

Study on the Application of Borel's Improved Lagrange Interpolation Formula

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Abstract: Because Lagrange interpolation formula can not converge uniformly to any continuous function, how to improve its convergence has become an important content in the study of Lagrange interpolation formula. Borel has pointed out that the polynomial obtained by Lagrange interpolation formula can't approximate the interpolated function well in some cases. How to improve Lagrange interpolation formula to better approximate the interpolated function is an important problem for mathematicians at that time, and Borel is one of them. In this paper, the application research of the improved Lagrangian interpolation formula by Bohr is deeply discussed by using the relevant original documents.

Keywords: Borel, Lagrange, Interpolation formula.

1. Introduction

Nowadays, science and technology are developing rapidly with each passing day. The endless stream of new problems requires people to constantly put forward corresponding new theories and methods to solve them. Approximation theory has become one of the research hotspots in computational mathematics. Using polynomial operators to approximate functions is a very effective tool in numerical methods, and Lagrange interpolation method is one of the commonly used and effective basic methods of polynomial operator approximation [1-2]. However, because Lagrange interpolation formula can not converge uniformly to any continuous function, how to improve its convergence has become an important content in the study of Lagrange interpolation formula. In this paper, the application research of the improved Lagrangian interpolation formula by Bohr is deeply discussed by using the relevant original documents.

2. Lagrange Interpolation Formula

Interpolation is a common method to find approximate expressions of functions [3-4]. People often approximate with the following Lagrange interpolation polynomial $L_n(x)$ when measuring data and getting tabular functions in practice.

Let the function $y = f(x)$ be continuous on the interval $[a, b]$, and at $n+1$ different points $(x_0, y_0), \dots, (x_n, y_n)$ on $[a, b]$, if there is a function value (y_0, y_n) , the Lagrange interpolation polynomial can be obtained:

$$L_n(x) = \sum_{i=0}^n Y_i \frac{(x-x_0)\dots(x-x_{i-1})(x-x_{i+1})\dots(x-x_n)}{(x_i-x_0)\dots(x_i-x_{i-1})(x_i-x_{i+1})\dots(x_i-x_n)} \quad (1)$$

This is a polynomial whose degree does not exceed n , and each addition in the sum formula is denoted as $L_i(x)$, which is called Lagrange factor.

For a given interpolation node group Θ , if we don't limit the degree of interpolation polynomial space, then the polynomial space satisfying the given interpolation condition

is not unique. People hope to find such a polynomial space, so that this polynomial space is the lowest among all interpolation-appropriate polynomial spaces on node group Θ [5-6].

We use R to represent the algebra of formal power series with real coefficients and introduce the mapping $R \rightarrow P^{(m)}$, which maps each power series to its non-zero minimal homogeneous term. The arbitrary center of this mapping for $f = \sum_{\alpha \in N_0^m} f_\alpha x^\alpha$ is given by the following form [7]:

$$\lambda(f) = \min_{n \in N_0} \left\{ p_n : p_n = \sum_{|\alpha|=n} f_\alpha x^\alpha, p_n \neq 0 \right\} \quad (2)$$

If the function $f(x)$ is defined on the interval $[a, b]$ and $y_i = f(x_i^{(n)}), 1 \leq i \leq n$ is taken, then the polynomial:

$$L_n(f; x) = \sum_{i=1}^n f(x_i^{(n)}) y_i^{(n)}(x) \quad (3)$$

Meet the interpolation conditions:

$$L_n(f; x_j^{(n)}) = f(x_j^{(n)}), 1 \leq j \leq n \quad (4)$$

We call $L_n(f; x)$ the Lagrangian interpolation formula of function $f(x)$ in the interval $[a, b]$ with $X = \{x_i^{(n)}\}, 1 \leq i \leq n$ as the interpolation node [8].

3. Application of Improved Lagrange Interpolation Formula by Borer

In scientific research and practical engineering design, almost all problems can be expressed by the quantitative relationship of some internal law with the function $y = f(x)$. However, idealized functional relationships are hard to expect in practical engineering applications.

Because algebraic polynomials have the characteristics of convenient numerical calculation and theoretical analysis, we choose algebraic interpolation [9-10] in practical engineering applications. That is, to find a polynomial whose degree is not more than n , and when $n = 2$ is parabolic interpolation, it is also a commonly used algebraic interpolation.

