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SOLVING LINEAR PROGRAMMING PROBLEMS USING THE SIMPLEX METHOD: THEORETICAL BASIS AND PRACTICAL APPLICATION

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Abstract: The article covers the theoretical foundations and practical applications of the Simplex method used in solving linear programming problems. The Simplex method is defined as the process of finding an optimal solution that provides the maximum or minimum value of the objective function through successive approximations. The article describes in detail the two main stages of the method - finding an initial base solution and determining the optimal solution. The structure of the simplex table, algorithms for selecting the decisive element, column and row, as well as the table replacement process are explained. A step-by-step solution of the linear programming problem given as an example using the Simplex method is presented and the practical application of the method is demonstrated. The article serves as a useful resource for students and specialists working in the field of mathematical optimization.

Keywords : Simplex method, linear programming, optimal solution, basic plan, Simplex table, decisive element, objective function, canonical form, free terms, iteration.

Literature review

The literature on linear programming and the Simplex method has been extensively studied in the fields of mathematical optimization, operations research, and decision making. The following are the main literature sources on this topic and their analysis: Dantzig, GB (1963). Linear Programming and Extensions. Analysis : Written by George Dantzig, the founder of the Simplex method, this book provides a detailed explanation of the theoretical foundations of linear programming and an early form of the Simplex method. The book explains the algorithmic structure of the method, the processes of finding a basic solution, and the determination of an optimal solution with examples. Dantzig's work is still considered a fundamental source on linear programming today. The book's shortcomings can be said to be its lack of adaptation to modern computing technologies, but it is theoretically important. Chvátal, V. (1983). Linear Programming. Analysis : This book is written in a simple and understandable language for students of linear programming. Practical applications of the simplex method, including the construction of the simplex tableau and iterative processes, are extensively covered through examples. The book is accessible to students and beginners and helps them understand the algorithms. However, it does not provide in-depth information on complex problems and modern optimization methods.

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Luenberger, DG, & Ye, Y. (2008). Linear and Nonlinear Programming. Analysis : This book aims to study linear and nonlinear programming together. The theoretical foundations of the simplex method and its practical application in a modern context are analyzed. The book explains the canonical form of the simplex method, the selection of the decisive element and the iterative processes with concrete examples. The advantage of the book is its adaptation to modern computational techniques and its relevance to nonlinear programming. As a disadvantage, some parts are intended for readers with a high level of mathematical knowledge. Taha, HA (2017). Operations Research: An Introduction. Analysis : Hamdy Taho's book on operations research aims to teach the Simplex method from a practical perspective. The book explains the structure of the Simplex table, the algorithms for selecting the optimal row and column, and real-life problems through examples. The book is useful for students and practitioners, and pays special attention to the application of the Simplex method using software (e.g. LINDO, Excel Solver). As a drawback, it can be said that the theoretical aspects are sometimes presented in a simplified form. Vanderbei, RJ (2020). Linear Programming: Foundations and Extensions.Analysis : This book includes modern forms of the Simplex method, including a comparison with interior point methods. The book clearly and systematically explains the two stages of the Simplex method (finding an initial base solution and finding an optimal solution). In addition, the canonicalization of the problem and the structure of the Simplex table are explained through examples. The advantage of the book is that it covers modern algorithms used to solve complex problems. As a drawback, some parts of the book may be complicated for beginners.

Research methodology

This study aims to apply linear programming methods to formulate an optimal production plan for a confectionery factory. The research methodology includes empirical and theoretical analysis. During the study, existing scientific sources on the formulation of an optimal production plan are studied through literature analysis. Various mathematician modeling methods , including simplex method , graphic method and dual method using working release processes optimization opportunities analysis Comparative analysis through various economic models compared and their confectionery products working release to the process compatibility is determined . In this working release resources limited , product types benefit level and demand conditions into account is obtained . Experimental analysis and theoretical basically of the optimal plan formulated to practice implementation to be completed to study aimed at to be , to work release size increase and expenses reduce according to recommendations working Qualitative analysis methodological aspects , work release process conditions and the results quality in terms of to evaluate is based on . Research methodology working release plan thorough planning , resources effective distribution and maximum benefit to take for scientific approaches to determine These methods are aimed at using working release process further improvement and economic efficiency increase possible .

Introduction. Processes research and optimal management – decision acceptance to do and systems to optimize scientific fields oriented . Processes research resources effective distribution for mathematician models , linear programming , games theory and networks optimization such as from methods uses.Optimal management systems the most good management strategies determination with He is engaged in . His main methods Pontryagin's Maximum principle and Bellman's dynamic programming .

