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GIBBS MEASURES FOR SOME MODELS DEFINED ON THE SECOND-ORDER CAYLEY TREE.

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Annotation: In this article, the existence and structural properties of Gibbs measures for certain spin models defined on the second-order Cayley tree are studied. The theoretical results obtained contribute to a deeper understanding of the phase structure of probabilistic models on Cayley trees and help to expand their physical interpretation.

Keywords: *Cayley tree, Gibbs measure, spin model, probabilistic model, phase structure, structural properties, physical interpretation, second-order model.*

Home part : Cayley trees occupy an important place in mathematical physics and probability theory, as they are infinite, cycle-free, and regular graphs. In particular, Cayley trees serve as convenient abstract models for analyzing spin systems Potts, and other probabilistic models. A second-order Cayley tree is a structure in which each vertex is connected to three neighboring vertices, allowing the recursive nature of interactions to be expressed clearly. In spin models, each vertex is assigned a specific spin value. These values are linked through interaction strength, external magnetic field, or an energy function. One of the main problems for such models is to determine the overall probability distribution of the system, known as the Gibbs measure. The Gibbs measure defines the probability over the entire configuration space based on all local interactions and is directly related to parameters such as temperature and external field. On the second-order Cayley tree, the determination of the Gibbs measure can be expressed through recursive relations. Since the state of each vertex depends on the states of its three neighbors, the structure of the measure itself has a recursive character. This allows the probabilities at each vertex to be expressed in terms of the conditional probabilities of neighboring vertices. As the temperature or interaction strength of the system changes, the structure of the Gibbs measure also changes. At low temperatures, stable spin states emerge in the system, leading to phase transition phenomena. During such transitions, multiple stable states — that is, multiple Gibbs measures may exist. At high temperatures, however, the influence of interactions decreases and the system possesses a single probability measure. The question of the existence and multiplicity of Gibbs measures on Cayley trees mathematically represents this phase transition phenomenon. On the second-order Cayley tree, this process becomes more delicate than on trees of other orders, since the interaction of each vertex with its three neighbors determines the system's symmetry and stability. Theoretical results concerning the existence of Gibbs measures are important not only from a mathematical standpoint but also from a physical one. These results help to understand the phase states of real physical systems, the minimization of their energy, and the conditions of stability. Models based on Cayley trees are considered effective tools for studying global phase behavior in systems with local interactions. The results obtained show that the existence of Gibbs measures for spin models on Cayley trees strongly depends on system parameters: depending on temperature, external field, and interaction strength, the system may have one or several phases. Thus, theoretical models constructed on the tree structure provide a simplified yet profound mathematical framework for analyzing the complex phase properties of physical systems.

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