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THE EFFECT OF INCENTIVE POLICY ON MANUFACTURER'S DISRUPTION RECOVERY IN BUILDING A RESILIENT SUPPLY CHAIN

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The outbreak of the epidemic and the complex international situation have caused supply disruptions to varying degrees. To survive the disruption, it is critical to think about how to build a resilient supply chain. We examine the manufacturer's recovery capacity under the fiscal interest-discount rate and tax discount rate. Motivated by the current incentive policies, such as the fiscal interest-discount policy and tax preference policy, we develop a game-theoretical game model focusing on a manufacturer aiming to maximize profits, a government aiming to maximize utility, and consumers. We have derived optimal decisions for products with varying degrees of scarcity after disruption by analyzing the perspectives of profit maximization and social welfare optimization for each entity in the supply chain. We find that for the recovery of moderately scarce products, the fiscal interest-discount policy will be the optimal choice for both the interrupted manufacturer and the government. However, as production scarcity increases, the manufacturer will prefer the tax preference policy.

Keywords: Resilient supply chain, Product scarcity, Fiscal interest-discount policy, Tax preference policy, Disruption recovery.

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1. INTRODUCTION

The escalating frequency of extreme events, from geopolitical tensions and pandemics to natural disasters, has starkly revealed the fragility of global supply chains and forced a critical re-examination of the long-standing paradigm in supply chain management: the tradeoff between cost-efficiency and resilience. Historically, global supply chains were optimized primarily for leanness and cost minimization through practices like single-sourcing. While effective in stable times, this efficiency-centric model has proven highly vulnerable to disruptions. Consequently, managers now face the imperative to build resilience without completely sacrificing competitive efficiency. The central challenge, therefore, lies in reconciling these two seemingly opposing objectives.

These disruptions can be categorized into human-induced and natural factors, each presenting distinct challenges. Human-induced factors, such as geopolitical tensions and trade restrictions, are often systemic and protracted. A prime example is the CHIPS and Science Act, which deliberately reshaped semiconductor supply chains (White House, 2022). Natural factors, such as earthquakes, expose acute operational vulnerabilities (Li *et al.*, 2022; Alptekinoğlu *et al.*, 2023; Fan *et al.*, 2023; Xu *et al.*, 2023; Nikkei Asia, 2023). For instance, the 2022 earthquake that halted production at a key supplier for Subaru Corporation demonstrates how a localized disruption can cripple a lean supply chain, revealing a critical "bottleneck risk" and underscoring the need for strategies like multi-sourcing and strategic stockholding (Michael, 2022).

Enhancing resilience against such diverse threats requires a multi-faceted approach. On one hand, technological empowerment through AI, IoT, and Blockchain can bolster operational resilience by improving forecasting, visibility, and traceability (Ivanov *et al.*, 2022; Dubey *et al.*, 2020; Saberi *et al.*, 2019). On the other hand, government intervention through policy instruments plays a critical strategic role, as evidenced by fiscal incentives from the U.S. and Japanese governments (Vuillemey, 2019; Zhao *et al.*, 2022). While technological tools are fundamental, their widespread adoption often requires external impetus, and government policies are crucial for sharing the financial burden of recovery, making resilience investments economically viable for firms (SC, 2022).

However, a critical gap persists in understanding how contingent factors shape policy effectiveness. While prior

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research has often studied firm-level recovery and government intervention in isolation, there is a lack of integrated models that specify the conditions under which different policies excel. This study addresses this gap by introducing a framework that posits product scarcity and manufacturer capacity recovery efficiency as two pivotal moderating variables. Consequently, we investigate the following research question: How do different levels of product scarcity and recovery efficiency jointly determine the preferences of the government and the manufacturer for fiscal interest-discount versus tax-preference policies?

To answer this, we develop a Stackelberg game model, modifying the newsvendor framework to analyze post-disruption recovery. We derive optimal decisions for both players under each policy and compare their outcomes. Our analysis reveals a complex interplay of preferences. For example, we find that under high product scarcity and low recovery efficiency, a goal conflict arises: the government prefers the fiscal interest-discount policy, whereas the manufacturer prefers the tax-preference policy. This study thus provides a contingent decision-making framework for policymakers and managers.

