

# Neural Network training using FFA and its variants for Channel Equalization

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**Abstract:** This Work is on the Artificial Neural Network (ANN) training and application in channel equalization. Here, we design a novel training strategy for neural networks. This training uses Firefly Algorithm (FFA) and its variants to train ANN. Then, these FFA trained ANNs are applied for equalization of nonlinear channels. As proved through simulations, the proposed methodology outperforms the existing ANN based equalization schemes.

**Keywords:** Equalization; Firefly Algorithm, Neural Network

## I. Introduction

### 1.1 A survey on channel Equalization

The study on the equalization of channel started during 1960's but was centered on the zero forcing equalizers, basic theories and structures. Use of adaptive filters in equalization started with development of the LMS algorithm [8]. However, Lucky [9] was the first person to design adaptive channel equalizers using LMS algorithm in 1965. Very soon, adaptive linear filters become popular in the application in channel equalization and efforts were made to eliminate the limitations of LMS and adaptive filters. However, for highly dispersive channels, even best trained linear equalizers fails to provide acceptable performance. This paves the way to the research in other techniques for equalization. In 1970's, the development of the MLSE equalizer [10] and also its viterbi implementation [11] was added to the literature. The IIR form of the linear adaptive equalizer also developed during the same time. Then, the equalizer started employing feedback [12] and was termed as DFE. Then, the adaptive equalizers of PAM systems were extended to other complex signaling systems as in [13]. Fast convergence and/or computational efficient algorithms also used in equalization during 1970's and 1980's. The Kalman filters [14], recursive least square (RLS) algorithm, fractionally spaced equalizers (FSE) [16] and RLS lattice algorithm [15] are some examples. A detailed review of equalizers up to 1985 can be found in [17]. In the late 1980's there was a beginning for use of artificial neural network (ANN) in equalization [18], discussed separately under intelligent equalizers.

In the literature, equalizers are classified into two categories, supervised and blind. However, this thesis discussed a third category "Intelligent equalizers" separately. Use of soft and evolutionary computing in equalization is put into this category.

### Supervised equalization

During the transmission, the channel distorts the transmitted signal. This distortions can be eliminated using a *training signal*, also known as *pilot signal*, transmitted periodically along with the information transmission. The receiver uses a replica of the training signal that can be made available at the receiver to update its parameters. The corresponding equalizers are termed as supervised equalizers.

There are two kinds of supervised equalization:

- (1) *symbol-by-symbol estimation* (also known as finite memory equalizers): detect the transmitted symbol using a fixed number of input samples. The MAP criterion based on Bayes's theory [19] provides a decision function that is optimum for these equalizers, and hence also termed as Bayesian equalizers [20]. and
- (2) *sequence estimation* (also known as infinite memory equalizers and MLSE [10]): detect the transmitted symbol using the past-received samples sequence and were implemented with the use of Viterbi Algorithm [11].

Bayesian equalizer with infinite memory may provide a better performance as compared to MLSE; however its large computational complexity limits this use. Hence, Bayesian equalizer with finite memory is used to provide performance that is comparable with that of the MLSE and also with reduced computational complexity [21]. Further advances on Bayesian equalizers can be found in [22].

### Intelligent Equalizers

Traditional equalizers have been taken over by the neural network based equalizers. NN based equalizers can provide significant improvement in performance for a large number of channels.

A variety of real-valued NN based adaptive equalizers can be found in the literature on equalization [23]. These equalizers using various kinds of ANN structures like RBFNNs, multi-layer perceptrons and modular networks, successfully equalize the nonlinear channels and outperform traditional linear equalizers. This can be proved from following examples, in [24] Chen et al. proved that MLP based equalizers can generate separation curves those are complex and also nonlinear and hence can equalize channels with high degree of nonlinearity. Authors in [25] present a programmable VLSI ANN processor for equalization that is very powerful and can be implemented through a chip configured as a four-layer perceptron. Research in [26] Introduces a functional-link ANN based decision DFE to overcome ISI, CCI and additive noise. The said structure proved to provide superior performance in terms of BER as compared to the conventional DFE, RBF equalizers, linear transversal equalizer (LTE) and MLP equalizers.

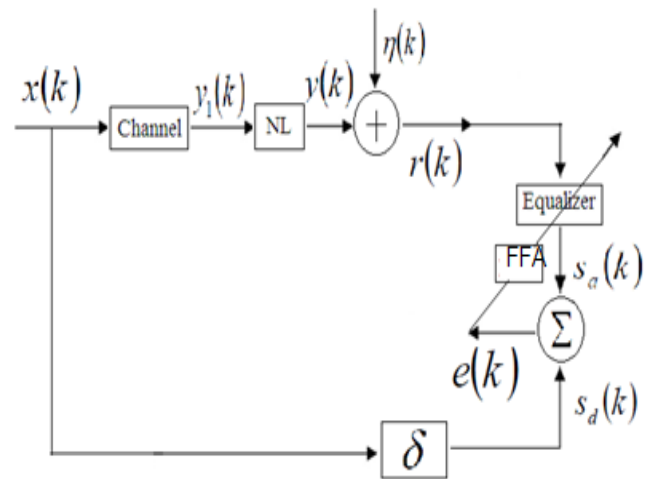
An analytical study on the performance for MLP-based receivers was proposed by De viciana and Zakhor in [27].

