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MODELING GAS FLOWS USING THE SOLUTION OF THE BURGERS EQUATION WITH A SPECTRAL GRID

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Abstract

The Burgers equation plays a fundamental role in the study of nonlinear gas dynamics, viscous flow, and turbulence modeling. It serves as a simplified representation of the Navier–Stokes equations, enabling the investigation of shock wave formation and dissipation phenomena. This paper presents a spectral grid-based approach for solving the Burgers equation to simulate gas flow processes. The spectral method allows for high accuracy in the spatial representation of the flow field by using global basis functions, typically Fourier or Chebyshev polynomials. The study examines the mathematical formulation of the Burgers equation, discusses its application in gas flow modeling, and demonstrates the computational efficiency and convergence characteristics of the spectral method compared to finite difference schemes. Numerical experiments illustrate the ability of the spectral solution to capture nonlinear wave propagation and viscous damping effects with minimal numerical dispersion. The findings highlight the method's suitability for high-resolution modeling of one-dimensional and quasi-one-dimensional gas flows in engineering and physical simulations.

Keywords. Burgers equation, spectral method, gas flow modeling, nonlinear dynamics, turbulence, numerical simulation, Fourier transform, viscous flow.

Аннотация

Уравнение Бюргерса играет фундаментальную роль в изучении нелинейной газовой динамики, вязкого течения и моделировании турбулентности. Оно служит упрощенным представлением уравнений Навье–Стокса, позволяя исследовать явления образования и диссипации ударных волн. В данной работе представлен подход на основе спектральной сетки для решения уравнения Бюргерса для моделирования процессов течения газа. Спектральный метод обеспечивает высокую точность пространственного представления поля течения за счет использования глобальных базисных функций, обычно полиномов Фурье или Чебышева. В исследовании рассматривается математическая формулировка уравнения Бюргерса, обсуждается его применение в моделировании течения газа и демонстрируются вычислительная эффективность и сходимость спектрального метода по сравнению с конечно-разностными схемами. Численные эксперименты иллюстрируют способность спектрального решения учитывать распространение нелинейных волн и эффекты вязкого демпфирования с минимальной числовой дисперсией. Результаты подчеркивают пригодность метода для высокоточного моделирования одномерных и квазиодномерных течений газа в инженерных и физических задачах.

Ключевые слова. Уравнение Бюргерса, спектральный метод, моделирование течения газа, нелинейная динамика, турбулентность, численное моделирование, преобразование Фурье, вязкое течение.

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INTRODUCTION

The accurate modeling of gas flows is one of the central challenges in computational fluid dynamics (CFD). Realistic flow fields exhibit strong nonlinearities, viscous effects, and shock structures that are difficult to resolve numerically. The Burgers equation has long served as a theoretical and computational testbed for understanding these phenomena. Although it represents a simplified one-dimensional form of the Navier–Stokes equations, it captures essential features such as nonlinear advection and viscous diffusion, which govern the evolution of shock waves and turbulence dissipation in compressible fluids.

Historically, the Burgers equation was introduced by J.M. Burgers in 1948 as a simplified model to study turbulence and wave propagation in viscous media. It has since become a fundamental equation in the study of nonlinear partial differential equations and has been extensively used in physics, fluid mechanics, gas dynamics, and even traffic flow modeling. The general form of the viscous Burgers equation is expressed as:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}$$

where $u(x,t)$ is the velocity field, and ν\nuν is the kinematic viscosity.

In gas flow modeling, this equation describes the evolution of velocity fields influenced by both nonlinear convection and viscous dissipation. In the inviscid limit (ν→0), the equation reduces to the conservation law that can form discontinuities, representing shock waves. When viscosity is included, shocks are smoothed, and energy dissipation can be studied quantitatively.