The rest of this paper is organized as follows. Section 2 reviews the related literature. Section 3 details the model formulation. Section 4 presents the analytical results, and Section 5 offers a comparative analysis and numerical experiments. Section 6 concludes with findings and future research directions.

2. LITERATURE REVIEW

This paper relates to two areas: (1) supply disruption recovery strategies, and (2) government strategies for supply chain recovery, and we concisely examine the related literature in the following.

The existing literature on supply chain disruption recovery strategies mainly focuses on two aspects: preventive mitigation strategies (Zhao *et al.*, 2023) and emergency relief operation (Xu *et al.*, 2023; Li *et al.*, 2021). This paper mainly focuses on recovery strategies after disruption, that is, the emergency relief operations. In the emergency relief operations stream, some studies focus on factors affecting disruption recovery. Eftekhar *et al.* (2021) studied post-disaster supply chain recovery in the presence of uncertainty in local supply, and they found that budget levels affect optimal decision-making compared to demand. Some other studies focus on flexible sourcing strategies for manufacturers to mitigate the risk of supply disruptions (Birge *et al.*, 2021; Goldschmidt *et al.*, 2021; Demirel *et al.*, 2018; Wang *et al.*, 2022; Berger *et al.*, 2023). Birge *et al.* (2021) examine sourcing strategies after disruptions and conclude that firms should choose multiple sourcing strategies, except when the firm's supplier default rate is low. Wang *et al.* (2022) focus on supply network configurations to synthesize supply and demand risks in the presence of unreliable suppliers. And some other scholars study post-disruption recovery by designing recovery models, mechanisms, and contracts (Chen *et al.*, 2015; Sawik, 2019; Azad and Hassini, 2019; Zhu *et al.*, 2021; Farahani *et al.*, 2021; Jain *et al.*, 2022; Cao *et al.*, 2022; Lai *et al.*, 2023; Aghajani *et al.*, 2023; Aldrighetti *et al.*, 2023). Farahani *et al.* (2021) investigate supply flexibility contracts by suppliers at risk of disruption and buyers with stochastic demand. Jain *et al.* (2021) developed a compound estimator of a supply chain's recovery rate, they demonstrate that procurement strategies play an important role in the rapid recovery of the supply chain. Aldrighetti *et al.* (2023) propose a new model for designing effective resilience investment portfolios in multi-level supply chains to determine the optimal combination of resilience investments.

Existing literature studies supply chain recovery after disruptions in terms of factors affecting capacity recovery, flexible sourcing policies, and the design of recovery models, mechanisms, and contracts. However, the study on promoting disruption recovery from the perspective of capacity enhancement is limited. This paper does this by giving incentives to stimulate capacity enhancement and thus promote disruption recovery. Specifically, we compare two common incentive policies, selecting product scarcity, and find that for different levels of scarcity, manufacturers exhibit different levels of capacity recovery under the two policies.

While the aforementioned firm-level strategies are crucial, their effectiveness is often constrained by the financial and operational limitations of individual enterprises, especially during large-scale systemic disruptions. When the costs of recovery exceed a firm's capacity, even the most sophisticated operational strategies may fail to ensure a timely rebound. This critical gap highlights the necessity for external support mechanisms that can alter the economic calculus of recovery investments. It is at this juncture that the role of government intervention becomes paramount.

The other related stream is government strategies for the recovery of supply chains. The existing literature examines the important role of government intervention and argues that well-designed regulatory policies can balance multiple competition objectives (Dou and Choi, 2024; Zhao *et al.*, 2023; Yu *et al.*, 2019). Then, some scholars examine the factors affecting government policies (Babich *et al.*, 2020; Hu *et al.*, 2022), as well as examine the impact of policies on a particular type of firm in the marketplace (Li *et al.*, 2023; Huang *et al.*, 2023; Dubey *et al.*, 2023; Xu *et al.*, 2022). Zhao *et al.* (2022) investigate a bank-supplier-retailer combinatorial game model considering four two-stage financing strategies.

