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# MATHEMATICAL MODEL OF THE TRANSPORT PROBLEM AND OPTIMAL SOLUTION METHODS

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**Abstract:**The transportation problem is an optimization problem aimed at finding a plan for transporting goods from the points of departure to the points of reception with minimal cost. Based on the given cargo stocks, requirements and transportation prices, the objective function $Z = \Sigma cij \cdot xij$ is minimized. An initial plan is created using the "Northwest Corner" and "Small Elements" methods, and the optimal plan is determined using the potential method. In the example, the minimum cost $Zmin = 375$ is achieved.

**Keywords:** Transportation problem, optimization, tariff matrix, base plan, Northwest corner method, Finite element method, potential method, objective function.

**Literature analysis on the issue of transportation**

The transportation problem occupies an important place in the field of mathematical programming and optimization. The literature listed below provides fundamental and practical knowledge on this topic. Akulich I. L. Mathematical programming in in examples and задачах (1996) – The book explains the transport problem through practical examples and problems, emphasizing methods for creating a basic plan and finding optimal solutions. Badalov FB Optimization Theory and Mathematical Programming (1989) – This resource in Uzbek describes the mathematical model of the transport problem and methods for solving it in a simple language, convenient for local readers. Kuznetsov A.V., Novikova G.I., Kholod N.I. Collection of problems on mathematical programming (1985) – As a collection of problems, it is useful for studying various aspects of the transport problem and developing practical skills. Kuritsky B. I. Search optimal decision with Excel tools (1997) – presents a modern approach to solving the transport problem in Excel, with high practical applicability. Safa e and K., Be knazarova N. Mathematical methods for verifying operations (1984) – explains the theoretical foundations of the transport problem and the method of potentials, but may be outdated.

**Research methodology**

The following methodology is used to solve the transportation problem. Mathematical modeling: The problem ($Z = \Sigma cij \cdot xij$) is formulated in the form of linear programming, based on the objective function and boundary conditions (cargo stocks, requirements). Initial plan: Northwest corner method: Load distribution starting from the upper left corner of the table. Small element method: Distribution starting from the cells with the lowest cost.

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Optimal solution: The difference in empty cells $\left(Sij = cij - (ui + vj)\right)$ is calculated using the potential method, the loads are redistributed using a closed loop, and the optimal plan is found until Sij ≥ 0. Analysis: Costs of plans $(Z)$ is compared, for example, $Zmin = 375$ to obtain . Tools like Excel or Python automate the calculation.

**Analyses and results**

The problem of finding the optimal plan for transporting goods from shipping points to given receiving points is called the transportation problem and is formulated as follows:

$a_1$, $a_2$,..., $a_m$ quantities of homogeneous cargoes $A_1, A_2,.., A_m$ at their respective points. $A_1, A_2,.., A_m$ We call these points of departure. These cargoes must be received by n $B_1, B_2,..., B_n$ points and their requirements are given accordingly . Let $b_1, b_2,..., b_n$ the cost of transporting each $x_{ij}$ unit of cargo $i-$ from the chith point of departure to the chith point of reception be known as c $j-$ ij . We must design a plan for transporting these cargoes in such a way that the demand points receive maximum satisfaction and the sum of the operations spent on transporting all cargoes is minimal.

We present the transportation problem conditionally in the form of a table. The table shows: receiving points, sending points, cargo stocks, cargo demand, and the cost of cargo units sent from each $i-$sending point $j-$ to each receiving point (i.e., the tariff matrix).

| Shipping punks | Pick-up points | | | | Cargo Zapatistas |
|---|---|---|---|---|---|
| | $B_1$ | $B_2$ | . . . . | $B_n$ | |
| $A_1$ | $c_{11}$ $x_{11}$ | $c_{12}$ $x_{12}$ | . . . . | $c_{1n}$ $x_{1n}$ | $a_1$ |
| $A_2$ | $c_{21}$ $x_{21}$ | $c_{22}$ $x_{22}$ | . . . . | $c_{2n}$ $x_{2n}$ | $a_2$ |
| . . . | . . . . | . . . . | . . . . | . . . . | ... |
| $A_m$ | with $_{m1}$ $x_{m1}$ | m2 $x_{m2}$ | . . . . | $c_{mn}$ $x_{mn}$ | $a_n$ |
| The need for luggage | $b_1$ | $b_2$ | . . . . | $b_n$ | e $a_i$ = e $b_j$ |

Here, $C = \{c_{ij}\}$ the matrix is called the tariff matrix or transportation costs. $X = \{x_{ij}\}$ and the matrix is called the transportation problem plan. Here is $x_{ij} - i$ the volume (number) of goods to be transported from point i to point j. The total cost associated with the transportation plan is expressed by the following objective function.

$$Z = c_{11}x_{11} + c_{12}x_{12} + \ . . . \ + c_{1n}x_{1n} + c_{21}x_{21} + c_{22}x_{22} + \ . . . \ + c_{2n}x_{2n} + \ . . .$$
$$c_{m1}x_{m1} + c_{m2}x_{m2} + \ . . . \ + c_{mn}x_{mn}.$$

Here, $x_{ij}$ the variables must satisfy the conditions (constraints) of load capacity, load demand, and non-negativity.