Gram-Schmidt orthogonal decomposition idea generated application of lattice polynomial perceptron (LPP) to equalization of 64-QAM channels, frequency-selective slow fading channel with ACI in [28] and found to outperform conventional equalizers like DFE.

The performance of cellular neural network [29] has been proposed for MLSE of signals in the presence of noise and ISI and applied with improved performance for equalization in [30]. They have addressed the hardware structure, model of the network and neuron in terms of BER performance, and found to be very efficient in realizing the MLSE receiver. Other kinds of hardware realization issues can be found from [31].

High degree of nonlinear dynamic characteristics of RNN [32] showing a rich and complex dynamical behavior [33] found application in channel equalization. The RTRL algorithm [34] was extended to the complex plane by Kechriotis et al. [35]. In equalization complex RTRL equalizer and linear TDL equalizers shows comparable performance for linear channels, but it outperforms for the channels with transfer function having spectral nulls or having severe nonlinear distortions. Also, RNNs outperform MLP equalizers for linear and nonlinear channels. The RTRL algorithm has been also applied in blind equalization, and shown to perform better than the CMA in all the channels.

As an alternative to gradient-based learning algorithms and also providing higher convergence speed than gradient-based methods, one training approach for RNN proposed in [36] based on the principle of discriminative learning [37] that minimizes an error function which is a direct measure of the error in classification.



**Figure 1.** Baseband Model of Digital Communication System

They used LS methods (most common in the signal processing applications) to fully RNNs and found to perform better than the RTRL algorithm in equalization.

A general ANN structure parameterizes the received signal to find conditional probability distribution function (PDF) of the transmitted proposed by Adali et al. in [38]. The PDF is estimated by minimizing the accumulated relative entropy (LRE) of the cost function. LRE equalizer provides high complex decision boundaries, and abrupt changes can be tracked in a nonlinear channel response where the MSE-based MLP fails.

The self-organizing map (SOM) has been connected either in cascade or in parallel with conventional equalizers such as DFE and LTE [39]. The adaptive decision was defined by  $m_i$  vectors of the SOM. Given that  $y(n)$  is the output of the DFE, then the error,  $e(n) = y(n) - m_i(n)$ , controls the adaptation behavior of the DFE. Hence, DFE compensates the dynamic linear distortions. But, SOM adaptively compensates for the nonlinear distortions. While applied to a nonlinear two-path channel, It was seen that the SOM-based equalizer outperforms conventional equalizers for different types of non-linearity and different levels of SNR.

ANNs have been applied in equalization of satellite UMTS channels in NEWTEST ACTS European Project [40]. A variety of NN structures and combinations of them has been applied in the project for the real-time trials. They have revealed that ANN approaches outperform classical equalizers for complicated modulation schemes like M-QAM modulations,  $M > 4$ ) are used [62].

In bulky signal processing system, a requirement of easy integration is always there. Because ANNs are more suitable for the requirement, there is abundant number of applications in nonlinear channel equalization. Since ANNs have resemblance with other schemes like coding and modulation techniques, signal processing, etc., ANNs found this multiple scale of applications. Following are some of very interesting finding in the literature.

A RBFNN based blind equalization scheme proposed in [41] makes use of simplex genetic algorithm (GA). A hybrid of GA and simulated annealing (SA) has been used in [64] for equalization. In both of these works, channel states were estimated using Bayesian likelihood cost function. However in

two models proposed in [42], CMA cost function has been used for blind equalization using complex-valued feed forward NNs avoiding the necessity of external phase correction. A DFE model classifier using Jordan NN along with delay estimation was proposed in [43].

In [44], A three-layer ANN was used for equalization. gradient algorithm was used for weight up-dation in the second layer followed by Kalman filter to estimate channel coefficients in the third layer. This ANN has improved estimation accuracy and the speed of convergence.

## 1.2 Motivation

Though ANNs perform well in equalization, but however associated with some of disadvantages like::

- The ANN structure becomes bulky because they are in no way related to the problem of equalization.
- High degree of non-linearity in ANN structure makes it difficult for performance analysis and comparison among parameters for adaptation. And also trial and error method is only available to select the parameters for training.
- There is no standard relation between the MLP and the optimal Bayesian equalizer.
- ANN equalizer does not guarantee to converge since it starts with random weights during training.
- The popular BP algorithm takes longer time to train ANN,
- The MLP associated with very large computational complexity.

Artificial neural network (ANN) for channel equalization has been used since long [45-48]. The performance of ANNs for non-linear problems makes them a popular choice in equalization. As discussed above, Back Propagation (BP) trained ANNs (1) fall in local minima (2) slow speed of convergence that depend on selection of parameters like momentum, learning rate and weights. Hence, evolutionary algorithms like Genetic Algorithm (GA) [49-51], Differential Evolution (DE) [57] and Particle Swarm Optimization (PSO) [52-54] used for ANN equalizer training [55-57].