Existing literature argues the important role of government regulation and analyzes how it affects firms' production operations; others study the tripartite game model of bank-supplier-retailer. However, there is little literature that synthesizes banks and governments in a comprehensive comparative analysis to study the role of government policies on supply chain disruption recovery. This paper aims to study the supply chain with the participation of banks and government, how manufacturers can enhance their capacity under different policies, and obtain the optimal policies applicable to different types of products. While the extant literature has made significant progress in understanding firm-level recovery strategies and government intervention policies in isolation, there is a critical lack of research that integrates these two streams to analyze their strategic interaction. Our study aims to bridge this gap by developing a unified game-theoretic framework that simultaneously considers the objectives and decisions of both the manufacturer and the government.

3. MODEL SETTINGS

This paper examines a two-stage decision-making process. In the following sections, we first establish a base model where the government's objective is to maximize its own utility, which incorporates tax revenues, supply chain stability considerations, and policy expenditure. This approach allows us to derive fundamental insights into the operational mechanics of each policy and the strategic interaction between the government and the manufacturer in a relatively parsimonious framework. Subsequently, in Section 6, we extend this base model to consider a broader objective: social welfare optimization. This extended model incorporates additional factors such as consumer surplus, providing a more comprehensive evaluation of policy performance from a societal perspective. This structured approach enables a gradual deepening of the analysis.

To analyze this problem, we employ a Stackelberg game framework where the government acts as the leader and the manufacturer as the follower. Importantly, we develop two separate but comparable sub-models to examine each policy in isolation, allowing for a clear comparative analysis rather than modeling their simultaneous interaction. The sequence of events common to both scenarios is: (1) the government sets the policy parameter; (2) the manufacturer determines the recovery effort level; (3) production and market outcomes are realized. The specific implementation mechanism is shown in Figure 1.

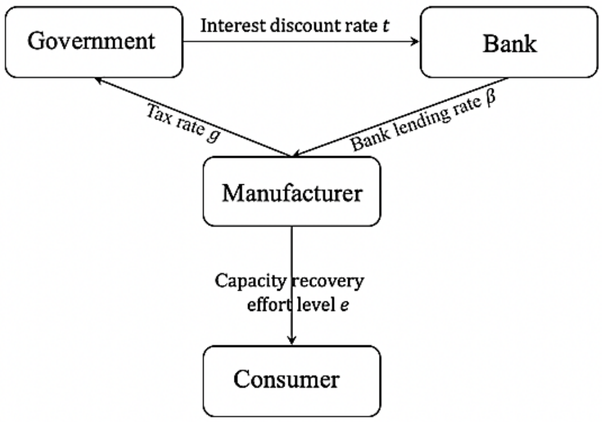

Figure 1. Fiscal interest-discount policy.

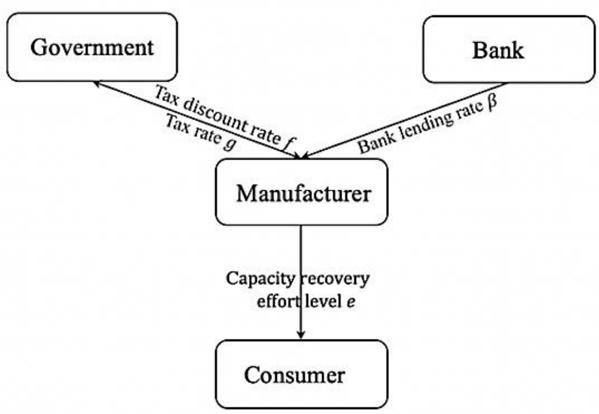

Figure 2. Tax preference policy.

3.1. General Model Setup and Assumptions

In this context, our study investigates manufacturers that have suffered supply disruptions and aims to find the optimal government incentive policy to stimulate capacity recovery and enhance supply chain resilience. We consider two incentive policies: the fiscal interest-discount policy and the tax-preference policy.

The manufacturer's goal is to maximize profit, and the government's goal is to maximize utility. To highlight the effectiveness of the two policies on post-disruption recovery, we assume that the manufacturer's initial capital after a disruption is zero (Huang *et al.*, 2020; Zhang and Zhang, 2022; Zhang *et al.*, 2023). Therefore, the credit needed from the bank includes both production cost and recovery cost.