Borel proved that one of the integer functions $g(x)$ with infinitely many given zeros can always be selected, and the series convergent to the interpolated function can be obtained by Lagrange interpolation formula [11]. However, when $g(x)$ is a polynomial, the series obtained by Lagrange interpolation formula does not necessarily converge to the interpolated function, which is exactly the problem to be solved by Borer in "On Approximating Continuous Functions by Polynomials" [12].

That is, when $z = a_1, B, a_m, f(a_n) = c_n, \varphi(z)$ is a polynomial, $\varphi(a_n) = 0, n = 1, 2, B, m$, determine the polynomial $f(z)$, and use Lagrange interpolation formula to get the polynomial of degree $m - 1$:

$$f(z) = \sum_{n=1}^m \frac{c_n \varphi(z)}{(z - a_n) \varphi'(a_n)} \quad (5)$$

When m tends to infinity, the polynomial obtained by Lagrange interpolation formula does not necessarily converge to the interpolated function $f(z)$.

In our eyes, the boundaries of the graphics displayed by the macro world are generally smooth. When the image information is saved in the computer, the graphics are transformed into a pixel matrix and presented to us. Through the observation of the pixel matrix, we find that it is not smooth on the boundaries of the graphics, but jagged, which is not smooth [13]. However, there will be no jagged information on the boundaries of the actual graphics, so it is necessary to smooth the jagged boundaries.

We can use the idea of numerical approximation to model the problem mathematically through the improved Lagrange interpolation formula of Borer, so as to smooth the jagged boundary and make the unreal bitmap information as close as possible to the real image [14].

In order to solve the problem conveniently, it is assumed that the probability of both cases is 50%, and each pixel is a particle. However, we only have bitmap information (as shown in Figure 1), and we need to find the real image boundary. The real image boundary is ever-changing, and there are many real boundaries corresponding to the same bitmap information. Therefore, we can use Borer's improved Lagrange interpolation formula to approximate the real boundary.

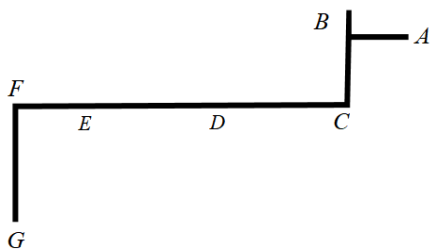


Figure 1. Bitmap information

i stands for pixel point, $i = A, B, C, D, E, F, G$; (x_i, y_i) represents the coordinates of the pixel point i . The approximation function is calculated by using Borer's improved Lagrange interpolation formula.

$$L(x) = \sum_{k=0}^n y_k \frac{w_{n+1}(x)}{(x - x_k) w'_{n+1}(x_k)} \quad (6)$$

Because the bitmap boundary information is divided into many breakpoints, if these points are interpolated by Lagrange in turn, the approximation effect will be reduced, so interpolation is carried out by segments, and two segment points are used as a group of points for interpolation. Taking two breakpoints as a group, the approximation function is calculated by using Borel's improved Lagrange interpolation formula. These functions are combined into segment functions to express them.

$$L(x) = \sum_{i=0}^n y_i l_i(x) \quad (7)$$

Although the jagged boundary can be smoothed by the above algorithm, the number of piecewise functions cannot be controlled within a reasonable range because of the piecewise function algorithm. Moreover, the model only improves the smoothness of the boundary curve, but can not completely eliminate the jagged boundary, so the jagged boundary still exists, but this problem can be solved by high-order interpolation method.

At present, data science and analytical technology lead economic innovation, competition and productivity. The leading process of data science is data acquisition and acquisition. There may be a lot of abnormal data in the obtained data, which may have a bad influence on the data analysis algorithm and cause the analysis results to be biased; Data cleaning is the most important part of data processing, which means deleting irrelevant data, duplicate data, smoothing noise data in original data, processing missing values, and processing abnormal values, so as to correct errors in data and ensure data consistency. Common data cleaning is the processing of missing values and abnormal values.

Data cleaning is to clean up errors in data. The data in a data warehouse is a collection of data oriented to a certain topic. Generally, the data is extracted from multiple business systems and contains historical data. Some abnormal data may contain useful information. These abnormal values can be mined and modeled on data sets with abnormal values without processing. Interpolation is used when some abnormal data or missing data cannot be deleted.

The basic idea of improving Lagrange interpolation formula by Borel is to give a polynomial that just passes through several known points on the two-dimensional plane, and construct a smooth curve by using the polynomial with the least degree, so that the curve passes through all known points.

Substitute the coordinates $(x_1, y_1), B, (x_n, y_n)$ of n points into the polynomial to obtain:

