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GEOMETRIC REPRESENTATION OF COMPLEX NUMBERS, TRIGONOMETRIC REPRESENTATION, OPERATIONS, MOIVRE'S FORMULA AND EXTRACTION OF ROOTS

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Abstract: This work is devoted to the study of theoretical and practical aspects of complex numbers. The algebraic and trigonometric representation of complex numbers, their geometric representation and representation on the Argand plane are covered. The work considers the main operations performed on complex numbers - multiplication, division, exponentiation. The Moivre formula is analyzed as a basic tool for exponentiating complex numbers and calculating their roots. Extracting n-th roots from a complex number, their geometric distribution on the Argand plane and explaining them through practical examples. This research provides a deeper understanding of the mathematical properties of complex numbers and their application in various scientific and technical fields. Trigonometric representation simplifies operations, provides visual analysis and facilitates the determination of properties of complex numbers through modules and arguments.

Keywords: complex number, algebraic representation, trigonometric representation, geometric representation, Argand plane, module, argument, multiplication, division, exponentiation, Moivre formula.

Complex numbers are one of the most important branches of mathematical analysis and algebra. They are widely used not only in the field of theoretical mathematics, but also in many practical areas such as physics, engineering, mechanics, electronics, wave theory, signal processing, control systems. The algebraic, trigonometric and exponential representations of complex numbers make them convenient for solving various mathematical problems. This thesis discusses in detail the geometric representation of complex numbers, trigonometric representation, operations performed on complex numbers in trigonometric form, MOIVRE'S FORMULA, and the process of extracting roots from a complex number.

Complex numbers are usually written in the form $z = a + bi$. Here:

- a is the real part of the number,

- b is the abstract part of the number,

- i is the abstract unit, which satisfies the equation $i^2 = -1$.

Complex numbers also include real numbers. Because if $b = 0$, then $z = a$, which is a simple real number. If $a = 0$, then $z = bi$ is called a purely abstract number. Although the algebraic

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representation of complex numbers is convenient for simple arithmetic operations, other representations are required for performing some complex operations.[1]

A special coordinate plane is used to represent complex numbers geometrically. This plane is called the Argand plane. In the Argand plane, the horizontal axis is the axis of real numbers, and the vertical axis is the axis of abstract numbers.

If $z = a + bi$, then it corresponds to the point (a, b) on the plane. A complex number can also be viewed as a vector. This vector is directed from the real axis to the abstract axis.

The modulus of a complex number is:

$$|z| = \sqrt{(a^2 + b^2)}$$

This value represents the length of the vector representing the complex number.

The argument of a complex number is:

$$\arg(z) = \alpha = \arctan(b / a)$$

The argument α is the angle made by the vector with the real axis. The argument takes a fundamental value in the range from $-\pi$ to π.

Every complex number can be written in trigonometric form as follows:

$$z = r (\cos \alpha + i \sin \alpha)$$

Here:

- r is the modulus of the complex number,

- α is the argument of the complex number.

The trigonometric representation more accurately expresses the geometric meaning of the number. In it, r is the distance, and α is the direction. It is this representation that significantly simplifies the operations of multiplication and division on complex numbers.[2]

Multiplication

If $z_1 = r_1 (\cos \alpha_1 + i \sin \alpha_1)$ va $z_2 = r_2 (\cos \alpha_2 + i \sin \alpha_2)$ If so, their product is:

$$z_1 z_2 = r_1 r_2 [\cos(\alpha_1 + \alpha_2) + i \sin(\alpha_1 + \alpha_2)]$$

That is, the moduli are multiplied, and the arguments are added.

Division

$$z_1 / z_2 = (r_1 / r_2) [\cos(\alpha_1 - \alpha_2) + i \sin(\alpha_1 - \alpha_2)]$$

Here, modules are divided, arguments are subtracted.

Raise to a level

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$z^n = r^n [\cos(n\alpha) + i \sin(n\alpha)]$

This formula is based on Moivre's formula.

Moivre's formula is very important in raising complex numbers to powers and finding roots.

$(\cos \alpha + i \sin \alpha)^n = \cos(n\alpha) + i \sin(n\alpha)$

This formula is very convenient for simplifying trigonometric expressions.

The nth root of a complex number is found as follows:

If z = r (cos α + i sin α) bo‘lsa, uning n ta ildizi mavjud:

$z_k = r^{(1/n)} [\cos((\alpha + 2\pi k)/n) + i \sin((\alpha + 2\pi k)/n)]$,
$k = 0, 1, 2, ..., n–1$.

Therefore, complex numbers have not just one, but n roots, as in the case of real numbers. For example, the square roots of the number –1 are i and –i.

Example 1. Find the trigonometric representation of the number z = 3 + 3i:

$r = \sqrt{(3^2 + 3^2)} = \sqrt{18} = 3\sqrt{2}$

$\alpha = \arctan(3/3) = \pi/4$

So:

$z = 3\sqrt{2} (\cos \pi/4 + i \sin \pi/4)$

Example 2. z = 2(cos 30° + i sin 30°), w = 4(cos 45° + i sin 45°).

Multiplication:

zw = 8 (cos 75° + i sin 75°)

Example 3. Square roots of the number 1 + i:

$z = \sqrt{2} (\cos \pi/4 + i \sin \pi/4)$

$z_k = 2^{(1/4)} [\cos((\pi/4 + 2\pi k)/2) + i \sin((\pi/4 + 2\pi k)/2)], k = 0,1$.

In conclusion, Complex numbers play an important role in mathematics, and their trigonometric representation simplifies many operations. The process of raising them to a power and extracting roots is facilitated by the Moivre formula. Despite the complexity of this topic, its practical application is very wide and is necessary in many areas.

References:

1. X.P. Bo‘riyev, “Algebra va analiz asoslari”, Toshkent, 2020.

2. G.M. Fikhtengolts, “Matematik analiz”, Moskva, 1982.

3. E. Kreyszig, “Advanced Engineering Mathematics”, Wiley, 2011.

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4. L. Euler, “Trigonometric Foundations of Complex Numbers”, Berlin, 1783.