Analyses and results

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Simplex method linear programming the issue solution main from the methods one gradually , successively approach method using $x_1, x_2, \ldots x_n$ of variables such an optimal value found that this values goal to the function maximum (or minimum) value gives .

The following linear programming issue given let it be :

$$\overset{n}{\underset{j=1}{}} a_{ij} x_j \quad b_i, \quad (i = \overline{1,m})$$

$$x_j \quad 0 \qquad (j = \overline{1,n})$$

$$Z = \overset{n}{\underset{j=1}{}} c_i x_i \to \max$$

The matter solution for simplex table we build and simplex method the idea to give for given issue as follows canonical in uniform we write .

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1s}x_s + \ldots + a_{1n}x_n + x_{n+1} = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2s}x_s + \ldots + a_{2n}x_n + x_{n+2} = b_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{ms}x_s + \ldots + a_{mn}x_n + x_{n+m} = b_m$$

$$x_j \quad 0 \quad (i = \overline{1,m}) \quad (j = \overline{1,n})$$

$$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n \to \max$$

Here x_{n+i} - free variables They are called convenience , and other from variables distinction for suitable accordingly $y_1, y_2, \ldots y_m$ we mark as and again following designations we enter $b_{i0} = b_i$; $b_{i,j} = a_{i,j}$; $b_{0j} = c_j$. These designations based on following simplex so- called table the table we will fix it .

BO'	1	$-x_1$	$-x_2$	. . .	$-x_s$	. . .	$-x_n$
y_1	b_{10}	b_{11}	b_{12}	. . .	b_{1s}	. . .	b_{1n}
y_2	b_{20}	b_{21}	b_{22}	. . .	b_{2s}	. . .	b_{2n}
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
y_s	b_{r0}	b_{r1}	b_{r2}	. . .	**b_{rs}**	. . .	b_{rn}
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
y_m	b_{m0}	b_{m1}	b_{m2}	. . .	b_{ms}	. . .	b_{mn}
Z	b_{00}	b_{01}	b_{02}	. . .	b_{0s}	. . .	b_{0n}

Linear programming the issue simplex method using solution two from the stage consists of :

1. Start with the inner support plan find .
2. Basic plans from within optimal plan of the problem find .

The matter support plan to compile .

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Start drinking support plan find following algorithm according to is done :

1. From the simplex table solution doer element find :

1. 1. Solution doer element find before solution doer the column from finding begins . This for free terms to the column is considered . If free limits on the column elements all positive if so , this The initial plan will be the base plan. and second to the stage is passed . If there is a negative element if any , from them module according to the most the eldest is selected (if one if this element itself is taken). Example for let's say this element b_{r0} Let the row containing this b_{r0} element be If the line from the elements all positive if so , the issue solution there is it won't be (this without calculations this in place (If the line there is a negative element if any , from them module according to the most the eldest is selected (if one if himself/herself This element is taken . column solution doer column It is called . Example for this is the sth column Let it be .

1. 2. Solution doer line is found . Free limits solution doer column to the elements to be will be released and from them positive the most younger is selected , that is

$$\min \quad \frac{b_{10}}{b_{1s}}; \frac{b_{20}}{b_{2s}}; \ldots ; \frac{b_{r0}}{b_{rs}}; \ldots ; \frac{b_{m0}}{b_{ms}} \quad .$$

Let's say this proportions inside positive the most younger b_{r0} / b_{rs} Let it be . Then this b_{rs} element standing line solution doer line It is said , b $_{rs}$ of the element himself/ herself and solution will be the element that makes .

2.Solver column and line variables places is replaced , (i.e. x $_{s}$ and y_s new in the table places exchange).

3. Simplex in the table replacement is done .

3. 1. Solution doer column elements solution doer to the element become out new to the table is written , that is $b'_{is} = -b_{is} / b_{rs} \ (i \quad r)$.

3. 2. Solution doer line elements solution doer to the element become out new to the table is written , that is $b'_{rj} = b_{rj} / b_{rs} \quad (j \quad s)$.