Taking the above into account, the mathematical model of the transportation problem can be written as follows.

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$$\sum_{j=1}^{n} x_{ij} = a_i, \quad (i = \overline{1, m})$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad (j = \overline{1, n})$$

$$x_{ij} \quad 0 \qquad (i = \overline{1, m}) \quad (j = \overline{1, n})$$

$$Z = \sum_{j=1}^{n} \sum_{i=1}^{m} c_{ij} x_{ij} \rightarrow \min$$

The mathematical formulation of the transportation problem is interpreted as follows: Given a boundary system, a non-negative condition, and an objective function, the task is to find a non-negative solution (plan) from the solution set of the system such that the objective function attains a minimum value.

The transportation problem is divided into two types, open and closed. If the sum of the cargo stocks is equal to the sum of the cargo demands, that is,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

the issue will be a closed type issue

If the sum of the cargo stocks is not equal to the sum of the required cargoes, i.e.

$$\sum_{i=1}^{m} a_i \qquad \sum_{j=1}^{n} b_j$$

The issue will be an open-ended issue.

**2. Methods for solving the transportation problem**

Solving the transportation problem consists of two stages.

1. Find a basic plan.

2. Finding the optimal plan from among the basic plans.

There are several ways to create a basic plan: "Northwest corner", "Small elements", "Fogel'", etc.

**"Northwest corner" method.**

The use of the "northwest corner" method when drawing up a preliminary cargo transportation plan is carried out as follows:

1. A tariff schedule is drawn up.

| | $b_1$ | $b_2$ | ..... .. . | $b_n$ |
|---|---|---|---|---|
| $a_1$ | $c_{11}$ | $c_{12}$ | . . . . . | $c_{1n}$ |
| $a_2$ | $c_{21}$ | $c_{22}$ | . . . . . | $c_{2n}$ |
| . . . . . | . . . . . | . . . . . | . . . . . | . . . . . |
| $a_m$ | $c_{m1}$ | $c_{m2}$ | . . . . . | $c_{mn}$ |

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2. Left on the side above corner , that is ( north - west ) corner ) from starting line according to or column according to we move to cell (1,1) a $_1$ and b $_1$ of the most the youngest we place , that is , $x_{11} = min(a_1, b_1)$.

3. If $a_1 > b_1$ if $x_{11} = b_1$ what we give , first column this with is closed , that is $x_{i1} = 0 \ (i = 2, m)$. ( First acceptance the one who does demand complete satisfied ).

4. We move along the first line to the cell (1;2), where $a_1 - b_1, b_2$ we place the smallest of , that is, $x_{12} = \min(a_1 - b_1, b_2)$.

5. $b_1 > a_1$ If so, the 1st line is closed, i.e. $x_{1j} = 0 \ (j = 2, n)$.

6. We proceed to fill in the adjacent cells (2.1), i.e. $x_{21} = min(a_2, b_1 - a_1)$.

7. We move on to filling in the cells of the second row or the second column, and so on. This process continues until the resources run out.

**"Small elements" method.**

Finding the base plan using the "small elements" method is done as follows:

1. Freight begins with filling in the box corresponding to the lowest shipping cost in the tariff table for the recipients.

2. The smallest of a $_i$ or b $_j$ is placed in the lowest tariff c $_{ij}$ box .

3. Then, when the row or receiving point requirement is met, the corresponding column is deleted.

4. If the cargo stocks at the shipping point are fully distributed and the recipient's request is fully fulfilled, the corresponding row and column will be deleted.

5. A further small definition is obtained from the remaining rows and columns. The process of distributing cargo stocks continues until the cargo stock is exhausted and the requirements are satisfied.

**Potential method.**

If an initial base plan is found using the above methods, finding the optimal plan is performed using the potential method.

The potential method for finding the optimal plan for the transportation problem is as follows:

1. Using the methods listed above, a basic cargo transportation plan is determined.

$_i$ and v $_j$ are determined for the points receiving and sending cargo, respectively .