Poor local search and the premature convergence of GA [58], fall into local minima and limited search space of PSO [59] and .sensitivity to the choice of control parameters of DE [60, 61] are still remain as limitations of these algorithms. On the other hand, Firefly Algorithm (FFA) [62] based on social behavior of fireflies is an attractive alternative for the purpose. In first part of this work, we make use of FFA to train ANN based equalizer. Then we extend the work used recently developed variants of FFA for a comparison with original version of FFA..

## II. The Problem

Figure 1 depicts a popular digital communication system.

A popular linear channel model is FIR model. In this model, transmitted sequence is binary, represented as  $x(k)$  at  $k^{th}$  time instance and corresponding output at the same instant is,  $y_1(k)$  as:

$$y_1(k) = \sum_{i=0}^{N-1} h_i x(k-i) \quad (1)$$

Here,  $h_i (i = 0, 1, \dots, N-1)$  and  $N$  respectively are the channel taps and  $N$  is the channel length. The block ‘NL’ denotes the nonlinearity inserted in the channel. One popular form of non-linear function is:

$$y(k) = F(y_1(k)) = y_1(k) + b[y_1(k)]^3 \quad (2)$$

Here,  $b$  is a constant. The block ‘NL’ output is:

$$y(k) = \left( \sum_{i=0}^{N-1} h_i x(k-i) \right) + b \left( \sum_{i=0}^{N-1} h_i x(k-i) \right)^3 \quad (3)$$

The channel output  $y(k)$  is added with noise,  $\eta(k)$  inserted in channel. The signal at the receiver is  $r(k)$  and as follows:

$$r(k) = y(k) + \eta(k) \quad (4)$$

Equalizer is used to recover the transmitted symbol,  $x(k-\delta)$ , from a-priory knowledge on the samples received, ‘ $\delta$ ’ being the associated transmission delay.

The desired signal can be represented as  $d(k)$  and can be defined as

$$d(k) = x(k-\delta) \quad (5)$$

The problem of equalization is a problem of classification [6-9], and the equalizer makes a partition the input space  $x(k) = [x(k), x(k-1), \dots, x(k-N+1)]^T$  into two distinct regions.

The Bays theory provides the optimal solution for this where the decision function is:

$$f_{bay}(x(k)) = \sum_{j=1}^n \beta_j \exp\left(\frac{-\|x(k) - c_j\|}{2\sigma^2}\right) \quad (6)$$

Since the sequence transmitted is binary, hence:

$$\beta_j = \begin{cases} +1 & c_j \in C_d^{(+1)} \\ -1 & c_j \in C_d^{(-1)} \end{cases} \quad (7)$$

Here,  $C_d^{(+1)} / C_d^{(-1)}$  and  $c_j$  respectively represent transmitted symbol,  $x(k-\delta) = +1/-1$  and  $\sigma^2$  is the noise variance.

In figure 1, the block ‘Equalizer’ is ANN. FFA and its modified forms are used to optimize the number of layers and neurons in each layer. For number of neurons in the input layer, it is N, same as number of taps.

The equalizer output is:

$$f_{RBF}(x(k)) = \sum_{j=1}^n w_j \exp\left(\frac{-\|x(k) - t_j\|^2}{\alpha_j}\right) \quad (8)$$

Here,  $t_j$  and  $\alpha_j$  respectively represent the centers and the spreads of the neurons in hidden layer (s). The vector  $w_j$  contains the connecting weights. The output from the equalizer of equation (6) uses the nonlinear function of equation (7), For optimal weights, the condition is  $t_j$  is equals to  $c_j$ .

The decision at equalizer output is:

$$\hat{x}(k - \delta) = \begin{cases} +1 & f_{ANN}(x(k)) \geq 0 \\ -1 & elsewhere \end{cases} \quad (9)$$

Also, the difference between the equalizer output (i.e,  $s_a(k) = \hat{x}(k - \delta)$ ) and desired signal

(i.e,  $s_d(k) = x(k - \delta)$ ) is termed as error,  $e(k)$ , and updates the weights.

For  $l$  is the number of samples then, Mean Square Error (MSE):

$$MSE = \frac{1}{l} E[e^2(k)] \quad (10)$$

Bit error rate (BER) is the ratio of error bits to transmitted bits: In this paper, MSE & BER are chosen as performance index.

### III. FFA, Modified forms and ANN Training

#### A. FFA.

FFA [62] is a population based optimization algorithm that mimics firefly. Firefly is known to be unisex and attracted towards each other according to intensity of lights they produce. Here, the population of FFA is formed by the solution vectors. The parameters of FFA are the weights ( $w$ ), spread parameters ( $\alpha$ ), center vector ( $c$ ) and the bias ( $\beta$ ).