We suppose that the recovery capacity is proportional to the manufacturer's capacity recovery effort level e, the recovery capacity is αe, the manufacturer's production cost is $c\alpha e$, where c is unit production cost. The credit amount is $c\alpha e + \frac{k}{2}e^2$, α is capacity recovery coefficient, e is capacity recovery effort level, k is capacity recovery cost coefficient (Ni *et al.*, 2010; Hu *et al.*, 2013). Additionally, we assume that the government collects tax on the manufacturer's sales with a tax rate of g. When the recovery capacity is larger than the demand, the manufacturer's sales d (d is the market demand). If the recovery capacity is less than the demand, the manufacturer's sales αe.

Therefore, the manufacturer's sale is $\min\{\alpha e, d\}$ (Zhao *et al.*, 2022), and the tax to be paid by the manufacturer is $gp\min\{\alpha e, d\}$ (p is unit sales price). Product scarcity is denoted as s $(0 < s < 1)$. Both the government and the manufacturer are risk-neutral and fully rational.

3.2. Scenario 1: Fiscal Interest-Discount Policy

In this scenario, the government subsidizes interest on credits to enterprises. This funding incentivizes manufacturers to increase capacity recovery effort to stabilize the market. For example, Jinshan District, Shanghai, provided interest discounts of up to 50% for enterprises affected by the pandemic (Jinshan District, 2022).

The decision-making sequence is as follows: first, the government sets the fiscal interest-discount rate t $(0 \leq t \leq 1)$, and then the manufacturer determines the recovery effort level e. The government bears the interest on the manufacturer's loan at lending interest rate $\beta (0 < \beta \leq 1)$. Without the policy, the interest is $\beta(c\alpha e + \frac{k}{2}e^2)$; with the policy, the interest becomes $\beta t\left(c\alpha e + \frac{k}{2}e^2\right)$. A lower t indicates a stronger policy.

The mechanism through which the fiscal interest-discount rate influences market stability can be precisely delineated as follows: The government's primary lever is the interest discount rate t. A lower t directly reduces the manufacturer's financial burden by decreasing the effective interest payment on loans from $\beta(c\alpha e + \frac{k}{2}e^2)$ to $\beta t\left(c\alpha e + \frac{k}{2}e^2\right)$. This reduction in financial cost lowers the marginal cost of investing in recovery effort e for the manufacturer. Consequently, the manufacturer is incentivized to choose a higher optimal recovery effort level e^*, as the cost-benefit tradeoff becomes more favorable. A higher e^* translates directly into a higher recovered capacity (αe^*). This increased capacity, in turn, enhances the manufacturer's ability to meet market demand d , thereby increasing the actual sales volume min $\{\alpha e^*, d\}$ and reducing shortages. The alleviation of the shortage is the quintessential manifestation of stabilizing the market. Thus, the government's determination of t critically shapes the manufacturer's recovery incentive and, ultimately, its capacity to restore market equilibrium.

3.3. Scenario 2: Tax-Preference Policy

In this scenario, the government provides tax preferences to support production recovery with discount rate f $(0 \leq f \leq 1)$. For example, South Korea and China stimulate the development of their semiconductor industry through tax preference (MOF, 2022). The decision-making sequence is similar to Scenario 1: first, the government sets the tax discount rate f, then the manufacturer determines the recovery effort level e $(e > 0)$. The actual tax paid by the manufacturer is $fgp\min\{\alpha e, d\}$, A higher f indicates a weaker tax preference.

The specific implementation mechanisms for both policies are shown in Figures 1 and 2, and all symbols are compiled in Table 1.

Table 1. Symbol and meaning of government's incentive policies.

