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DERIVATIVE OF A FUNCTION AND ITS APPLICATIONS IN ECONOMICS

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ANNOTATSIYA: Ushbu maqolada hosila tushunchasining iqtisodiy muammolarni hal qilishdagi ahamiyati yoritilgan. Xususan, funksiyaga tangens chizig‘i tenglamasini tuzish, tezlik va tezlanishni hisoblash, ishlab chiqarish xarajatlari, funksiyaning elastikligi, daromad va narxga nisbatan talab elastikligi kabi iqtisodiy masalalarga hosilaning qo‘llanilishi tahlil qilinadi. Maqolada oliy ta'limda matematikani iqtisodiy yo‘nalishda o‘qitish uslubiy yondashuvlar bilan boyitilishi lozimligi ko‘rsatib o‘tilgan.

KALIT SO‘ZLAR: Hosila, iqtisodiy masalalar, funksiyaning elastikligi, talab elastikligi, narx, daromad, matematik tahlil, o‘qitish metodikasi

АННОТАЦИЯ:В данной статье рассматривается значение понятия производной функции при решении экономических задач. В частности, анализируется применение производной при построении уравнения касательной к графику функции, расчёте скорости и ускорения, затрат на производство, эластичности функции, эластичности спроса по доходу и цене. В статье подчёркивается необходимость методического обогащения преподавания математики в экономических направлениях высшего образования.

КЛЮЧЕВЫЕ СЛОВА:Производная, экономические задачи, эластичность функции, эластичность спроса, цена, доход, математический анализ, методика преподавания

ANNOTATION. This article highlights the significance of the derivative concept in solving economic problems. Specifically, it analyzes the use of derivatives in constructing tangent line equations, calculating speed and acceleration, production costs, function elasticity, and the elasticity of demand with respect to income and price. The article emphasizes the need for methodological enrichment of teaching mathematics in economics-oriented higher education programs.

KEYWORDS. derivative, economic problems, function elasticity, demand elasticity, price, income, mathematical analysis, teaching methodology

INTRODUCTION. The introduction of new standards of higher education in the Republic of Uzbekistan is a necessity dictated by the current situation and the rapid development of science and technology, as well as the integrated development of all sectors of the national economy. It is the new standards that set the goal of forming a personality with a set of qualities that allow a person to be successful in the 21st century.

The purpose of higher education is to create conditions for student self-realization in the educational process, to develop the ability to pose practical problems in one’s specialty, the ability to choose methods for solving a given problem, to solve, analyze and apply solutions to practical problems, using the latest achievements of science and technology. To achieve these goals, we consider it advisable to use professionally oriented modern

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pedagogical technologies in teaching all subjects at universities, taking into account the specifics of the subject and specialty.

The subject of mathematics has its own special place in this process. Here you should definitely take into account how much attention the government and state of Uzbekistan pays to the issue of education in general, as well as the teaching of mathematics at all levels of education and the integration of science and production.

The emergence of students' interest in mathematics depends to a large extent on the methodology of its teaching, on the chosen style of communication with students, and on the extent to which the applied problems of this particular specialty are demonstrated.

When solving many problems in science, technology, and economics, the concept of a derivative function is encountered. For example, when drawing up an equation for a tangent to a curve, when calculating speed and acceleration, when studying and plotting functions, when calculating production costs, elasticity of a function, elasticity of demand relative to income, relative to price, and others. Therefore, studying the concept of derivative is one of the most important issues in mathematics.

METHODS. Definition 1. Let $f(x)$ be a function defined outside some neighborhood of the point x_0. The derivative of the function $f(x)$ at the point x_0 is the number equal to

$$\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0} \qquad (1)$$

If this limit exists and is denoted by $f'(x_0)$.

To find the derivative of a given function, you must perform the following work:

1) Let's assign an increment Δx to the free variable x, then the function y takes on an increment Δy, i.e.
$y+\Delta y=f(x+\Delta x)$.

2) Let's find the increment of the function Δy :

$\Delta y=f(x+\Delta x)-y=f(x+\Delta x)-f(x)$.

3) Find the ratio of the increment of the function to the increment of the argument:

$\frac{\Delta y}{\Delta x}=\frac{f(x+\Delta x)-f(x)}{\Delta x}$.

4) Let us calculate the limit of the ratio of the increment of the function to the increment of the argument as $\Delta x\to 0$, i.e.

$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$.

If this limit exists, then it is called the derivative of the function $f(x)$ at the point x_0 and is denoted by y' or $f'(x_0)$, i.e.