The mean vector  $c_i$  of the  $i^{\text{th}}$  neuron of hidden layers is represented by  $c_i = (c_{i1}, c_{i2}, \dots, c_{im})$ , hence, the parametric vector  $t_i$  of each firefly with  $IJ + I + MI + J$  parameter is:

$$t_i = \begin{pmatrix} w_{11}^i, w_{12}^i, \dots, w_{IJ}^i, \alpha_1^i, \alpha_2^i, \dots, \alpha_I^i, c_{11}^i, c_{12}^i, \\ \dots, c_{1m}^i, \dots, c_{I1}^i, c_{I2}^i, \dots, c_{Im}^i, \beta_1^i, \dots, \beta_J^i \end{pmatrix} \quad (11)$$

In this work, a firefly represents a specific ANN equalizer. In the training process using FFA, the vector  $t_i$  of firefly of corresponding ANN optimizes the fitness function [18]:

$$f(t_i) = \frac{1}{1 + MSE} = \frac{1}{1 + \frac{1}{Q} \sum_{k=1}^Q \|d(k) - y(k)\|^2} \quad (12)$$

Here,  $d(k)$  and  $y(k)$  are the desired and actual output for training samples  $x_i$  of ANN designed by vector  $t_i$  and that of equalizer. The number of samples used in the training is represented by  $Q$ .

#### B. Variants of FFA

In FFA, each of the fireflies is attracted all brighter fireflies of the population that results in oscillations during the search and leading to higher complexity. To overcome this problem, a variant of FFA, termed as NaFA, where a firefly attracted by

brighter firefly only in the neighborhood of the concerned firefly is proposed and validated through benchmark functions in [63]. A hybrid algorithm consisting FFA and Recursive Least Square (RLS) algorithm, termed here as FFARLS, also proposed in [64]. In another modified form of FFA, ODFFA [65], initialization has been done using opposition based learning and position is updated using dimensional based approach. A nonlinear time-varying step strategy for firefly algorithm (NTSFA) also proposed and validated in [66].

In this paper, we first use original version of FFA to train ANN based equalizer and compared with other ANN based schemes. The we use these modified versions of FFA in place of FFA for the same purpose. In all the cases, training method followed is the same and outlined in the next section.

