	$\boldsymbol{I_3}$	1/2
C	1/4	1/2	1	$\boldsymbol{I_4}$	1/2	1/2	1/2
D	1/3	2	$\mathbf{1/}\boldsymbol{I_4}$	1	2	3	2
E	$\mathbf{1/}\boldsymbol{I_2}$	2	2	1/2	1	1/2	1/2
F	1/3	$\mathbf{1/}\boldsymbol{I_3}$	2	1/3	2	1	1/3
G	1/2	2	2	1/2	2	3	1

Using the proposed method, the best values of I_1, I_2, I_3 and I_4 are 5, 3, 1, and 1/4, respectively. Using this value, the CR is 0.0453. The proposed method, using the same computational device was able to find this solution within 0.28 seconds.

6. Discussion and conclusion

While previous research discusses several approaches to deal with an incomplete pairwise comparison, for example by reducing the number of questions (Maharani et al. (2019), other researchers have proposed several methods for completing the incomplete pairwise comparison matrix by creating an algorithm to provide a suggested preference

ISSN 1936-6744
https://doi.org/10.13033/ijahp.v16i1.1136

value that can be input into the incomplete pairwise comparison matrix (Ichihashi & Turksen,1993; Nishizawa, 1997; Shiraishi et al.,1998; Hu & Tsai, 2006; Gomez-Ruiz et al., 2010; Bernroider et al., 2010; Srdjevic et al., 2014; Pan et al., 2014; Chen et al., 2015; Jandova, 2016; Zhou et al., 2018; Maleki, 2020; Tekile et al., 2021; Sharabiani, 2023). The research proposed in this article attempts to solve incomplete pairwise comparison matrices by providing an algorithm based on PSO to find a suggested preference value.

Based on the case examples solved by the proposed PSO-based algorithm, the proposed algorithm can suggest preference values for incomplete pairwise comparison matrices with satisfactory CR values. These values can be taken into consideration by decision makers to give a final preference value in a pairwise comparison matrix.

In addition, the proposed algorithm was also tested using a larger pairwise comparison matrix with different unknown pairwise comparison values. From the results the following conclusions can be drawn:

a. The purpose of this research is to complete the incomplete pairwise comparison of the AHP by providing the suggested preference value that can obtain a CR ≤ 0.1.

b. The decision maker can use the suggested value resulting from the proposed method or use this value as a basis for their consideration.

c. The time needed to complete the pairwise comparisons to obtain CR values for complete pairwise comparisons and incomplete pairwise comparisons is influenced by the size of the matrix and the number of unknown pairwise comparison values.

ISSN 1936-6744
https://doi.org/10.13033/ijahp.v16i1.1136

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