



THE IMPORTANCE OF STUDYING $y = \sin(ax^2)$ THE FUNCTION IN STRENGTHENING STUDENTS' KNOWLEDGE OF TRIGONOMETRIC FUNCTIONS

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Аннотация: В данной статье рассмотрены задачи определения свойств и построения графиков функции вида $y = f(ax^2 + bx + c)$, когда f была тригонометрической функцией. В качестве примера изучены свойства функций $y = \sin(ax^2 + bx + c)$, по этим свойствам сделаны научно-методические заключения. Для построения графики функций использованы современные

Ключевые слова: функция, сложная функция, квадратичная функция, тригонометрической функций, графики функции

Annotation: This article discusses the problem of determining the properties and graphing functions of the form $y = f(ax^2 + bx + c)$, when f there was a trigonometric function. As an example, the properties of functions a have been studied, and scientific and methodological conclusions have been made on these properties. To build the graphics functions used modern graphics programs GeoGebra and Maple.

Key words: Function, complex function, quadratic function, trigonometric functions, function graphs.

Funksiya tushunchasi matematikaning asosiy tushunchalaridan bo'lib, uning xossalari va grafigini chizish masalasi ko'p o'rganilgan. Ammo adabiyotlarda murakkab funksiya tushunchasi ta'riflari berilgani bilan argumenti kvadrat funksiya iborat murakkab funksiyalarni xossalari aniqlash va grafigini chizish masalalari keltirilmagan. $y = f(ax^2 + bx + c)$ ko'rinishdagi funksiyalarni f trigonometrik funksiya bo'lgandagi xossalari aniqlash va grafigini chizish masalasi muhimdir. Chunki ko'plab amaliy jarayonlarni ifodalovchi funksiyalar tarkibida trigonometrik funksiyalar ishtirok etadi. Xususan $f(x) = \sin x$ bo'lganda funksiya $y = \sin(ax^2 + bx + c)$ ko'rinishda bo'lib, ushbu funksiya a, b, c parametrlarning turli xildagi qiymatlarida turlicha ko'rinish va xossalarga ega bo'ladi. Ushbu parametrlarning turli qiymatlarida yuzaga keladigan hollarni o'rganish najasida bir qancha ilmiy-uslubiy xulosalar chiqarish mumkin. Biz quyida maqola hajmini inobatga olgan holda $a \neq 0, b = 0, c = 0$ holni o'rganish bilan chegaralanamiz. Ushbu turdagi funksiyalarning grafiklarini zamonaviy grafik chizuvchi dasturlar GeoGebra hamda Maple dasturlari yordamida chizish mumkin.

Bizga $y = \sin(ax^2)$ ($a \neq 0$) funksiya berilgan bo'lsin.

I. $f(x) = \sin(ax^2)$ funksiyaning aniqlanish sohasi va qiymatlar sohasi mos ravishda $D(x) = (-\infty; \infty)$, $E(y) = (-1; 1)$ ga teng;

II. $f(x) = \sin(ax^2)$ funksiya uzluksiz.

III. Funksiyaning juft, toqligi hamda davriyligini aniqlash:

a) $f(x) = \sin(ax^2)$ funksiya juft funksiya, ya'ni:

$$f(-x) = \sin a(-x)^2 = \sin ax^2 = f(x);$$

b) davriy funksiya emas. Haqiqatdan,

$$f(x) = \sin(a(x+T)^2) = \sin(ax^2 + 2axT + aT^2)$$

$f(x) = f(x+T)$ tenglik bajarilishi uchun $aT^2 + 2axT = 2\pi k, k \in \mathbb{Z}$ bo'lishi kerak. Endi $aT^2 + 2axT = 2\pi k, k \in \mathbb{Z}$ kvadrat tenglamani yechsak

$$aT^2 + 2axT - 2\pi k = 0, k \in \mathbb{Z} \quad T_{1,2} = \frac{-2ax \pm \sqrt{4a^2x^2 + 8a\pi k}}{2a}$$

yechimlarga ega bo'lamiz. Bu yechimlar esa funksiya davri bo'la olmaydi, chunki noma'lum x ga bog'liq bo'lib qoldi.