#### C. Training

In the proposed training method, ANN formulates guidelines for optimal structure as an administrator. Then FFA plays the role of the teacher to optimize the structure. This teacher once again teaches the ANN that plays the role of student. In this way, ANN acts both as an administrator and a student. The methodology follows following flow chart:

```

Initialize ANN_administrator
for  $i = 1, 2, \dots, N$ 
    generate FFA_teacher (i)
    for ANN_student  $j = 1, 2, \dots, N$ 
        start ANN_student
        end
    end
when solution is not found
    calculate update
    set maximum number of iterations
    for (FFA_teacher  $i = 1, 2, \dots, N$ )
        while (iterations < maximum)
            for(ANN_student  $j = 1, 2, \dots, N$ )
                test ANN_student (j)
            end
            for ANN_student  $j = 1, 2, \dots, M$ 
                Update weights of ANN_student (j)
            end
        end
    end
return global best
end
update global best
end

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### IV. Simulations

For comparison, original GA, PSO and FFA are considered. Simulation parameters considered are as given in table 1. Here, parameters used for FFA are same as used in [62].

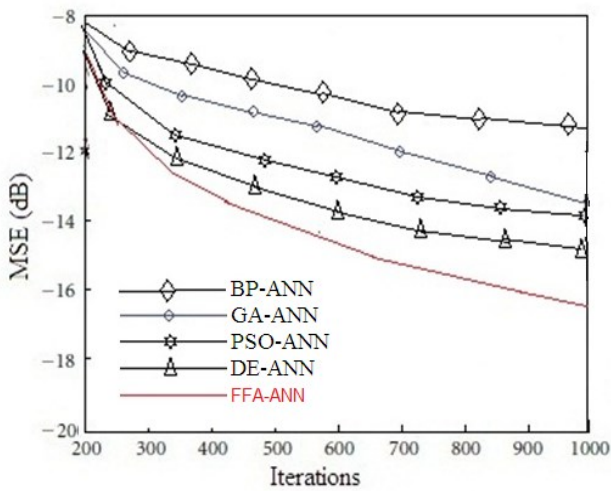


Fig. 2. MSE performance for channel (13)

For simulations, we have chosen following two popular channels :

$$H(z) = 0.24 + 0.93z^{-1} + 0.26z^{-2} \quad (13)$$

$$H(z) = 0.303 + 0.9029z^{-1} + 0.304z^{-2} \quad (14)$$

Nonlinearity of equation (2) introduced.

For the purpose of comparison we considered following:

- ANN-equalizers of [67] represented by BP-ANN
- ANN-equalizers of [55], represented by GA\_ANN and PSO\_ANN
- ANN-equalizers of [57], represented by DE-ANN

MSE was computed with a fixed Signal to Noise Ratio (SNR) of 10dB. MSE plots are depicted in figures 2 and 3 for channels of equations (13) and (14) respectively. Corresponding BER plots are shown in figures 4 through 5 Table 1. Simulation Parameters

GA		PSO		FFA	
Parameter	Value	Parameter	Value	Parameter	Value
Max No. of iterations	1000	Max No. of iterations	1000	Max No. of iterations	1000
Population size	50	Population size	50	Population size	50
Mutation ratio	0.03	Coefficient C1	0.7	Attractiveness	1
Crossover ratio	0.9	Coefficient C2	0.7	Light Absorption Coefficient	2
Mutation type	Uniform				
Crossover type	Single point				

The figures reveal that:

- Channel of equation (13): MSE of proposed FFA-ANN and DE-ANN are comparable till 300 iterations and then FFA-ANN performance is better. But, FFA-ANN performance is better than other equalizers in all conditions. Also, BER of FFA-ANN becomes less than  $10^{-5}$  at SNR of 10dB outperforms other ANN-equalizers.

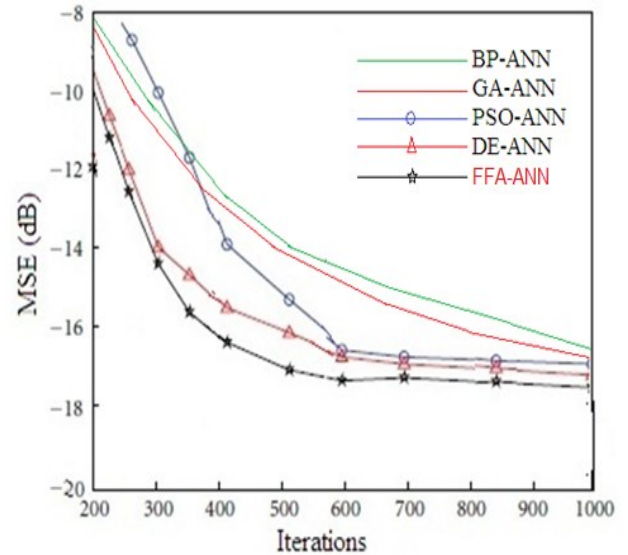


Figure. 3. MSE performance for channel (14)

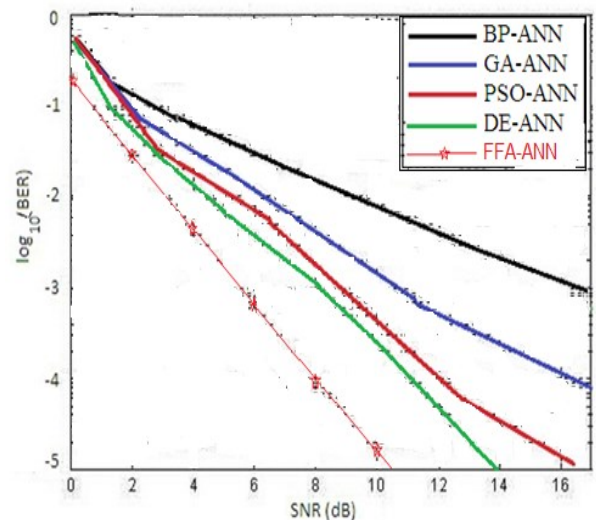


Figure. 4. BER performance for channel (13)

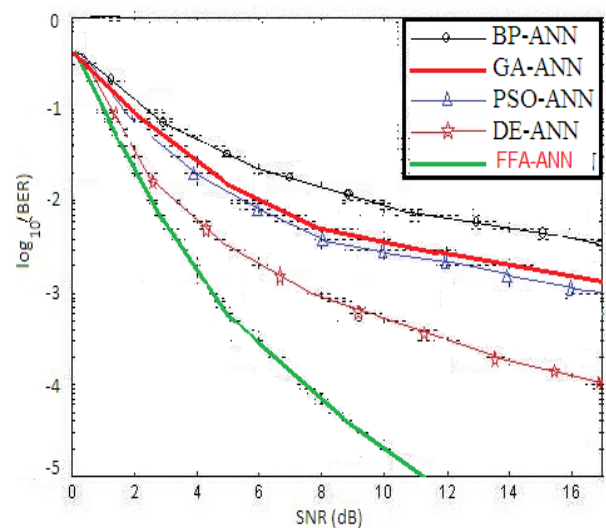
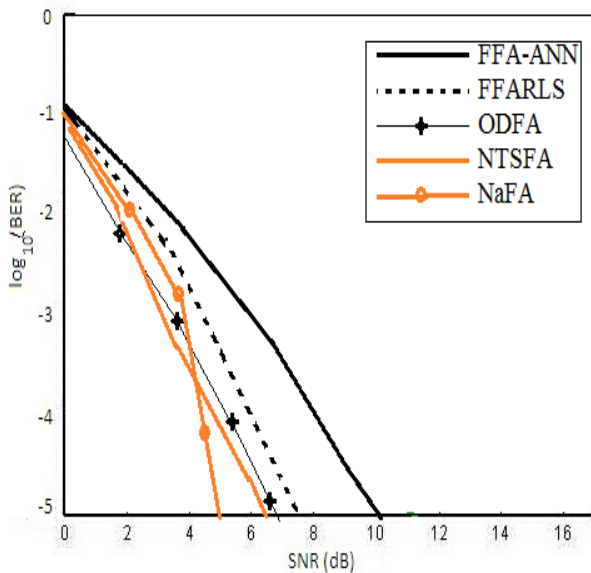


Figure. 5. BER performance for channel (14)

- Channel of equation (14): MSE of proposed FFA-ANN and DE-ANN, are comparable up to 600 iterations. In BER after SNR of 2dB, FFA-ANN outperforms other ANN based equalizers like those of channel of equation (12).



**Figure 6.** Comparison among forms of FFA in training ANNs based equalizers via BER performance for channel (13)

### Comparison among Forms of FFA

A comparison study was undertaken through BER performance for channel of equation (13). Here, all variants of FFA were used to train ANN based equalizer following the methodology discussed in previous section. Corresponding results were plotted in figure 6.

A study on figure 6 shows that all the variants perform better than original version of FFA. NaFA was found to be best among all having a better convergence speed

### V. Conclusion

This paper proposed a FFA to train neural network based equalizer. Then, the work was extended using improved and modified versions of FFA. Superior performance of proposed equalizers as compared with existing neural networks based equalizers was evidenced from simulations conducted on three different channels.

