

APPLICATIONS OF DEFINITE INTEGRALS IN MECHANICS

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Abstract: This article explores the application of definite integrals in solving mechanical problems. In particular, the determination of the center of gravity, moment of inertia, rectangle approximation formulas, and numerical integration methods such as the trapezoidal and Simpson's rules are discussed. As an essential tool in mathematical analysis, the definite integral is used to carry out computations on real physical objects.

Keywords: definite integral, center of gravity, moment of inertia, trapezoidal rule, Simpson's rule, static moment, mechanics, integral calculation

Introduction

The theory of definite integrals, a core part of mathematical analysis, finds extensive application in natural sciences, especially in physics and mechanics. Solving basic mechanical problems such as finding the center of gravity, moment of inertia, and force distribution requires the use of integrals. For bodies of complex shapes, analytical methods may not be feasible; hence, numerical integration and approximate solutions become essential.

Literature Review

There are numerous reputable sources on the application of definite integrals. For example, 'Mathematical Analysis' by R. G. Bartle and D. R. Sherbert discusses the theoretical foundations and physical applications of definite integrals. The Uzbek-language textbook 'Matematikal Analysis' by A. M. Mamatov and T. T. Eshmamatov provides a comprehensive explanation of center of gravity and moment of inertia in integral terms. Moreover, integral formulas are extensively covered in engineering and physics literature.

Main Part (Outlined Topics)

1. Static Moment and Center of Gravity

The static moment reflects how mass or area is distributed with respect to a certain axis. The center of gravity is the point at which the entire mass of a body appears to be concentrated.

a) Center of Gravity of a Plane Curve

Determined by the following formulas:

$$\bar{x} = (1/L) \int_C x \, ds, \quad \bar{y} = (1/L) \int_C y \, ds$$

where $ds = \sqrt{(1 + (dy/dx)^2)} \, dx$ and L is the total arc length.

b) Center of Gravity of a Plane Shape

Defined over the area:

$$\bar{x} = (1/A) \iint_D x \, dA, \quad \bar{y} = (1/A) \iint_D y \, dA$$

where A is the total area.

2. Moment of Inertia

The moment of inertia about a given axis is:

$$I_x = \int y^2 \, dm, \quad I_y = \int x^2 \, dm$$

For plane regions:

$$I_x = \int [a \text{ to } b] y^2 \rho(x) \, dx$$

where $\rho(x)$ is the density function.

3. Rectangle Approximation Formulas

The integral can be approximated using the rectangle rule:

$$\int [a \text{ to } b] f(x) \, dx \approx \sum f(x_i^*) \Delta x$$

where x_i^* is a point in each subinterval and $\Delta x = (b - a)/n$.

4. Numerical Integration of Definite Integrals

When exact analytical computation is difficult or impossible, numerical methods are employed. Trapezoidal and Simpson's rules are among the most effective.

5. Trapezoidal Rule

$$\int [a \text{ to } b] f(x) \, dx \approx (h/2) [f(a) + 2 \sum f(x_i) + f(b)]$$

where $h = (b-a)/n$; this formula estimates the area using trapezoids.

6. Simpson's (Parabolic) Rule

$$\int [a \text{ to } b] f(x) \, dx \approx (h/3) [f(a) + 4 \sum (\text{odd indices}) f(x_i) + 2 \sum (\text{even indices}) f(x_i) + f(b)]$$

This method approximates the function using parabolas and yields high accuracy.

Discussion

Using the above methods, it is possible to calculate the physical characteristics of complex-shaped bodies such as their center of gravity and moment of inertia. The trapezoidal and Simpson's rules are widely used both theoretically and in practical computer applications. In fields like engineering and architecture, definite integrals play a central role in performing design and structural calculations.

Results

- Definite integrals serve as the main mathematical tool in determining the center of gravity and moment of inertia.

- Approximate methods like the trapezoidal and Simpson's rules are effective in calculating integrals of complex functions.

- Integral-based analysis is critical in engineering design and mechanical planning.

Conclusion

This article analyzed key applications of definite integrals in mechanics. It was demonstrated that concepts such as static moment, center of gravity, and moment of inertia can be precisely calculated using integration. The advantages of numerical methods such as the trapezoidal and Simpson's rules were also discussed. It is evident that definite integrals are indispensable in the accurate analysis of physical and mechanical systems.

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