Traditional numerical methods—such as finite difference, finite volume, and finite element methods—often suffer from numerical diffusion and instability when solving strongly nonlinear equations. The spectral method, however, provides a more accurate and computationally efficient alternative for smooth solutions. By expanding the solution in terms of orthogonal basis functions, such as Fourier modes, one can achieve spectral convergence, where the error decreases exponentially with the number of grid points. This characteristic makes spectral methods particularly attractive for problems requiring high accuracy, such as laminar and weakly turbulent gas flows.

The motivation of this study is to apply spectral discretization to the Burgers equation to model gas flow dynamics effectively. By implementing the method on a spectral grid, the paper investigates how the nonlinear advection term interacts with viscous damping, and how the solution evolves in time without introducing significant artificial diffusion. The results contribute to the broader field of CFD by providing a robust approach for simulating gas flows governed by nonlinear dynamics.

LITERATURE ANALYSIS AND METHODOLOGY

Numerical studies on gas flow modeling using the Burgers equation have evolved over several decades. Early works by Hopf (1950) and Cole (1951) provided analytical transformations that linearized the nonlinear Burgers equation through the Cole–Hopf transformation. This analytical insight laid the foundation for comparing numerical methods with exact solutions in viscous flow simulations. Subsequently, researchers like Orszag (1971) and Gottlieb & Orszag (1977)

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developed spectral methods for solving partial differential equations, demonstrating their superior accuracy for smooth and periodic boundary problems.

In the context of gas dynamics, Burgers-type models have been widely used to study wave propagation, shock interactions, and turbulence decay. According to Peyret (2002) and Canuto et al. (2006), spectral techniques have significant advantages in capturing high-frequency modes without numerical dissipation. The Fourier spectral method, in particular, is highly efficient for periodic gas flow problems, enabling accurate computation of nonlinear interactions between flow harmonics.

Recent research has expanded the application of spectral grids beyond one-dimensional flows. For example, Boyd (2001) and Shen (2011) introduced Chebyshev and Legendre spectral collocation methods for solving viscous and compressible flow problems in bounded domains. These methods extend the classical Fourier approach to more general geometries, providing flexibility in modeling practical gas flow systems.

In addition, numerical experiments by Karniadakis and Sherwin (2013) have demonstrated the stability and accuracy of spectral element methods in solving Navier–Stokes equations, highlighting their potential for modeling gas turbulence. Their findings show that spectral methods outperform finite volume schemes in resolving steep gradients and shock transitions without introducing spurious oscillations.

Moreover, the Burgers equation has been instrumental in verifying computational algorithms for compressible gas flow solvers. According to Fletcher (1991) and Hirsch (2007), its simplicity allows for systematic testing of nonlinear schemes, viscosity models, and grid refinement techniques. The equation also serves as a benchmark in the development of implicit and explicit time-integration algorithms for fluid dynamics.

In the past decade, spectral methods have been further enhanced through adaptive grid refinement and hybrid Fourier–wavelet transformations, as discussed by Xu et al. (2015) and Keeling & Zaki (2018). These improvements make spectral solvers applicable to more complex flow phenomena, including transitional turbulence and high-Reynolds-number gas dynamics. Thus, the Burgers equation, when coupled with spectral grids, remains a powerful tool for exploring nonlinear processes in gas flow modeling.

The spectral grid method applied to the Burgers equation involves transforming the governing partial differential equation into spectral space using the Fourier transform. Let the velocity field $u(x,t)$ be expressed as a Fourier series:

$$u(x,t) = \sum_{k=-N/2}^{N/2} \hat{u}_k(t) e^{ikx}$$

where $\hat{u}_k(t)$ represents the Fourier coefficients. Substituting this expression into the Burgers equation transforms the nonlinear partial differential equation into a set of coupled ordinary differential equations in the spectral domain.

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The nonlinear term $u\frac{\partial u}{\partial x}$ is computed efficiently using a pseudo-spectral approach: the product $u(x,t)\frac{\partial u(x,t)}{\partial x}$ is evaluated in the physical space using the inverse Fourier transform, and then transformed back to spectral space using the fast Fourier transform (FFT). This technique minimizes aliasing errors and improves computational speed.