### References

- [1] Y. Chen, F. Schaich, T. Wild, Multiple access and waveforms for 5g: Idma and universal filtered multi-carrier, IEEE 79th Vehicular Technology Conference (VTC Spring), 2014, pp. 1-5.
- [2] X. Xiong, J. Hu, F. Yang, X. Ling, Effect of channel estimation error on the performance of interleave-division multiple access systems, 11th International Conference on Advanced Communication Technology, 03, ICACT 2009, pp. 1538-1542.
- [3] G. Wunder, P. Jung, M. Kasparick, T. Wild, F. Schaich, Y. Chen, S. Brink, I. Gaspar, N. Michailow, A. Festag, L. Mendes, N. Cassiau, D. Ktenas, M. Dryjanski, S. Pietrzyk, B. Eged, P. Vago, F. Wiedmann, '5G now: non-orthogonal, asynchronous waveforms for future mobile applications', IEEE Communications Magazine 52, 2014, pp.97-105.
- [4] B. R. Peterson, D. D. Falconer, 'Minimum Mean Square Equalization in Cyclostationary and Stationary Interference-Analysis and Subscriber Line Calculations, IEEE Journal on Selected Areas in Communication, vol. 9, 1991, pp. 931-940.
- [5] J. H. Winter, Optimum Combining in Digital Mobile Radio with Co-channel Interference, IEEE Journal on Selected Areas in Communication, vol. SAC-2, 1984, pp. 528-539.
- [6] J. G. Proakis, Adaptive Equalization for TDMA Digital Mobile Radio, IEEE Transactions on Vehicular Technology, vol. 40, 1991, pp. 333-341.
- [7] K. Feher, MODEMS for Emerging Digital Cellular-Mobile Radio system, IEEE Transactions on Vehicular Technology, vol. 40, 1991, pp. 355-365.
- [8] B. Widrow, M. E. Hoff(Jr), Adaptive Switching Circuits, in IRE WESCON Conv., vol. 4, 1960, pp. 94-104.
- [9] R. W. Lucky, Automatic Equalization of Digital Communication, Bell System Tech. J, vol. 44, 1965, pp. 547-588.
- [10] G. D. Forney, Maximum-Likelihood Sequence Estimation of Digital Sequences in the Presence of Intersymbol Interference, IEEE Transactions on Information Theory, vol. IT-18, 1972, pp. 363-378.
- [11] G. D. Forney, The Viterbi Algorithm, Proceedings of the IEEE, vol. 61, 1973, pp. 268-278.
- [12] D. A. George, R. R. Bowen, J. R. Storey, An Adaptive Decision Feedback Equalizer, IEEE Transactions on Communication Technology, vol. COM-19, 1971, pp. 281-293.
- [13] D. D. Falconer, Joint Adaptive Equalization and Carrier Recovery in Two-dimensional Digital Communication Systems, Bell System Technical Journal, vol. 55, March 1976, pp. 317-334.
- [14] D. Godard, Channel Equalization Using Kalman Filter for Fast Data Transmission, IBM Journal Res. Development, vol. 18, May 1974, pp. 267-273.
- [15] J. Makhoul, A Class of All-Zero Lattice Digital Filters, IEEE Transactions on Acoustics, Speech and Signal Processing, vol. ASSP-26, August 1978, pp. 304-314.
- [16] J. R. Treichler, I. Fijakow, and C. R. Johnson, Fractionally Spaced Equalizers - How Long Should They Really be?, IEEE Signal Processing Magazine, May 1996, pp. 65-81.
- [17] S. U. H. Qureshi, Adaptive Equalization, Proceedings of the IEEE, vol. 73, September 1985, pp. 1349-1387.
- [18] S. Haykin, Neural Networks - A Comprehensive Foundation. New York: Macmillan, 1994.
- [19] R. O. Duda and P. E. Hart, Pattern Classification and Scene Analysis. John Wiley and Sons, 1973.
- [20] S. Chen, B. Mulgrew, and S. McLaughlin, Adaptive Bayesian Equalizer with Decision Feedback, IEEE Transactions on Signal Processing, vol. 41, September 1993, pp. 2918-2927.
- [21] J. F. Hayes, T. M. Cover, J. B. Riera, Optimal Sequence Detection and Optimal Symbol-by-Symbol Detection: Similar Algorithms, IEEE Transactions on Communications, vol. COM-30, 1982, pp. 152-157.
- [22] A. Elkhazin, K. N. Plataniotis, S. Pasupathy, BER analysis of Bayesian equalization using orthogonal hyperplanes' Elsevier Signal Processing Vol 86, 2006, pp. 1992-2000.
- [23] J. Choi, S. Bang, B. Sheu, A programmable analog VLSI neural network processor for communication receivers, IEEE Transactions on Neural Networks 4 (3), 1993, pp. 484-495.