3. The sum of potentials in empty cells is $c\check{y}_{ij} = u_i + v_j$.

4. The difference between the rates $c_{ij}$ and the empty cells $c\check{y}_{ij}$ is calculated.

The resulting plan is optimal if all empty cells have the same difference . $S_{ij} > 0$

6. If it is in one of the empty cells , the variable value x $S_{ij} < 0$ ij is entered in the non-empty cells , that is, the above difference is minimal. For this cell, a closed contour is formed using the non-empty cells and the loads are redistributed in this contour. As a result, we get a new transportation plan.

This process $S_{ij} > 0$ continues until there is no difference and the final cargo transportation plan is optimal.

**Example.**

Solve the transportation problem given in the table below.

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| b k / oh my | 40 | 25 | 20 | 50 |
|---|---|---|---|---|
| 60 | 5 | 4 | 1 | 2 |
| 40 | 4 | 2 | 6 | 3 |
| 35 | 7 | 3 | 5 | 4 |

Solution: We find the initial base plan using the "Northwest corner" and "Small elements" methods. According to the "Northwest corner" method, $X_{1,1} = min(60, 40) = 40$ we place the number in the (1,1) cell of the table, and the next $X_{12} = min(60-40, 25) = 20$ number in the (1,2) cell. Thus, the load at the first point is completed, and the next cells (1,3) and (1,4) are closed.

We start distributing the cargo at the next point. $X_{22} = min(40,5) = 5$ We place the number in the (2,2) cell. Thus, the requirements of the 1st and 2nd applicants are satisfied, that is, the 1st and 2nd columns are closed. $X_{23} = min(35, 20) = 20$ It is placed in the (2,3) cell. The requirement of the 3rd applicant is fulfilled. We place the remaining cargo in the (2,4) cell, that is, $X_{24} = min(15,50) = 15$ and the cargo at the second dispatch point is finished. We start distributing the cargo at the 3rd dispatch point.

The cells (3,1), (3,2), (3,3) are closed, that is, the demands of applicants 1,2 and 3 are satisfied. $X_{34} = min(35,35) = 35$ We write in cell (3,4). Thus, the loads are completely distributed, that is, we have the following plan.

$$X = \begin{matrix} 40 & 20 & 0 & 0 \\ 0 & 5 & 20 & 15 \\ 0 & 0 & 0 & 35 \end{matrix}$$

The objective function value $Z = 595$ is .

We will now find the base plan of the problem using the "Least Elements" method.

Solution: We find the smallest cost by column or row.

This element is located in the cell (1;3) in the row, i.e. c $_{13}$ =1. Therefore, $X_{13} = min(60, 20) = 20$ we place the load in this cell. The requirement of the third applicant is satisfied. Therefore, the 3rd column is not considered in further calculations. We find the next smallest element. This element is located in the cells (1,4) and (2,2), i.e. $c_{14} = 2$ and $c_{22} = 2$. We place the loads in these cells. $X_{14} = min(60-20, 50) = 40,\ X_{22} = min(40, 25) = 25$. The requirement of the second applicant is satisfied, therefore, the 2nd column is not considered in further calculations.

The next smallest elements are located in cells (2,4) and (3,2), i.e. $c_{24} = 3$ and $c_{32} = 3$. We place loads in these cells. $X_{24} = min(40-25, 50-40) = 10$. The cell (3,2) is not considered, because this column is excluded. We look for the next smallest element, this is element $c_{21} = 4$. We place the load in this cell. $X_{21} = min(15-10,\ 40) = 5$. The last smallest element $c_{31} = 7$. We also place the load in this cell $X_{31} = min(35, 40-5) = 35.$. As a result, we have distributed the loads and obtained the initial base plan, i.e.

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$$X = \begin{matrix} 0 & 0 & 20 & 40 \\ 5 & 25 & 0 & 10 \\ 35 & 0 & 0 & 0 \end{matrix}$$

We calculate the objective function $Z = 415$.

Now we find the optimal plan for the problem. To find the optimal plan, we use the method of potentials. Suppose the initial base plan was found using the "Least Elements" method. We write it in the form of a table below.

| $b_k$ / $a_i$ | 40 | 25 | 20 | 50 | u |
|---|---|---|---|---|---|
| 60 | 5 | 4 | 1<br>20 | 2<br>40 | 0 |
| 40 | + 4<br>5 | - 2<br>25 | 6 | 3<br>10 | 1 |
| 35 | - 7<br>35 | + 3 | 5 | 4 | 4 |
| V | 3 | 1 | 1 | 2 | |

Non-empty cells in the table satisfy the following condition.