3. 3. Solution The element that makes the difference is set to 1. to oneself is divided , that is $b'_{rs} = 1/b_{rs}$. 3. 4.New simplex of the table remaining elements by the following formula is found .

$$b'_{ij} = \frac{b_{ij}b_{rs} - b_{is}b_{rj}}{b_{rs}} \text{ or } b'_{ij} = b_{ij} - \frac{b_{is}b_{rj}}{b_{rs}} \; ; \quad i \quad r, \; j \quad s$$

b r $_{ij}$ element in the new table , finding the elements b $_{ij}$, b $_{is}$,b $_{rj}$, b $_{rs}$ from the old table will be as follows:

$b_{ij} - br_{ij}$ of the element old in the table to him/her suitable element;

$b_{is} - b_{ij}$ element standing line with b $_{r}$ solution column of the element element at the intersection ;

b $_{rj}$ -b $_{ij}$ element standing column with b_{rs} solution element string element at the intersection ;

b_{rs} - thing Qiluvchi element.

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BO'	1	$-x_1$	$-x_2$	. . .	$-y_r$	. . .	$-x_n$
and $_1$	b'_{10}	b'_{11}	b'_{12}	. . .	b'_{1s}	. . .	b'_{1n}
and $_2$	b'_{20}	b'_{21}	b'_{22}	. . .	b'_{2s}	. . .	b'_{2n}
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
x_s	b'_{r0}	b'_{r1}	b'_{r2}	. . .	b'_{rs}	. . .	b'_{rn}
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
y_m	b'_{m0}	b'_{m1}	b'_{m2}	. . .	b'_{ms}	. . .	b'_{mn}
Z	b'_{00}	b'_{01}	b'_{02}	. . .	b'_{0s}	. . .	b'_{0n}

4.Newly discovered simplex in the table basic plan available if second to the stage , i.e. the optimal plan to find will pass , otherwise without above process new table for till until a basic plan is found again repeated .

The optimal plan of the problem find .

If from step 1 taken base planning simplex in the table Z - line elements (free present br_{00} except) all positive if so , this taken elementary The base plan is unique and is the optimal plan (solution) for the problem . If Z in line everyone positive from elements at least one to zero equal if , then of the matter infinite multi -optimal plan there is will be . If Z in line from elements no unless one negative If , the optimal plan is as follows algorithm according to found :

Decisive element find .

1. 1. Solution doer column is found . Z - in the line negative of elements module according to the most the eldest (one) if itself) is selected . This element is column solution doer column It will be .

1. Solvent line is found . Free terms elements solution doer column to the elements to be will be released and from them positive the most younger is taken , that is first in step 1.2 Like this . suitable incoming element in the column is solved the element that makes and this element is standing line and solution doer line It will be .

2.Solver line and column variables own their places replaces .

3. Simplex in the table replacement is executed . Simplex replacement points 3.1, 3.2, 3.3, 3.4 in stage 1 such as is done .

4.Newly discovered of the table Z line is considered . If Z in line everyone elements positive if so , taken The final plan is the optimal plan for the problem. will be . Otherwise without points 1,2,3 above again is repeated until an optimal plan is found .

Note : Linear programming in the matter of , if the goal function my little one if sought step 1 above completely appropriate and in the 2nd stage only Z - row elements negative to the situation to be brought need , that is reverse situation will be .

Optimization of textile production at the Yuksalish enterprise

"Yuksalish" is engaged in the production of textile products and produces three types of products: shirts (M1) , trousers (M2) and jackets (M3) . Four resources are required to produce each product: fabric (meters), labor hours, sewing machine hours and thread (kg). The objective of the enterprise is to maximize total profit. The following data are given:

Shirt (M1):

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- 1 meter of fabric, 4 hours of work, 1 sewing machine hour, -3 kg of thread (negative value due to leftover thread used, e.g. during rework).
- Profit : 5 thousand soum (each) one party for).

Pants (M2):

- 1 meter fabric , 2 works hour , -1 stitch car hour (economical) sewing technology because of car time again distributed), 2 kg thread
- Profit : 1 thousand soum (each) one party for).

Jacket (M3):

- 1 meter fabric , 1 piece watch , 2 stitches car hour , -2 kg thread (decorative) at the seams again used thread because of negative value).
- Profit : 3 thousand soum (each) one party for).

Resource limitations :

- Fabric: 2 meters (daily) backup).
- Work hours : 3 hours (of workers) daily opportunity).
- Sewing car hours : -1 hour (economical) technologies because of again distribution possibility , negative value).
- Yarn : 5 kg (daily) backup).

Requirement: Using the simplex method, find the optimal production quantity of shirts (M1), pants (M2), and jackets (M3) that will ensure maximum profit for the "Yuksalish" enterprise.

