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Active Contours without Edges and without Reinitialisation

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Abstract

This paper is concerned with the use of the level set methods in image segmentation. It is known that a reinitialisation procedure is sometimes needed for the correct function of these methods. This step, however, is problematic both from the theoretical and from the practical point of view. In this paper, we present a modification of the well known Chan and Vese method that does not require the reinitialisation. It means that, during the iterative process, it is not necessary to switch between the usual and reinitialisation steps. The method is based on formulating the image segmentation problem as a problem of minimising a functional with a constraint. The solution is done by making use of the Lagrange multipliers method. We present the theory needed for implementing as well as results of practical testing.

Keywords: *image segmentation, level set method, reinitialisation, Lagrange multipliers*

1. Introduction

The level set methods, introduced by the pioneering work by Osher and Sethian [11], are increasingly considered by the computer vision community. Image segmentation is one of their important application areas. Over the last several years, a number of variational and level set approaches have been proposed [2, 3, 4, 8]. Up to these days, the level set methods are still being developed [14, 15]. The approach proposed by Chan and Vese (e.g., [3, 4]), however, still remains respected [6].

In the level set methods, it is sometimes necessary to reinitialise the level set function [1, 2, 7, 9, 10]. It is due to the fact that, during the iterative cycles, the size of the gradient of the level set function may become too small at certain points, which may cause numerical instabilities during computation. The remedy may be done by reinitilisation. The iterative steps solving the Euler-Lagrange partial differential equation (PDE) are stopped for a while and another PDE is solved that should remove the flat areas of the level set function. The reinitialisation may be required repeatedly from time to time. As was pointed out in [7], it may be regarded as a disagreement between the theory of the level set method and its implementation. The question when and how the reinitialisation should exactly be applied still remains unanswered.

In the Chan and Vese method, the need for reinitialisation is involved also even from the theoretical point of view if the theoretical Heaviside function is considered (see the next section). In this case, infinitely many level-set functions exist that minimise the functional and comply with the corresponding PDE solving the problem. All of them take the zero values at the same points. Only the same sign of their values is required at the remaining points; the value itself does not matter. It follows that the problem is not well posed in this case. Fortunately, a certain regularised form of the Heaviside function is used in practice, which causes that the method behaves better.

In this paper, we present a modification of the Chan and Vese method. We formulate the image segmentation problem as a problem of minimisation of a functional with a constraint replacing the reinitialisation equation. The problem is then solved by making use of the Lagrange multipliers method. This is a difference in comparison with the approach presented in [7], where the reinitialisation equation is adjoined, with a certain weight, directly to the functional. It follows that in the method we propose, the reinitialisation step is not needed since the requirement that the level set function should nowhere be flat is incorporated directly into the method. From the theoretical point of view, the modification also ensures the uniqueness of the solution even for the theoretical Heaviside function. The rationale of the method is the following: The functional is minimised with a constraint that requires the level set function to measure the distance from the object boundary similarly as it was done previously by the separate reinitilisation PDE.

The paper is organised as follows. In the following section, the Chan and Vese method is briefly summarised. The reinitialisation process is discussed in Section 3. Section 4 contains the description of the method we propose. In Section 5, some experimental results are presented.

2. Original Chan and Vese's Method

In this section, for convenience of the reader, we will briefly summarise original Chan and Vese's approach. Let $u(x)$ be the brightness function of the input image. The image is defined over a two-dimensional area, denoted by Ω. It is being assumed that the image contains objects and a background. Theoretically, the objects as well as the background are supposed to be of constant brightness, denoted by c_o and c_b, respectively. Let C stand for the collection of closed curves in the image that separate the objects from the background. In Chan and Vese's method, the following functional is minimised in which $\mu \geq 0$, $\nu \geq 0$, and $\alpha_o, \alpha_b \geq 0$ are parameters of suitably chosen values

$$\begin{aligned} F(c_o, c_b, C) = \mu \cdot \mathrm{Length}\,(C) \\ +\nu \cdot \mathrm{Area}\,(\mathrm{inside}\,(C)) \\ +\alpha_o \int_{\mathrm{inside}(C)} (u(x) - c_o)^2 \, dx \\ +\alpha_b \int_{\mathrm{outside}(C)} (u(x) - c_b)^2 \, dx \, . \end{aligned} \tag{1}$$