Parameter	Description
d	Market Demand, $d > 0$
p	Unit Sales Price, $p > c$
c	Unit Production Cost, $c > 0$
α	Capacity Recovery Coefficient, $\alpha > 0$
k	Capacity Recovery Cost Coefficient, $k > 0$
s	Product Scarcity, $0 < s < 1$
β	Bank Lending Rate, $0 < \beta \leq 1$
g	Tax Rate, $0 < g \leq 1$
Decision Variables	**Description**
t	Interest Discount Rate, $0 \leq t \leq 1$
f	Tax Discount Rate, $0 \leq f \leq 1$
e	Capacity Recovery Effort Level, $e > 0$
Objective Function	**Description**
π_M^I	Manufacturer's Profit with the Government's Fiscal Interest Discount
π_G^I	Government Utility with Government's Fiscal Interest Discount
π_M^T	Manufacturer's Profit with Government's Tax Preference
π_G^T	Government Utility with the Government's Tax Preference

4. ANALYSIS OF INCENTIVE POLICY

4.1 Scenario 1: Fiscal interest-discount policy

The government sets the interest discount rate as t with utility maximization, and the manufacturer sets the capacity recovery effort level as e with the objective of profit maximization. When the government provides a fiscal interest-discount policy after production disruptions, the manufacturer's profit function is

$$\pi_M^I(e) = (1-g)p\min\{\alpha e, d\} - (1+\beta t)\left(c\alpha e + \frac{k}{2}e^2\right). \tag{1}$$

This function captures the essential tradeoff between post-tax revenue and loan-financed recovery costs. The first term, $(1-g)p\min\{\alpha e, d\}$, represents the after-tax sales revenue. The second term, $(1+\beta t)\left(c\alpha e + \frac{k}{2}e^2\right)$, constitutes the total loan-financed cost of recovery. The function of a government utility is

$$\pi_G^I(t) = (s+gp)\min\{\alpha e, d\} - (1-t)\beta\left(c\alpha e + \frac{k}{2}e^2\right). \tag{2}$$

The government's utility function, π_G^I, is designed to capture its dual role as a fiscal manager and a guardian of public welfare. It comprises three elements: (1) Tax Revenue ($gp \cdot \min\{\alpha e, d\}$), which represents the direct fiscal income; (2) Social Stability Benefit ($s \cdot \min\{\alpha e, d\}$), a term that monetizes the broader economic value of stabilizing the supply of a critical good, where the parameter 's' reflects the product's scarcity and societal importance; and (3) Policy Expenditure ($(1-t)\beta\left(c\alpha e + \frac{k}{2}e^2\right)$), which accounts for the direct cost of the intervention. This formulation allows us to model the government's tradeoff between stimulating recovery and managing public spending.

From the perspective of the manufacturer's decision on recovery, the relationship between the optimal recovery effort level e^{I*} and the government's interest discount rate t is summarized as follows. All proofs are attached in the Appendix.

Lemma 1. : Given t, when $0 \le t \le \frac{p\alpha^2 - dk - c\alpha^2 - gp\alpha^2}{(dk + c\alpha^2)\beta}$, the manufacturer's optimal capacity recovery effort level is $e^{I1*} = \frac{d}{\alpha}$, e^{I1*} is independent of t; When $\frac{p\alpha^2 - dk - c\alpha^2 - gp\alpha^2}{(dk + c\alpha^2)\beta} \le t \le 1$, $e^{I2*} = \frac{\alpha((1-g)p - c - ct\beta)}{k(1+t\beta)}$, e^{I2*} decreases as t increases.

Lemma 1 shows that the optimal capacity recovery effort level e is not influenced by the government's interest discount rate t. When t is relatively low, the manufacturer's optimal capacity recovery effort level e^{I1*} is independent of the government's interest discount rate t. The manufacturer's capacity recovery has already met the demand ($e^{I*}\alpha = d$), which is only influenced by the demand d and the capacity recovery coefficient α. Therefore, in this case, the government could set an upper subsidy limit in the fiscal interest-discount policy. Otherwise, it will cause overcapacity and waste of social resources. When t is relatively high, the change of manufacturer's optimal capacity recovery effort level e^{I2*} is related to the government's interest discount rate t. The manufacturer's capacity is lower than the market demand. After the production disruptions, the manufacturers incentivized by the interest discount and are willing to make efforts to restore production capacity. Therefore, in this case, the government could subsidize the firms under a fiscal interest-discount policy.

It should be noted that manufacturers are willing to take loans to recover production only if their profit is greater than zero after taking loans, that is, $\pi_M^I(e) = (1-g)p\min\{\alpha e, d\} - (1+\beta t)\left(c\alpha e + \frac{k}{2}e^2\right) > 0$. Under this constraint, we obtain Proposition 1.