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}=f'(x_0)=y' \qquad (2)$$

If limit (2) exists, then $\Delta y\to 0$ as $\Delta x\to 0$. This means that $f(x)-f(x_0)\to 0$ or $f(x)\to f(x_0)$. Consequently, the derivative at a point exists only for functions that are continuous at the point x_0 (and even then not for all).

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Limit (2) may not exist at all or may exist for some values of x and not exist for other values. At those points x where limit (2) does not exist, the derivative of the function is not defined.

For example, find the derivative of the function

$$y = f(x) = x^2$$

Solution.

1) Let's set x the increment Δx, then $y = f(x)$ takes the increment Δy, i.e. $y + \Delta y = f(x + \Delta x) = (x + \Delta x)^2$.
2) Let's find the increment of the function Δy:

$$\Delta y = (x + \Delta x)^2 - y = (x + \Delta x)^2 - x^2 = x^2 + 2x\Delta x + \Delta x^2 - x^2 = 2x\Delta x + (\Delta x)^2$$

3) Let's find the ratio $\frac{\Delta y}{\Delta x}$:

$$\frac{\Delta y}{\Delta x} = \frac{2x\Delta x + (\Delta x)^2}{\Delta x} = 2x + \Delta x.$$

4) Let's calculate $\frac{\Delta y}{\Delta x}$:

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x) = 2x.$$

This means $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ exists and therefore the derivative of the function $y = x^2$ will be: $y' = f'(x) = 2x$.

2. Geometric meaning of the derivative.

As we mentioned above, the concept of derivative is one of the most important concepts in mathematical analysis. Many theoretical and applied questions, being similar to each other in their content, come down to the same mathematical problem - finding the limit of the ratio of the increment of a function to the increment of the argument, provided that the latter tends to zero, i.e. to finding the derivative.

Let us consider the most important of these problems—the problems of the tangent to a curve and the speed of alternating motion.

Problem about the tangent to the graph of a curve

Consider the graph of the function $y = f(x)$, defined in the circle of the point x_0

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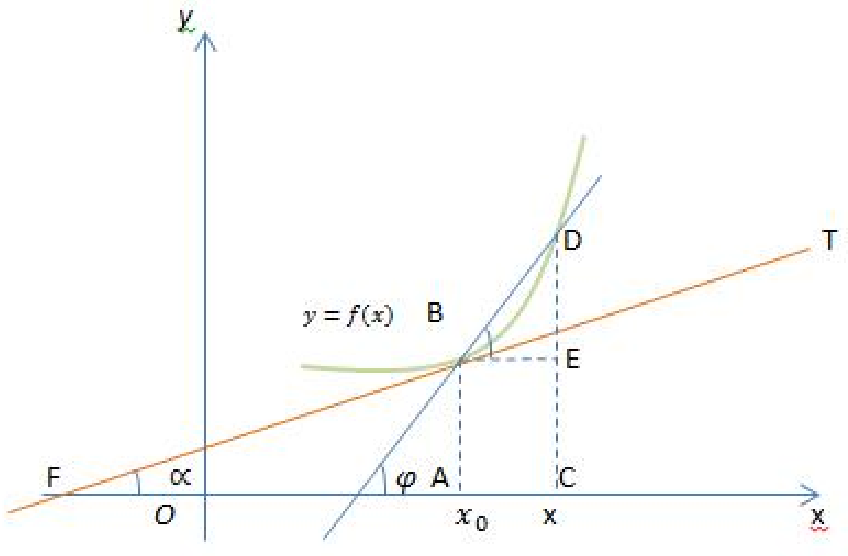

At point B we bring the tangent to the curve $y = f(x)$. It forms an angle α with the positive direction of the ox axis. On the curve $y = f(x)$ we take an arbitrary point D and give a secant BD. It forms an angle φ with the positive direction of the ox axis, i.e.

$$\angle BDE = \varphi \text{ и } \angle TFO = \alpha$$

A-priory

$f'(x_0) = \lim\limits_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$

From the drawing we can write: $OA = x_0, OC = x\ \ AC = OC - OA = x - x_0 = \Delta x.$

$$B = f(x_0), CD = f(x)\ \ DE = CD - CE = CD - AB = y - y_0 = f(x) - f(x_0) = \Delta y, BE = AC = \Delta x.$$

From $\frac{ED}{BD} = tg\varphi$, $\frac{\Delta y}{\Delta x} = tg\varphi$ then $\frac{\Delta y}{\Delta x} = \frac{f(x) - f(x_0)}{x - x_0}$ is the ratio of the legs of the right triangle BDE , i.e. tangent of the angle φ. If $x \to x_0$, then point D tends to point B along the curve, and straight line BD tends to take the position of the tangent BT to the curve at point B.