For minimising the functional from Eq. (1), a function $\phi(x), x \in \Omega$, is introduced that takes a value of $\phi > 0$ inside the objects, $\phi = 0$ on their boundaries, and $\phi < 0$ outside the objects. Using the Heaviside function defined by

$$H(z) = \begin{cases} 1, & \text{if} \quad z \geq 0 \\ 0, & \text{if} \quad z < 0 \end{cases} \quad , \tag{2}$$

the functional from Eq. (1) may be rewritten into the form of (we omit the argument x of u and ϕ for brevity from now on)

$$\begin{aligned} F(c_o, c_b, \phi) = \mu \int_\Omega |\nabla H(\phi)| dx + \nu \int_\Omega H\,(\phi)\, dx \\ +\alpha_o \int_\Omega (u - c_o)^2 \, H\,(\phi)\, dx \\ +\alpha_b \int_\Omega (u - c_b)^2 \, (1 - H\,(\phi))\, dx \, . \end{aligned} \tag{3}$$

Keeping ϕ fixed and minimising the value of $F(c_o, c_b, \phi)$ with respect to the constants c_o, c_b, it is easy to find for them the following expressions

$$\begin{aligned} c_o(\phi) &= \frac{\int_\Omega u H(\phi) dx}{\int_\Omega H(\phi) dx} \, , \\ c_b(\phi) &= \frac{\int_\Omega u\,(1 - H\,(\phi))\, dx}{\int_\Omega (1 - H\,(\phi))\, dx} \, . \end{aligned} \tag{4}$$

It can be easily seen that the values of $c_o(\phi)$, $c_b(\phi)$ have the meaning of average brightness of original image over the areas that are regarded, in segmentation, as the objects ($\phi \geq 0$) and the background ($\phi < 0$), respectively.

Keeping c_o, c_b fixed and minimising $F(c_o, c_b, \phi)$ with respect to ϕ, the associated Euler-Lagrange equation may be obtained that takes the form of

$$\begin{aligned} \delta\,(\phi) \Bigg[\mu \mathrm{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) - \nu \\ -\alpha_o (u - c_o)^2 + \alpha_b (u - c_b)^2 \Bigg] = 0 \quad \text{in} \quad \Omega \, . \end{aligned} \tag{5}$$

For practical computation, the authors introduce a regularised version of H and its derivative as follows (ε is a suitably chosen value)

$$\begin{aligned} H_\varepsilon\,(z) &= \frac{1}{2} \left(1 + \frac{2}{\pi} \arctan \left(\frac{z}{\varepsilon} \right) \right) \, , \\ \delta_\varepsilon(z) = H'_\varepsilon(z) &= \frac{1}{\pi} \cdot \frac{\varepsilon}{\varepsilon^2 + z^2} \, . \end{aligned} \tag{6}$$

Introducing $\phi\,(t, x)$ by parametrising the descent direction by time $t \geq 0$, and taking $\phi\,(0, x) = \phi_0\,(x)$ (a chosen initial contour), a system is obtained for solving ϕ iteratively that can be written in the form of

$$\begin{cases} \frac{\partial \phi}{\partial t} = \delta_\varepsilon\,(\phi) \left(\mu \mathrm{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) - \nu \right. \\ \left. -\alpha_o (u - c_o)^2 + \alpha_b (u - c_b)^2 \right) \quad \text{in} \quad \Omega \, , \\ \phi\,(0, x) = \phi_0\,(x) \quad \text{in} \quad \Omega \, , \\ \frac{\delta_\varepsilon(\phi)}{|\nabla \phi|} \frac{\partial \phi}{\partial \vec{n}} = 0 \quad \text{on} \quad \partial\Omega \, , \end{cases} \tag{7}$$

where $\vec{n}$ denotes the exterior normal to the boundary $\partial\Omega$ of Ω, and $\partial\phi/\partial\vec{n}$ denotes the normal derivative of ϕ at the boundary. The initial value of the level set function is often obtained in such a way that an initial curve (i.e., an initial estimation of object boundaries) is placed into image. The values of ϕ_0 are then computed as the signed distances from this initial boundary curve.