IV. Funksiya grafigining koordinata o'qlari bilan kesishish nuqtalarini topish:

Oy o'qi bilan: $x = 0$ da $f(x) = 0$;

Ox o'qi bilan $f(x) = 0$ bo'lganda:

a) $a > 0$ bo'lganda $y(x) = \sin ax^2 = \sin |a|x^2$

b) $a < 0$ bo'lganda $y(x) = \sin ax^2 = -\sin |a|x^2$ ekanligidan

$$y(x) = \sin |a|x^2 = 0, \quad |a|x^2 = \pi k,$$

$$x_{1,2} = \pm \sqrt{\frac{\pi k}{|a|}}, \quad k \in \mathbb{Z}^+ \setminus \{0\}.$$

Shunday qilib, funksiya grafigi koordinata o'qlari bilan $M \left(\sqrt{\frac{\pi k}{|a|}}, 0 \right)$ va $N \left(-\sqrt{\frac{\pi k}{|a|}}, 0 \right)$, $k \in \mathbb{Z}^+ \setminus \{0\}$

nuqtalarda kesishadi.

V. Funksiyaning ishorasi saqlanadigan oraliqlarni aniqlaymiz, aniqlanish sohasini nuqtalar yordamida funksiya nolga teng bo'ladigan intervallarga ajratamiz. Bu intervallarning har birida funksiyaning ishorasini tekshiramiz:

Malumki, $y = \sin x$ funksiyaning qiymatlari $x \in (2\pi k, \pi + 2\pi k)$ oraliqda musbat, grafigi esa Ox o'qi ustida joylashadi; $x \in (\pi + 2\pi k, 2\pi + 2\pi k)$ oraliqda esa funksiya qiymatlari manfiy grafigi Ox o'qi ostida joylashadi. Ushbu xossalarni qo'llab, quyidagilarga ega bo'lamiz:

a) $a > 0$ bo'lganda:

$$1. \quad 2\pi k < ax^2 < \pi + 2\pi k, \quad \frac{2\pi k}{a} < x^2 < \frac{\pi + 2\pi k}{a}, \quad k \in \mathbb{Z}$$

Bu tengsizlikda intervallar usulidan foydalanib, yechimlarni topamiz:

$$x \in \left[-\sqrt{\frac{\pi + 2\pi k}{a}}, -\sqrt{\frac{2\pi k}{a}} \right] \cup \left[\sqrt{\frac{2\pi k}{a}}, \sqrt{\frac{\pi + 2\pi k}{a}} \right], \quad k \in \mathbb{Z}^+ \setminus \{0\}$$

oraliqda funksiya qiymati musbat, grafigi Ox o'qi ustida joylashadi;

$$2. \quad \pi + 2\pi k < ax^2 < 2\pi + 2\pi k, \quad \frac{\pi + 2\pi k}{a} < x^2 < \frac{2\pi + 2\pi k}{a}, \quad k \in \mathbb{Z}$$

Bu tengsizlikda ham intervallar usulidan foydalanib, yechimlarni topamiz:

$$x \in \left[-\sqrt{\frac{2\pi + 2\pi k}{a}}, -\sqrt{\frac{\pi + 2\pi k}{a}} \right] \cup \left[\sqrt{\frac{\pi + 2\pi k}{a}}, \sqrt{\frac{2\pi + 2\pi k}{a}} \right], \quad k \in \mathbb{Z}^+ \setminus \{0\}$$

oraliqda funksiya qiymati manfiy, grafigi Ox o'qi ostida joylashadi.

b) $a < 0$ bo'lganda esa ushbu $\sin(-x) = -\sin(x)$ xossaga ko'ra ushbu holdagi funksiya grafigi yuqorida ko'rilgan funksiya grafigiga Ox o'qiga nisbatan simmetrik bo'lib, sohaga bog'liq xossalar teskarisiga almashadi.

VI. Funksiyaning monotonlik oraliqlarini topish va ekstremumga tekshirish

$f(x) = \sin(ax^2)$ funksiya hosilasini olib nolga tenglab ishlasak:

a) $a > 0$ bo'lganda

$$f' = (\sin ax^2)' = a \cdot 2x \cos ax^2 = 0$$

$$x_1 = 0, \quad ax^2 = \frac{\pi}{2} + \pi k \quad x_{2,3} = \pm \sqrt{\frac{\pi}{2a} + \frac{\pi k}{a}} = \pm \sqrt{\frac{\pi + 2\pi k}{2a}} = \pm \sqrt{\frac{(2k+1)\pi}{2a}} \quad k \in \mathbb{Z}^+ \setminus \{0\}$$

bo'lib, bunda x_1, x_2, x_3 nuqtalar $f(x)$ funksiyaning statsionar nuqtalari.