- [24] S. Chen, C. Cowan, P. Grant, Orthogonal least squares learning algorithm for radial basis function networks, IEEE Transactions on Neural Networks 2 (2), 1991.

- [25] S. Chen, G. Gibson, C. Cowan, P. Grant, Adaptive equalization of finite non-linear channels using multi layer perceptron, *Signal Processing* 20, 1990, pp. 107-119.
- [26] A. Hussain, J. Soraghan, T. Durrani, A new adaptive functional-link neural network-based DFE for overcoming co-channel interference, *IEEE Transactions on Communications*. 45 (11), 1997, pp. 1358-1362.
- [27] G. De Veciana, A. Zakhor, Neural net-based continuous phase modulation receivers, *IEEE Transactions on Communications*.. 40 (8), 1992, pp. 1396 - 1408
- [28] Z. Xiang, G. Bi, 'A new polynomial perceptron based 64QAM cellular mobile communications receivers', *IEEE Transactions on Signal Processing*. 43 (12), 1995, pp. 3094-3098.
- [29] L. Chua, L. Yang, Cellular neural networks: theory, *IEEE Transactions on Circuits Systems* 35, 1988, pp. 1257-1272.
- [30] B. Sheu, S. Bang, R. Chang, A cellular neural network with optimized performance for wireless communication receivers, *Proceedings of WCNN'95*, Washington, DC, 1995, pp. II-660-II-664.
- [31] S.H. Bang, B.J. Sheu, J. Choi, Programmable VLSI neural network processors for equalization of digital communication channels, *Proceedings of International Workshop on applications of Neural Networks to Telecommunications*, LEA Publishers, 1993, pp. 1-12.
- [32] R. Williams, D. Zipser, A learning algorithm for continually running fully recurrent neural networks, *Neural Computation* 1, 1989, pp: 270-280.
- [33] G. Kechriotis, E. Zervas, E. Manolakos, Using recurrent neural networks for adaptive communication channel equalization, *IEEE Transactions on Neural Networks* 5 (2), 1994, pp. 267-278.
- [34] R. Parisi, E. Di Claudio, G. Orlandi, B. Rao, Fast adaptive digital equalization by recurrent neural networks, *IEEE Transactions on Signal Processing* 45 (11), Nov'1997, pp:2731-2739.
- [35] B. Juang, S. Katagiri, Discriminative learning for minimum error classification, *IEEE Transactions on Signal Processing* 40, 1992. 3043 - 3054
- [36] T. Adali, X. Liu, M.K. Sonmez, Conditional learning with neural networks and its application to channel equalization, *IEEE Transactions on Signal Processing*, 45 (4), 1997. Pp. 1051 - 1064
- [37] T. Kohonen, E. Oja, O. Simula, A. Visa, J. Kangas, Engineering applications of the self-organizing map, *IEEE Proc.*, Oct'1996, pp. 1358-1384.
- [38] A. Guntsch, M. Ibnkahla, G. Losquadro, M. Mazzella, D. Roviras, A. Timm, EU's R&D activities on third generation mobile satellite systems (S-UMTS), *IEEE Communication Magazine*., 1998, pp.104-110
- [39] S. Bouchired, Equalization of time-varying non-linear channels using neural networks: application to the satellite mobile channel, Ph.D. Thesis, INP, Toulouse, France, 1999.
- [40] H. Lin, K. Yamashita, Hybrid simplex genetic algorithm for blind equalization using RBF networks, *Mathematics and Computers in Simulation*, 59, 2002, pp: 293-304
- [41] W. Pedrycz, C. Han., Nonlinear Channel Blind Equalization Using Hybrid Genetic Algorithm with Simulated Annealing, *Elsevier Mathematical and Computer Modelling* 41, 2005, pp: 697-709.
- [42] R. Pandey, Complex-Valued Neural Networks for Blind Equalization of Time-Varying Channels, *International Journal of Signal Processing* Vol 1, 2004, ISSN: 1304-4494
- [43] W. Chagra F, Bouani, Equalization with decision delay estimation using recurrent neural networks, *Advances in Engineering Software*, Vol. 36, 2005, pp: 442-447.
- [44] Ling Z., et al. MIMO Channel Estimation and Equalization Using Three-Layer Neural Networks with Feedback, *Tsinghua science and technology*, Vol 12, 2007, pp 658-662
- [45] M. N. Seyman, N. Taşpınar, Channel estimation based on neural network in space time block coded MIMO-OFDM system, *Digital Signal Processing*, 23 (1) (2013) 275-280.
- [46] X. Ruan, Y. Zhang, Blind sequence estimation of MPSK signals using dynamically driven recurrent neural networks, *Neurocomputing*, 129 (2014) 421-427