The term $\nu \int_\Omega H\,(\phi)\, dx$ in the functional from Eq. (3) may be regarded as a bit problematic since it prefers small total size of objects, which need not correspond to what is really required. The objects that are extremely small are usually noise. Chan and Vese also admit that they usually choose $\nu = 0$, which eliminates the influence of that term. As a remedy, we have introduced in [13] a modified method in which the mentioned term is replaced by the term expressing the possibly known total size (area) of objects that are to be found. Let A_R be that size. The size, denoted by A, of the objects that can be computed from the values of ϕ is given by the expression

$$A(\phi) = \int_\Omega H\,(\phi)\, dx \, . \tag{8}$$

The requirement that the size of objects should be A_R may be taken into account by minimising the value of the expression

$$\left(\frac{A(\phi)}{A_R} - 1\right)^2 \tag{9}$$

that takes the value of zero if the size of objects is A_R, and the value greater than zero otherwise. The minimisation of the expression from Eq. (9) is equivalent to minimisation of its integral over Ω. Therefore, we replace the term $\nu \int_\Omega H(\phi)\, dx$ in the functional from Eq. (3) with the following term

$$\frac{\nu}{2} \int_\Omega \left(\frac{A(\phi)}{A_R} - 1\right)^2 dx\,. \tag{10}$$

The term in the Euler-Lagrange equation that corresponds to the new size term from Eq. (10) of functional can be easily determined. By computing the derivative of the expression behind the integration sign in Eq. (10), we obtain

$$\frac{\partial}{\partial \phi}\left[\frac{\nu}{2}\left(\frac{A(\phi)}{A_R} - 1\right)^2\right] = \frac{\nu}{A_R}\left(\frac{A(\phi)}{A_R} - 1\right)\frac{\partial A(\phi)}{\partial \phi}\,. \tag{11}$$

From Eq. (8), it is clear that $\partial A(\phi)/\partial\phi = \partial H(\phi)/\partial\phi = \delta(\phi)$. Therefore, at the point $x \in \Omega$, the value of the derivative we seek for simply is $\nu\delta(\phi)(A(\phi)/A_R - 1)/A_R$. The Euler-Lagrange equation modified with respect to the new size term may now be written in the form of

$$\delta(\phi)\left[\mu \mathrm{div}\left(\frac{\nabla\phi}{|\nabla\phi|}\right) - \frac{\nu}{A_R}\left(\frac{A(\phi)}{A_R} - 1\right) - \alpha_o(u - c_o)^2 + \alpha_b(u - c_b)^2\right] = 0\,. \tag{12}$$

This equation may now be used for solving ϕ instead of Eq. (5).

3. Reinitialisation

Reinitialisation is a process that is sometimes needed in the original Chan and Vese method. If, during the computation, the size of the gradient of the level set function becomes too small at certain points, the first term in the Euler-Lagrange equation causes the instability of computation. The reinitialisation should be carried out if this situation is threatening.

Let it be pointed out that the need for reinitialisation is incorporated in the functional from Eq. (3) itself. If the theoretical Heaviside function is used (which is the case for which the meaning of the functional is well understood), infinitely many level-sets functions exist that minimise that functional. All of them take the values of zero at the same set of points. Only the same sign of their values (not the value itself) is required at the remaining points. It follows that, in this case, the problem is not well posed if only the functional from Eq. (3) is used. Fortunately, the theoretical Heaviside function is not used in practical computation. Thanks to the regularisation from Eq. (6), the method behaves better than it has just been outlined. The need for reinitialisation, however, is reported by many authors [1, 2, 7, 9, 10, 12]. Obviously, the size of ε and the image data itself may play the role.

During reinitialisation, the process of seeking for ϕ by making use of Eq. (5) is interrupted. Zero values of ϕ remain fixed. For other values, only the sign remains fixed; the value itself may be modified. Often, the new values of ϕ are obtained by solving the equation

$$|\nabla\phi| - 1 = 0\,. \tag{13}$$

The above equation ensures that ϕ measures the distance from the boundary of objects (i.e., from the points at which ϕ has value of zero). The equation is solved iteratively. After finishing the process of reinitialisation, Eq. (5) is used for solving ϕ again.

4. Including Reinitialisation into the Method

In this section, we show that the condition from Eq. (13) may be included into the functional from Eq. (3) and into the corresponding subsequent equations. From the theoretical point of view, this ensures the uniqueness of solution even for the theoretical Heaviside function, which is a remedy of the problem that was discussed in the previous section. From the practical point of view, it guarantees that the reinitialisation step is not needed at all. In essence, we use the approach in which the functional from Eq. (3) is minimised with a constraint. Similarly as in the case of reinitialisation, the constraint will require ϕ to measure the distance from the object boundary. The problem is solved by making use of the Lagrange multipliers method.