$r = 3 + 4 - 1 = 6$

We determine the potential of the shipper and the demander and obtain the following equations.

$u_1 + v_3 = 1;\ u_1 + v_4 = 2\ ;\ u_2 + v_1 = 4; u_2 + v_2 = 2;\ u_2 + v_4 = 3\ ;\ u_3 + v_1 = 7$

It is known that the number of equations is 1 less than the number of unknowns, that is, 1 of the unknowns is free and it can take any value. For example, let's say $u_1 = 0$. Then the remaining potentials are determined as follows

$u_1 = 0,\ v_3 = 1,\ v_4 = 2,\ u_2 = 1,\quad v_2 = 1,\ v_1 = 3,\ u_3 = 4.$

Empty in cells $s_{ij}$ value following formula with we will determine $S_{ij} = c_{ij} - (u_i + v_j)$. In that case

$S_{11} = 5 - (0 + 3) = 2;\ S_{12} = 4 - (0 + 1) = 3;\ S_{23} = 6 - (1 + 1) = 4;$

$S_{32} = 3 - (4 + 1) = -2; S_{11} = 5 - (4 + 1) = 0;\ S_{34} = 4 - (4 - +2) = -2.$

The resulting plan cannot be optimal, because there are also negative ones in $S_{32} = S_{34} = -2$ the $s_{ij}$ s. We create a closed contour (cycle) for these cells. For cell (3,2), the contour is (3,2),(3,1),(2,1),(2,2). We draw the contour clockwise or counterclockwise, starting from cell (3,2), sequentially placing + and - signs. We choose the smallest of the negative cells $min(25;35) = 25$, i.e. $x_{22} = 25$. We redistribute the loads in the cells. When distributing, the overall balance should not be disturbed in these cells and the costs should be minimal. We add the load in the negative cell (2,2) to the load in the next positive cell. In this case, $5 + 25 = 30$ there will be a load in cell (2,1). In order not to disturb the balance, we load 25 units of the load in cell (3,1) to cell (3,2). Thus, we have a new plan. We define the potentials for this table and calculate the $s_{ij}$ in the empty cells:

$S_{11} = 5 - (0 + 3) = 2;\ S_{12} = 4 - (0 + 1) = 3;\ S_{23} = 6 - (1 + 1) = 4;$

$S_{22} = 2 - (2 + 1) = 0; S_{33} = 5 - (4 + 1) = 0;\ S_{34} = 4 - (4 - +2) = -2;$

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| $b_k$ / $a_i$ | 40 | 25 | 20 | 50 | u |
|---|---|---|---|---|---|
| 60 | 5 | 4 | 1<br>20 | 2<br>40 | 0 |
| 60 | 5 | 4 | 1<br>20 | 2<br>40 | 0 |
| 40 | + 4<br>30 | 2<br>0 | 6 | - 3<br>10 | 1 |
| 35 | - 7<br>10 | 3<br>25 | 5 | + 4 | 4 |
| and | 3 | 1 | 1 | 2 | |

The newly obtained plan is also not optimal, because S $_{34}$ =-2. We construct a closed contour and redistribute the loads within this contour, resulting in the following plan.

| b $_k$ / oh $_{my}$ | 40 | 25 | 20 | 50 | she is |
|---|---|---|---|---|---|
| 60 | 5 | 4 | 1<br>20 | 2<br>40 | 0 |
| 40 | 4<br>40 | 2<br>0 | 6 | 3 | 1 |
| 35 | 7 | 3<br>25 | 5 | 4<br>10 | 2 |
| V | 3 | 1 | 1 | 2 | |

The resulting plan is an optimal plan because all empty cells have positive S $_{ij}$ . Therefore, the optimal plan is as follows.

$$X = \begin{matrix} 0 & 0 & 20 & 40 \\ 40 & 0 & 0 & 0 \\ 0 & 25 & 0 & 10 \end{matrix}$$

The value of the objective function $z_{min} = 375$.

**Conclusion**

The transportation problem is a linear programming problem aimed at optimizing the delivery of goods from the point of shipment to the point of reception with minimal cost. The mathematical model is the objective function $(Z = \Sigma cij \cdot xij)$ and boundary conditions. The initial plan is found by the "Northwest Corner" or "Small Elements" methods, and the optimal solution is found by the potential method. $Zmin = 375$ In the example, was achieved. The problem is of great importance in economics and logistics and is effectively solved using modern software tools.

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