Proposition 1. There is a threshold $\bar{t}$ and $0 \le \bar{t} \le 1$. When $t \le \bar{t}$ and $dk < \sqrt{2}c\alpha^2$, the manufacturer will make efforts for the production recovery; when $t > \bar{t}$, the manufacturer will not do that.

Proposition 1 shows that after severe production disruptions, if the production recovery cost is greater than the manufacturer's revenue, the manufacturer may not improve capacity and even stop production, which is not suitable for securing market supply. Therefore, to incentivize manufacturers to recover production, the government's fiscal interest discount needs to be greater than a certain threshold to play an incentive role.

From the perspective of the government's fiscal interest discount, on the premise of considering the manufacturer's recovery, the government sets the interest discount rate with the government utility function $\pi_G^I(t) = (s +$

$gp)min\{\alpha e, d\} - (1-t)\,\beta\,(c\alpha e + \frac{k}{2}e^2)$, which leads to proposition 2.

Proposition 2. If $t^{I0} < \frac{2p-4gp-2s+3p\beta-3gp\beta}{(p+gp+2s)\beta}$, the government profit is a strictly convex function, and there is a maximum value. The government's optimal interest discount rate is,

$$t^{I*} = \begin{cases} \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta} & if 0 \le t^{I0} \le \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta} \\ t^{I0} & if \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta} \le t^{I0} \le 1. \end{cases},$$

where $t^{I0} = ((1-g)p(p+gp+2s)\beta)/(-27H2+3\sqrt{3}H1)^{1/3} - \frac{1}{\beta} - \frac{(-9H2+\sqrt{3}H1)^{1/3}}{3^{2/3}c^2\beta^3}$.

This proposition shows that the government's optimal interest discount rate is not always related to the degree of product scarcity: when $t^{I*} = \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta}$, the government's optimal interest discount rate is independent of the degree of product scarcity s. In this condition, the manufacturer's capacity meets the demand ($\alpha e = d$) and there is no supply gap. Thus, the manufacturer's optimal recovery level is independent of the government's interest discount rate. Furthermore, the government will not consider the degree of product scarcity when formulating the fiscal interest-discount policy. However, when $t^{I*} = t^{I0}$, the government's optimal interest discount rate is related to the degree of product scarcity s and the manufacturer's capacity cannot meet the demand. We analyze the government's optimal interest discount rate and obtain Lemma 2.

Lemma 2. If $t^{I0} < \frac{2p-4gp-2s+3p\beta-3gp\beta}{(p+gp+2s)\beta}$ and $0 \le t^{I0} \le \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta}$, the government's optimal interest discount rate t^{I*} decreases as d increases and increases as α increases.

As market demand rises, the government collects more tax. Additionally, the government must raise the fiscal interest discount rate. Furthermore, with the increase in the capacity recovery coefficient, manufacturers are more likely to recover their capacity, allowing the government to lower the fiscal interest discount. We obtain the manufacturer's optimal profit with a fiscal interest discount in Proposition 3.

Proposition 3. When the government incentivizes manufacturers through a fiscal interest discount, and in the condition that $t^{I0} < \frac{2p-4gp-2s+3p\beta-3gp\beta}{(p+gp+2s)\beta}$ and $0 \le t^{I0} \le \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta}$, the manufacturer's optimal profit function is $\frac{d^2(1-g)kp}{2(dk+c\alpha^2)}$, which will decrease as α increases.

Proposition 3 demonstrates that the profit of the manufacturer decreases as the capacity recovery coefficient increases. The manufacturer's recovery capacity has met the demand, and the manufacturer's revenue remains unchanged. On the one hand, the manufacturer's capacity recovery effort level decreases as the capacity recovery coefficient increases, which also reduces the recovery cost and loan interest. On the other hand, the government's fiscal interest discount decreases as the capacity recovery coefficient increases, leading to a lower interest discount for the manufacturer. The negative impact of the decrease in the government's fiscal interest discount is greater than the positive impact of the decrease in the manufacturer's capacity recovery effort level, resulting in a decrease in the manufacturer's profit. We further examine the government's optimal utility with a sensitivity analysis in Proposition 4.