From the point of view of the steps that will follow, it is advantageous to reformulate the constraint from Eq. (13) into the integral form as follows

$$C(\phi) = \frac{1}{2}\int_\Omega (|\nabla\phi| - 1)^2 dx = 0\,. \tag{14}$$

In calculus of variations, the condition in this integral form is usually called an isoperimetric constraint. It is also well known fact that minimising the functional from Eq. (3) with the constraint of this type may be done by minimising the augmented functional that is of the form

$$\tilde{F}(c_o, c_b, \phi, \lambda) = F(c_o, c_b, \phi) + \frac{\lambda}{2}\int_\Omega (|\nabla\phi| - 1)^2 dx\,, \tag{15}$$

where λ is a Lagrange multiplier, which is an unknown and, in this case, scalar quantity. The minimisation of $\tilde{F}$ is now carried out with respect to c_o, c_b, ϕ, and λ.

Similarly as in the original approach, the solution we propose is iterative. In each iteration, we proceed in the following three steps: (i) finding the new values for c_o, c_b; (ii) determining the new value for ϕ; (iii) determining the new value for λ. In the sequel, the latter two steps are discussed in more details. The first step (determining c_o, c_b) need not be discussed since it is exactly the same as in the original approach, i.e., Eq. (4) is valid also in this case.

For determining the new value of ϕ, the Euler-Lagrange equation corresponding to the functional from Eq. (15) must be deduced. Clearly, the new equation may be written in the form of the sum

$$\chi(\phi) + \lambda\psi(\phi) = 0\,. \tag{16}$$

Recall that χ stands for the terms in the original approach (left-hand side of Eq. (5)); ψ is a new term corresponding to the new term that was added to $\tilde{F}$. Using the standard technique of deducing the Euler-Lagrange equations, the following formula is obtained for ψ after some effort

$$\psi(\phi) = \operatorname{div}\left(\nabla\phi - \frac{\nabla\phi}{|\nabla\phi|}\right)\,. \tag{17}$$

The new Euler-Lagrange equation specified in Eqs. (16, 17) is used in exactly the same way as Eq. (5) in the original method. The computational cost of determining the terms in Eq. (17) is low since the similar terms have already been present also in the original equation.

The final step in each iteration cycle is to determine the new value of λ. Generally speaking, the value of λ should be chosen in such a way that $\partial\tilde{F}/\partial\lambda = 0$. It gives $C(\phi) = 0$, which, in fact, is Eq. (14). To stress that ϕ depends on λ during the solution, we will use the notation $C(\phi(\lambda)) = 0$. At the beginning of computation, this condition need not be satisfied. But during the iterations, we will reduce the value of $C(\phi(\lambda))$ (recall that C is non negative). It can be done, e.g., by the steepest descent method, i.e., between the particular iterations, we change the value of λ according to the following formula (k and $k+1$ stand for the indexes of iteration, η is a chosen constant)

$$\lambda_{k+1} = \lambda_k - \eta\frac{\partial C(\phi(\lambda))}{\partial\lambda}\,. \tag{18}$$

In order to be able to compute the new value of λ, the derivative $\partial C(\phi(\lambda))/\partial\lambda$ is needed. It can be computed as a derivative of the right-hand side of Eq. (14). By making use of Eq. (16), by taking into account the fact that $\phi_{k+1} = \phi_k + \Delta t(\partial\phi/\partial t)$ and by introducing the abbreviations of the type $\phi_x \equiv \partial\phi/\partial x$ for the derivatives, we obtain the following result

$$\begin{aligned}\frac{\partial C(\phi(\lambda))}{\partial\lambda} &= \int_\Omega (|\nabla\phi| - 1)\frac{\partial|\nabla\phi|}{\partial\lambda}dx \\ &= \int_\Omega (1 - \frac{1}{|\nabla\phi|})(\phi_x\frac{\partial\phi_\lambda}{\partial x} + \phi_y\frac{\partial\phi_\lambda}{\partial y})dx \\ &= \Delta t\int_\Omega (1 - \frac{1}{|\nabla\phi|})(\psi_x\phi_x + \phi_y\psi_y)dx \\ &= \Delta t\int_\Omega (1 - \frac{1}{|\nabla\phi|})(\nabla\phi\cdot\nabla\psi)dx\,. \end{aligned} \tag{19}$$

Again, it can be seen that the value of the derivative can be easily determined with a low computational cost. The values of $\nabla\phi$ and $|\nabla\phi|$ are used also in the original method. The value of $\nabla\psi$ may be easily computed from Eq. (17).