Next, we have

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{D \to B} tg\varphi = tg\alpha$$

So, the geometric meaning of the derivative is as follows:

The derivative of the function at the point x_0 is equal to the tangent of the angle formed by the tangent to the curve $y = f(x)$ at the point $[x_0,\ f(x_0)]$ and the positive direction of the ox axis.

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DISCUSSION. The most important functions found in economics

When solving various economic problems, the following functions are encountered.

1) **Linear function $y = ax + b$**

The graph of this function is a straight line and it is defined in the interval $(-\infty; \infty)$. Example: production costs at an industrial enterprise where homogeneous products are manufactured can be divided into two groups:

a) variable costs proportional to the volume of production, for example material costs.

b) fixed costs, i.e. those that mainly do not depend on the volume of production, for example, the costs of maintaining administrative buildings, their heating, etc. If fixed costs are denoted by b, and variable costs per unit of production are denoted by a, then with a production volume of x units, the total production costs in a given period will be $y = ax + b$.

If linear is expressed by the formula $y = ax$, then it is said that it determines direct proportionality between y and x.

2) Power function $f(x) = x^{\alpha}$.

a) If α is a natural number, then the interval of definition is $(-\infty; \infty)$.

b) If α is a negative number, then substituting $\alpha = -n$, we get: $f(x) = x^{\alpha} = x^{-n} = \frac{1}{x^n}$. Here the function is defined for all values of $x \neq 0$.

c) If α is a reverse natural number, then substituting $\alpha = \frac{1}{n}$, then $f(x) = x^{\alpha} = x^{\frac{1}{n}} = \sqrt[n]{x}$ is defined $(-\infty; \infty)$.

3) Exponential function $f(x) = a^x$.

If $a > 0$, defined on $(-\infty; \infty)$. If $0 < a < 1$– decreases, $a > 1$, increases.

Example: If 1 zloty is invested at compound interest at a rate of 5%, then its value after X years will be: $y = 1{,}05^x$. If $a = e$, then $y = e^x$or $y = \exp x$.

4) Logarithmic function $f(x) = \log_a x \ (a > 1)$.

It is defined only by $x > 0$. Logarithmic functions with base $0 < a < 1$ are not used in practice. If $a = e$, then $f(x) = \log_a x = \ln x$.

Example: The Italian economist Pareto formulated a theorem about the distribution of income in any society. If y denotes the number of persons with income not less than x, then

$y = \frac{a}{x^m}$, where a and m are constants.

For low incomes, Pareto's law does not apply. Let the distribution of income in some society be determined by the equation $y = \frac{2\,000\,000\,000}{x^{1,5}}$ $(a = 2\,000\,000\,000,\ m = 1{,}5)$.

Find:

a) the number of persons who have an income exceeding 100 000;

b) the lowest income among the 100 richest individuals.

Solution.

a) We have, $x = 100\,000$

$y = \frac{2\,000\,000\,000}{100\,000^{1,5}}$;

$$\lg y = \lg \frac{2\,000\,000\,000}{100\,000^{1,5}} = \lg 2\,000\,000\,000 - 1{,}5 \lg 100\,000 = 9{,}3010 - 1{,}5 \cdot 5 = 1{,}8010$$

$\lg y = 1{,}8010; y = 63{,}2.$

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Therefore, 63 people have an income exceeding 100,000.

b) We have, $100 = \frac{2\,000\,000\,000}{x^{1,5}}$; $x^{1,5} = 20\,000\,000;\ 1,5\lg x = \lg 20\,000\,000 = 7.3010$
$\lg x = 4,8673;\quad x = 73700.$

That. the lowest income among the 100 richest persons (that is, the income of the hundredth person, counting the richest) is 73,700.

Production costs for homogeneous products are a function of the quantity of production x. Therefore we can write:

$$K = k(x)$$

Let us assume that the quantity of production increases by Δx. Product x+Δx corresponds to production costs

$$K(x + \Delta x)$$

Consequently, the increment in the quantity of products Δx corresponds to the increment in production costs

$$\Delta K = K(x + \Delta x) - K(x)$$

The average increase in production costs is:

$$\frac{\Delta K}{\Delta x} = \frac{K(x + \Delta x) - K(x)}{\Delta x}$$

This is an increase in production costs per unit increase in the quantity of production. Limit

$$\lim_{\Delta x \to 0} \frac{\Delta K}{\Delta x} = K'(x)$$

called the marginal cost of production.