- [47] A. Rizaner, Radial basis function network assisted single-user channel estimation by using a linear minimum mean square error detector under impulsive noise, *Computers & Electrical Engineering*, 39 (4) (2013) 1288-1299.
- [48] H.K. Sahoo, P.K. Dash, N.P. Rath, NARX model based nonlinear dynamic system identification using low complexity neural networks and robust  $H_\infty$  filter, *Applied Soft Computing*, 13 (7) (2013) 3324-3334
- [49] D. J. Montana and L. Davis, Training Feedforward Neural Networks Using Genetic Algorithms, *Machine Learning*, pp. 762-767, <http://ijcai.org/Past%20Proceedings/IJCAI-89-VOL1/PDF/122.pdf>
- [50] A. Blanco, M. Delgado, M.C. Pegalajar, A real-coded genetic algorithm for training recurrent neural networks, *Neural Networks* 14 (2001) 93-105.
- [51] D. Kim, H. Kim, D. Chung, A Modified Genetic Algorithm for Fast Training Neural Networks, *Lecture Notes in Computer Science* 3496(2005), 660-665.
- [52] A.S. Rakitianskaia, A. P. Engelbrecht, Training feedforward neural networks with dynamic particle swarm optimization, *Swarm Intelligence*, 6, (3) (2012) 233-270.
- [53] I. Vilovic, N. Burum, D. Milic, , Using particle swarm optimization in training neural network for indoor field strength prediction, *ELMAR, International Symposium* . (2009).275,278,
- [54] R. Su; L. Kong; S. Song; P. Zhang; K. Zhou; J. Cheng, A New Ridgelet Neural Network Training Algorithm Based on Improved Particle Swarm Optimization, *Third International Conference on Natural Computation*, 3, (2012).411-415.
- [55] R. K. Jatoth, M. S. Vaddadi, S. Anoop, An intelligent functional link artificial neural network for channel equalization, *ISPR'A'09 Proceedings of the 8th WSEAS international conference on Signal processing*, (2009) 240-245
- [56] G. Das, P.K. Patnaik, S. K. Padhy, Artificial Neural Network trained by Particle Swarm Optimization for non-linear channel equalization, *Expert Systems with Applications*, 41 (7) (2014) 3491-3496
- [57] G. R. Patra, S. Maity, S. Sardar, S. Das, Nonlinear Channel Equalization for Digital Communications Using DE-Trained Functional Link Artificial Neural Networks, *Communications in Computer and Information Science* 168 (2011) 403-414.
- [58] N. Karaboga, Digital IIR filter design using differential evolution algorithm, *EURASIP Journal on Applied Signal Processing*, 8 (2005) 1269-1276.
- [59] V. Bergh, F. Engelbrecht, A new locally convergent particle swarm optimizer, *Proceedings of IEEE International Conference on Systems, Man, and Cybernetics* 2002, 96-101
- [60] J. Liu, J. Lampinen, On setting the control parameter of the differential evolution method, *Proc. 8th Int. Conf. Soft Computing (MENDEL 2002)*, 2002, pp. 11-18.
- [61] S. Das, P. N. Suganthan, Differential evolution: A survey of the state-of-the-art, *IEEE Trans. on Evolutionary Computation*, 15 (1) (2011) 4-31
- [62] X.S. Yang Firefly Algorithm, *Stochastic Test Functions and Design Optimization*, *Int. J. Bio-inspired Computation*., 2 (2); (2010), 78-84.
- [63] H. Wang, W. Wang, X. Zhou, H. Sun, Firefly algorithm with neighborhood attraction, *Information Sciences*, Volumes 382-383, (2017), 374-387

- [64] S. K. Singh, N. Sinha, A. K. Goswami, Robust estimation of power system harmonics using a hybrid firefly-based recursive least square algorithm, *International Journal of Electrical Power & Energy Systems*, 80 (2016), 287-296
- [65] O. Verma, D. Aggarwal, T. Patodi, Opposition and dimensional based modified Firefly algorithm, *Expert Systems with Applications*, 44, (2016), 168-176
- [66] S. Yu, S. Zhu, X. Zhou, An Improved Firefly Algorithm Based on Nonlinear Time-varying Step-size. *International Journal of Hybrid Information Technology*, 9,(2016), 397-410
- [67] H. Zhao, X. Zeng, J. Zhang, T. Li, Y. Liu, D. Ruan, Pipelined functional link artificial recurrent neural network with the decision feedback structure for nonlinear channel equalization, *Information Sciences* 181 (2011) 3677–3692