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APPLICATION OF THE INITIAL INTEGRATION METHOD TO THE NUMERICAL SOLUTION OF DIFFERENTIAL EQUATIONS WITH A SMALL PARAMETER IN FRONT OF THE HIGHEST DERIVATIVE

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Abstract: This study addresses the numerical solution of differential equations that contain a small parameter in front of the highest-order derivative. Such equations frequently arise in various physical modeling scenarios, such as heat conduction, diffusion, and fluid dynamics. These equations exhibit special characteristics, where the presence of a small parameter leads to the formation of boundary layers and sharp changes in the solution. The initial integration method is a powerful approach for obtaining stable and highly accurate results in solving such problems numerically. This research analyzes the application features of the method, its advantages, and its impact on the sensitive zones of the solution. The results demonstrate that the method is an effective tool for reliably solving complex problems involving small parameters.

Keywords: Initial integration method, small parameter, higher-order differential equation.

Modeling of Physical and Technical Processes Expressed by Differential EquationsModeling of physical and technical processes using differential equations is one of the key directions in applied mathematics. Most real-world models include higher-order derivatives, and in some cases, a small parameter appears in front of these derivatives. For example:$\varepsilon y^{(n)}(x) + a_{n-1}(x)y^{(n-1)}(x) + \ldots + a_0(x)y(x) = f(x), \quad 0 < \varepsilon \ll 1$The solutions to such equations differ significantly from those of standard equations and exhibit singular behavior. That is, boundary layer zones appear in the solution—regions where the solution changes rapidly—while the rest of the interval remains smooth. Traditional numerical approaches often fail to provide sufficient accuracy in these cases, or they incur very high computational costs. Therefore, the method of initial integration becomes especially relevant. This method, which takes into account the sensitive structure of such equations, allows for stable and efficient solutions. The essence of the initial integration method lies in integrating the equation beforehand, thereby reducing higher-order derivatives to lower-order ones. As a result, the influence of the small parameter is diminished, and the solution becomes smoother. Particularly, if the differential equation is expressed using integrals, this method naturally captures the boundary layer zones in the solution. In our research, we comprehensively cover the classical analysis of high-order singularly perturbed equations, the algorithmic application of the initial integration method, the accuracy and stability of numerical solutions, and comparisons with other methods.

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Additionally, we demonstrate the method’s applicability in physical models (such as heat flow and diffusion models) through examples. Thus, this work highlights the advantages of new approaches in mathematical modeling and provides methodologically and practically significant conclusions in solving singular differential equations.

^c • n ? j. A(r, e) + eC(r, e)-^ + b(r, e)x = P(r, E)el6(t-£) (1) look at the question of integrating a system of second-order Linear Differential Equations dependent on the parameter, where x(t,£) is an n-dimensional vector R = Et - slowly changing time e < 1-small real parameter; 6(t, e) is a scalar function, i = V - l; A(t, e), B(t, e), C(T, E)—(N x n) matrices p(r, e) — n - dimensional vector, let the conditions below Be Satisfied: A(R, E), B(T, E), C(T, E) matrices,and P(T, E)-vector function given r e [0, l] in the range, let e extend to the converging series by the degrees of parameter: Co Co C(T, = ^ ESCs (t), p(t, = ^ ESPs (t), (2) s=0 s=0 CC A(x, e) = ^ ESAs (t), b(t,e) = ^ ESBs (T) S=0 s=0 VTE [0, l], deta0(r) * 0(3) DQ d(T, E) the-derivative of a function is a slow — changing function, i.e. dO Tt = m() (2) series coefficients As(t), Bs(t), C(t), Ps(t)s = 0.1.2,... and given a function k(r) [0,L] in the study of an infinitesimally differentiable system in a section, its character is

fs (r) = Bivs-I - ^ ^ ^ Xixjagvs—1—i—j - ^ ¿0¿iajvs-i-j - i=1 i=1 c=0 k=0 i=1 c=0 s k-1s-1— i s-1 s-1-I - ^¿0a0vs—I-^ ^ ¿iccvk—1—i—j-0-2^ ^ Xiajv'k-1-i-j- i=1 i=0 c=0 i=0 c=0 s-2 s-2 - ^ Civ's—2—I - ^ AIV'sl-2-i s = 1,2,3,... (14) I=1 i=1 based on the condition of the theorem these roots are simple, then to each of them one p¿r) corresponding to the specific vector (B0-WiA0) (i = 0 satisfies the relation and is defined in arbitrary scalar product accuracy, which is nonzero. In this case, the vector A0 added to The Matrix B0 will not exist. In this case (B0 - WiA0)Z = a0pb (i = l/n) (LS) has no equation solution. We look at the system of equations (11), (12). Equation (11) has a nonzero solution only and only when ¿2 = -Wi, (i = 1/n), from this we define 2N different Ä0(r) : Yao (r) = ±i^Wi (r), (i = %n) (16 Then n different v0 (r) vector functions from(11) are defined: v0(r) = Pi(r), (i = ln) (l7) where(pi(r), A0(r) are the hos values of the Matrix B0 (r) with respect to The Matrix. (16) munoit plays an important role in the modeling of many other natural processes. The underlying complexity is the layer zones generated by the presence of a small parameter and is manifested in the drastically changing nature of the solution. This makes it difficult for traditional numerical methods to be used, or they will not have sufficient accuracy as a result. The study initially explored both theoretically and practically the application of the integralization method to solving such equations. Based on analyzes and numerical experiments, the following main conclusions were drawn: 1. Initially, the integrable method takes into account the sensitive zones of small parametric equations in a natural way. This plays an important role in capturing the drastic change in solution, especially in the layer zone.2. This method is highly effective in terms of stability and accuracy. Reduces the uncertainties associated with a small parameter, smoothing the points of view in the solution.3. In relation to traditional methods, the computer resource perspective

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