Similarly, if we denote by $U(x)$ the revenue from the sale of x units of goods, then the limit

$$\lim_{\Delta x \to 0} \frac{\Delta U(x)}{\Delta x} = U'(x)$$

we will call it marginal revenue.

Example 1. Production costs K depend on the volume of production x according to the formula:

$$K = 100x - \frac{1}{30}x^3$$

Determine marginal costs if the production volume is: a) 5 units; b) 10 units of production.

Determine marginal costs if the production volume is: a) 5 units; b) 10 units of production.

We have: $K' = 100 - \frac{1}{10}x^2$.

$K'(5) = 100 - \frac{1}{10} \cdot 25 = 97,5$

$K'(10) = 100 - \frac{1}{10} \cdot 100 = 90.$

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This means that with a production volume of 5 units of production, the cost of producing the next (sixth) unit of production is 97.5; with a production volume of 10 units they will be 90.

Elasticity of function. In many problems, it is more convenient to calculate the percentage increase (relative increment) of the dependent variable corresponding to the percentage increase of the independent variable. This brings us to the concept of elasticity of a function (sometimes called relative derivative).

If the function $y = f(x)$ is given, then

$$\lim_{\Delta x \to 0}\left(\frac{\Delta y}{y};\frac{\Delta x}{x}\right) = \lim_{\Delta x \to 0}\frac{x}{y}\cdot\frac{\Delta y}{\Delta x} = \frac{x}{y}\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = \frac{x}{y}f'(x) = \frac{x}{y}\cdot\frac{dy}{dx}$$

It is called the elasticity of the function $y = f(x)$ with respect to the variable x and denotes $E_x(y)$

$$E_x(y) = \frac{x}{y}\cdot\frac{dy}{dx}$$

Elasticity with respect to x is an approximate percentage increase in a function (increase or decrease), corresponding to an increase in the independent variable by 1%.

Elasticity of demand relative to price. The functional relationship between demand for a given product and its price (provided that the price of other goods, consumer income and the structure of needs are constant values) allows the price to be brought into line with demand, properly defined. However, in many economic studies it is necessary to determine not the quantity of demand, but the change in demand caused by a certain change in price, i.e. The elasticity of demand relative to price is determined.

Let us assume that demand q depends on price p:

$$q = f(p)$$

Let Δp be the price increment and Δq the corresponding demand increment.

$\frac{\Delta p}{p}$ – relative price change,

$\frac{\Delta q}{q}$ - relative change in demand.

The elasticity of demand relative to price is called the limit

$$E_p(q) = E_c = \lim_{\Delta p \to 0}\left(\frac{\Delta q}{q};\frac{\Delta p}{p}\right) = \frac{p}{q}\lim_{\Delta p \to 0}\left(\frac{\Delta q}{\Delta p}\right) = \frac{p}{q}\cdot\frac{dq}{dp}$$

$$E_c = \frac{p}{q}\cdot\frac{dq}{dp}$$

The price elasticity of demand approximately determines how the demand for a given good will change if its yen increases by 1%.

a) If E_$E_c > 1$, i.e. if an increase in price by 1% corresponds to a decrease in demand by more than 1%, they say that demand is elastic, and if $E_c = 1$, i.e. a decrease in demand by 1%, then demand is neutral.

b) If E_$E_c < 1$, then demand is inelastic.

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Because the demand function in most cases is a decreasing function, then $\frac{dq}{dp} < 0$ and therefore the elasticity of demand should be written as follows:

$$E_c = -\frac{p}{q} \cdot \frac{dq}{dp}$$

Example. If the demand function is $q = 10 - p$, then the elasticity of demand is:

$$E_c = -\frac{p}{q} \cdot \frac{dq}{dp} = -\frac{p}{10 - p}(-1) = \frac{p}{10 - p}$$

If, for example, $p = 2$, then $E_c = \frac{2}{10-2} = \frac{1}{4}$.

This means that at price 2, a 1% increase in price will cause a decrease in demand by $\frac{1}{4}$ %. Therefore, the elasticity of demand in this case is inelastic. It is also possible to show the elasticity of total and average costs.

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