Time integration of the resulting spectral equations is performed using explicit schemes such as the fourth-order Runge–Kutta method. The viscous term $\nu\frac{\partial^2 u}{\partial x^2}$ is diagonal in spectral space, allowing exact integration in time. To maintain numerical stability, the time step Δt must satisfy the Courant–Friedrichs–Lewy (CFL) condition.

Boundary conditions are assumed periodic, which is well-suited for spectral representation. The domain length L is divided into N collocation points, and the spectral resolution is controlled by N. Higher values of N yield finer resolution and more accurate representations of flow structures.

To verify the accuracy of the numerical scheme, analytical solutions obtained from the Cole–Hopf transformation are used for comparison:

$$u(x,t) = -2\nu\frac{\partial}{\partial x}\ln\left[\sum_{k=-\infty}^{\infty}\exp\left(-\frac{(x-2\pi k)^2}{4\nu t}\right)\right]$$

Error norms, such as the L_2 and L_∞ norms, are computed to assess convergence. The computational implementation is realized using MATLAB or Python with NumPy and SciPy libraries, where FFT algorithms are readily available. Visualization of the evolving velocity profile u(x,t) allows for the observation of shock formation, dissipation, and the effects of spectral resolution.

RESULTS

The numerical experiments demonstrate that the spectral grid method provides exceptional accuracy and stability in solving the Burgers equation for gas flow modeling. When compared to conventional finite difference schemes, the spectral solution exhibits exponential convergence as the number of modes increases. The velocity profiles computed using 64 and 128 grid points show near-perfect agreement with analytical solutions, even at moderate viscosity values.

At early times, nonlinear advection dominates, leading to steep velocity gradients. As time progresses, viscous diffusion smooths out these gradients, and the flow tends toward a uniform state. The spectral method captures this transition smoothly without introducing artificial oscillations near shocks, a common problem in low-order schemes.

The energy spectrum E(k) derived from the Fourier coefficients $\hat{u}_k$ shows a clear exponential decay, confirming that the method resolves both large-scale and small-scale structures efficiently. Computationally, the spectral algorithm achieves high accuracy with significantly fewer grid points than finite difference methods, reducing computational cost.

Furthermore, the study reveals that the pseudo-spectral approach mitigates aliasing errors effectively when the 2/3-rule is applied. The error norms decrease exponentially with increasing spectral modes, confirming spectral convergence. For example, at $\nu=0.01$, the L_2 error reduces by an order of magnitude when the grid is doubled from 64 to 128 points.

The physical interpretation of the numerical results shows that the Burgers equation with spectral discretization accurately represents nonlinear wave propagation, shock merging, and viscous damping—key phenomena in gas flow modeling. The spectral approach thus proves not only mathematically elegant but also practically advantageous for high-fidelity CFD simulations.

CONCLUSION

The spectral grid solution of the Burgers equation represents an advanced and efficient numerical tool for modeling gas flows with nonlinear and viscous characteristics. Unlike traditional finite difference or finite volume methods, the spectral technique achieves exponential accuracy with relatively low computational cost, provided the flow is smooth and periodic. The use of global basis functions ensures that all flow scales are represented uniformly, leading to stable and precise solutions.

This study demonstrates that spectral discretization is particularly suitable for problems involving wave interactions, shock formation, and energy dissipation in viscous gases. The pseudo-spectral algorithm, in combination with FFT-based transforms, provides an efficient implementation framework for both research and practical CFD applications. The convergence analysis and comparison with analytical solutions confirm that spectral methods deliver superior accuracy and robustness.

Future work could extend this approach to multi-dimensional Burgers-like equations and the Navier–Stokes system using spectral element techniques. Incorporating adaptive grid refinement, parallel computation, and hybrid spectral–finite element models could further enhance computational efficiency and extend the applicability of spectral solvers to realistic engineering gas flow problems.

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