$x_1 = 0$ nuqtada funksiya lokal minimumga erishadi. Agar,

$k -$ juft son bo'lsa $x_{2,3}$ nuqta $f(x)$ funksiyaning maksimum nuqtasi bo'ladi;

$k -$ toq son bo'lsa $x_{2,3}$ nuqta $f(x)$ funksiyaning minimum nuqtasi bo'ladi.

$$x \in \left[-\sqrt{\frac{(4k+3)\pi}{2a}}; -\sqrt{\frac{(4k+1)\pi}{2a}}\right] \cup \left[0; \sqrt{\frac{\pi}{2a}}\right] \cup \left[\sqrt{\frac{(4n-1)\pi}{2a}}; \sqrt{\frac{(4n+1)\pi}{2a}}\right] \quad n \in \mathbb{Z}^+, k \in \mathbb{Z}^+ \setminus \{0\}$$

Oraliqda $f(x) > 0$ bundan funksiya grafigi o'suvchi;

$$x \in \left[-\sqrt{\frac{(4n+1)\pi}{2a}}; -\sqrt{\frac{(4n-1)\pi}{2a}}\right] \cup \left[-\sqrt{\frac{\pi}{2a}}; 0\right] \cup \left[\sqrt{\frac{(4k+1)\pi}{2a}}; \sqrt{\frac{(4k+3)\pi}{2a}}\right] \quad n \in \mathbb{Z}^+, k \in \mathbb{Z}^+ \setminus \{0\}$$

Oraliqda $f(x) < 0$ bundan funksiya grafigi kamayuvchi;

b) $a < 0$ bo'lganda

$$f' = (\sin ax^2)' = a \cdot 2x \cos ax^2 = 0$$

$$x_1 = 0, \quad ax^2 = \frac{\pi}{2} + \pi k \quad x_{2,3} = \pm \sqrt{\frac{\pi}{2|a|} + \frac{\pi k}{|a|}} = \pm \sqrt{\frac{\pi + 2\pi k}{2|a|}} \quad k \in \mathbb{Z}^+ \setminus \{0\}$$

bo'lib, bunda x_1, x_2, x_3 nuqtalar $f(x)$ funksiyaning kritik nuqtalari. Agar,

$k -$ juft son bo'lsa $x_{2,3}$ nuqta $f(x)$ funksiyaning minimum nuqtasi bo'ladi;

$k -$ toq son bo'lsa $x_{2,3}$ nuqta $f(x)$ funksiyaning maksimum nuqtasi bo'ladi.

$$x \in \left[-\sqrt{\frac{(4k+3)\pi}{2a}}; -\sqrt{\frac{(4k+1)\pi}{2a}}\right] \cup \left[0; \sqrt{\frac{\pi}{2a}}\right] \cup \left[\sqrt{\frac{(4n-1)\pi}{2a}}; \sqrt{\frac{(4n+1)\pi}{2a}}\right] \quad n \in \mathbb{Z}^+, k \in \mathbb{Z}^+ \setminus \{0\}$$

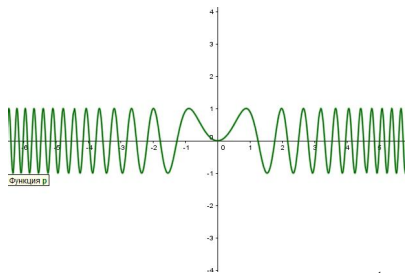
oraliqda $f(x) < 0$ bundan funksiya grafigi kamayuvchi;

$$x \in \left[-\sqrt{\frac{(4n+1)\pi}{2a}}; -\sqrt{\frac{(4n-1)\pi}{2a}}\right] \cup \left[-\sqrt{\frac{\pi}{2a}}; 0\right] \cup \left[\sqrt{\frac{(4k+1)\pi}{2a}}; \sqrt{\frac{(4k+3)\pi}{2a}}\right] \quad n \in \mathbb{Z}^+, k \in \mathbb{Z}^+ \setminus \{0\}$$

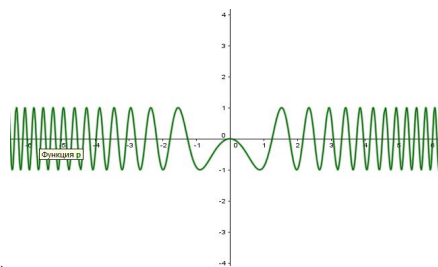
oraliqda $f(x) > 0$ bundan funksiya grafigi o'suvchi;

VII. $f(x) = \sin(ax^2)$ funksiya grafigi asimptotalarga ega emas. Funksiya grafigining simmetriya o'qi esa $x=0$ chiziq ya'ni Oy o'qi bo'ladi chunki u juft funksiya.

VIII. Funksiya grafigini chizish.



1- chizma ($f(x) = \sin(2x^2)$)



2 - chizma ($f(x) = \sin(-2x^2)$)

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