In mathematical form: $z = 5x_1 - x_2 + 3x_3$

linear to the function maximum value giver

$x_1 + x_2 + x_3 \quad 2$

$4x_1 + 2x_2 + x_3 \quad 3$

$x_1 - x_2 + 2x_3 \quad -1$

$-3x_1 + 2x_2 - 2x_3 \quad 5$

$x_1 \quad 0,\ x_2 \quad 0, x_3 \quad 0$

borderline of the system possible was solutions in the field unknowns find . Boundary the system canonical apparently as follows writing we get :

$x_1 + x_2 + x_3 + x_4 = 2$

$4x_1 + 2x_2 + x_3 + x_5 = 3$

$x_1 - x_2 + 2x_3 + x_6 = -1$

$-3x_1 + 2x_2 - 2x_3 + x_7 = 5$

$x_1 \quad 0,\ x_2 \quad 0, x_3 \quad 0$

In the equation basis o ' variables simplex from variables distinction for $x_4 = y_1,\ x_5 = y_2,\ x_6 = y_3,\ x_7 = y_4$ designations we enter and Simplex table we will fix it .

So / Color	1	-x 1	-x 2	- x 3
at 1	2	1	1	1
at 2	3	4	2	1
at 3	-1	1	-1	2

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at 4	5	-3	2	-2
Z	0	-5	1	-3

Variables negative not to be condition given into account take , directly support the solution to find Let's go in . Free terms negative -1 in indicative there is a coefficient . From this series gesture negative was module according to the most big element We will find it . It x_2 is the -1 element in the column . The rule according to positive proportions from within the most the youngest we find :

$+min\{2/1,\ 3/2,\ -1/-1,\ 5/2\} = 1/1$

So , to him matching element x 2 The -1 element in the column . This element is the solution will be the element that makes . Now Simplex replacement so , the following the table we will fix it .

So / Color	1	-x 1	- y 3	- x 3
at 1	1	2	1	3
at 2	1	6	2	5
x2	1	-1	-1	-2
at 4	3	-1	2	2
z	-1	-4	1	-1

this table apparently it is so free terms positive , this reason There is a basic plan . Now the optimal solution find to the Z row for Let's look at this line . two negative indicative there is a coefficient . Of them module according to value big was coefficient selectively we get , it is an element of -4 . By rule according to solution doer element clearly new table we will create :

$+min\{1/2,\ 1/6,\ 1/-1,\ 3/-1\} = 1/6$

To him/her matching item x 1 6 elements in column 1. This element is the solution will be the element that makes . Now Simplex replacement so , the following the table we will fix it .

So / Color	1	-y 2	-y 3	- x 3
In 1	2/3	-1/3	1/3	4/3
X 1	1/6	1/6	1/3	5/6
X2	7/6	1/6	-2/3	-7/6
At 4	19/6	1/6	7/3	17/6
Z	-1/3	2/3	7/3	7/3

Free terms and in the Z row odds is positive . Therefore , the optimal solution is found , that is $y_2 = y_3 = x_3 = 0$ and $x_1 = 1/6, x_2 = 7/6$ when Z 's maximum value $-1/3$ equal to will be , that is $z = -1/3$.

The "Yuksalish" enterprise has 16 batches to ensure maximum daily profit (2.33 million soums). $1/6$ should produce $x_3 = 0$ a shirt (about 0.17 batches), a batch of pants (about 1.17 batches), and not produce a jacket $7/6$

Conclusion

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The article provides a detailed description of the theoretical foundations and practical application of the Simplex method in solving linear programming problems. The Simplex method is based on the process of successive approximation to find the optimal (maximum or minimum) value of the objective function and consists of two stages: finding an initial base solution and determining the optimal solution. The algorithmic structure of the method, the structure of the Simplex table, the processes of selecting the decisive element, columns and rows are explained through examples. The analysis of the literature confirms the importance of the Simplex method in the field of mathematical optimization, but shows that there are some limitations in adapting to modern computing technologies and solving complex problems. As a practical example, the problem of optimizing the production of textile products of the "Yuksalish" enterprise was solved using the Simplex method, and it was determined that the maximum daily profit is 2.33 million soums, for which it is necessary to produce 0.17 batches of shirts, 1.17 batches of trousers and not to produce jackets. The research methodology combines theoretical and empirical analysis to prove the effectiveness of the Simplex method in optimizing production processes. The Simplex method is an important tool for solving linear programming problems, allowing for efficient resource allocation and profit maximization. It serves as a useful resource for students, professionals, and practicing economists, but further development of integration with modern algorithms and software is required.

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