5. Experimental Results

The method was implemented and tested. Naturally, we can expect that from the point of view of image segmentation itself, the new and the original method should give similar results since the terms in the functional directly responsible for discriminating the objects from the background remained unchanged. The difference, however, is expected in the way how the sizes of the gradient of the level set function evolve during computation. For comparing the successfulness of segmentation, we used the value of the functional $F(\phi)$ from Eq. (3). For measuring how the sizes of the gradient evolve, we introduced two quantities: (i) The quantity C^- that is similar to the quantity from Eq. (14) except that the integration is carried out only over the area where the gradient size is less than one, (ii) the relative area A (in %) of image in which the size of the gradient is less than 0.5. The quantity C^- tells how much the values of the gradient size drop below the desired value of 1. The value of A tells what is the area in which the gradient size is small.

Firstly, a synthetic test image depicted in Figure 1 was used. All parameters were the same both for the original and the modified method ($\mu = 1$, $\nu = 0$, $\alpha_o = \alpha_b = 5$, $\epsilon = 5$, $\Delta t = 10$). The value of λ_0 was chosen $\lambda_0 = 0.01$; the constant η from Eq. (18) was set to 0.00001. The circle inscribed into the image with the diameter of 80% of image size was chosen as an initial boundary curve. The initial values of the level set function were computed as a distance from this curve. The values of C^-, A, and F are stated in the following table for the original and new method (the indices stand for the number of iterations; the values of C^- and F are normalised to one pixel).

	C^-_{1000}	C^-_{10000}	A_{1000}	A_{10000}	F_{10000}
Old	0.054	0.159	5.46	26.61	0.1244
New	0.019	0.002	0.03	0.00	0.1246

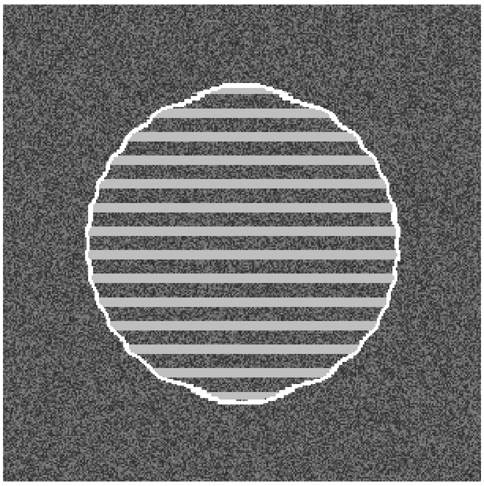

Figure 1. An artificial test image with the result of segmentation

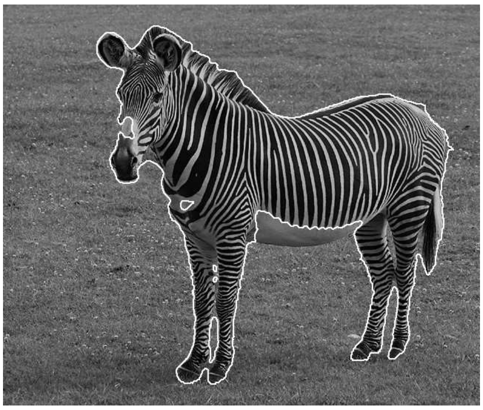

Figure 2. A real-life test image

As can be seen, the quality of segmentation remains almost unchanged in the modified method. The size of the gradient of the level set function tends to be generally more distant from the ”dangerous small values” as follows from the values of C^- and A. Similar results have also been obtained for the real-life images (Figure 2). In this case, the size of object was considered to be known, i.e., Eq. (12) was used.

6. Conclusion

In this paper, a modification of the Chan and Vese level set method for image segmentation has been presented. The modification is based on introducing the condition preserving the level set function from becoming too flat during the iterative process. Thus, the reinitilisation step that is reported by the majority of authors as sometimes necessary can be avoided. Moreover, also the theoretical ambiguity of the solution has been removed. The functional is minimised with a constraint that requires the level set function to measure the distance from the object boundary similarly as it was done previously by the separate reinitilisation PDE. The method was implemented and tested.

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