Proposition 4. If $t^{I0} < \frac{2p-4gp-2s+3p\beta-3gp\beta}{(p+gp+2s)\beta}$ and $0 \le t^{I0} \le \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta}$, the optimal utility of the government is $\pi_G^{*1}(t^*; e^*)$ which decreases as the interest rate β increases and increases as the price p increases.

Proposition 4 demonstrates that as the interest rate rises, the government's expenditure on interest discount also increases, leading to a decrease in government utility. Additionally, with the increase in price, the government's tax revenue also increases. This higher price serves as an incentive for manufacturers, resulting in a decrease in the government's fiscal interest discount and an increase in government utility. We further explain the connection between the manufacturer's optimal capacity recovery effort level and the relevant parameters.

Lemma 3. If $t^{I0} < \frac{2p-4gp-2s+3p\beta-3gp\beta}{(p+gp+2s)\beta}$ and $0 \le t^{I0} \le \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta}$, the manufacturer's optimal capacity recovery effort level is only related to d and α, it increases as the d increases and decreases as the α increases.

When the government sets a higher interest discount, the manufacturer's optimal capacity recovery effort level increases in response to the higher demand. Moreover, the demand incentive raises profit expectations, leading manufacturers to increase their recovery investment. Additionally, if manufacturers become more efficient in production recovery, their capacity recovery effort level may decrease. Therefore, the government should consider market demand and capacity recovery efficiency when formulating incentive policies.

After analyzing the scenarios where the government uses fiscal interest-discount policy to subsidize manufacturers with production disruptions, we have drawn the following conclusions: In the scenario of the government's fiscal interest discount, when the manufacturer's expected recovery capacity is less than the demand, the manufacturer's capacity recovery effort level, the government's fiscal interest discount, the manufacturer's profit, and the government utility all increase as the degree of product scarcity increases. In the scenario that the manufacturer expects the recovery capacity to meet the demand, as the degree of product scarcity increases, the government's interest discount and the manufacturer's capacity recovery effort level are unrelated to the degree of product scarcity. Therefore, the government should give enterprises a specific limit of interest discount to avoid wasting resources. Moreover, the government needs to consider the impact of market demand and capacity recovery efficiency. In addition to the degree of product scarcity, if the product price is high, manufacturers will be incentivized to increase the capacity recovery effort level, and the government can lower the interest discount according to the actual situation.

4.2 Scenario 2: Tax-preference policy

After the production disruptions, the government takes the tax preference as an emergency measure, and the manufacturer's profit function is

$$\pi_M^T(e) = (1-fg)p\min\{\alpha e, d\} - (1+\beta)\left(c\alpha e + \frac{k}{2}e^2\right). \tag{3}$$

The first term represents the sales revenue deducts taxes paid, and the second term represents production cost, recovery cost, and interest.

The government's utility function is

$$\pi_G^T(f) = (s + gfp)\min\{\alpha e, d\}. \tag{4}$$

This function means the government revenue plus the tax paid after actual tax preference. The government's tax preference expenditure is $(1-f)g\min\{\alpha e, d\}$.

From the perspective of the manufacturer's decision on recovery, the relationship between the manufacturer's optimal capacity recovery effort level e^* and the government's tax discount rate f is summarized by Proposition 5.

Proposition 5. Given f, when $0 \le f \le \frac{\alpha^2(p-c-c\beta)-dk(1+\beta)}{gp\alpha^2}$, the manufacturer's optimal capacity recovery effort level is $e^{T1*} = \frac{d}{\alpha}$ and e^{T1*} is independent of f; when $\frac{\alpha^2(p-c-c\beta)-dk(1+\beta)}{gp\alpha^2} \le f \le 1$ and $e^{T2*} = \frac{\alpha(p-fgp-c-c\beta)}{k(1+\beta)}$, e^{T2*} decreases as f increases.

Proposition 5 states that the optimal capacity recovery effort level is not influenced by the government's tax discount rate f. When f is low, the manufacturer's optimal capacity recovery effort level is independent of the government's tax discount rate f. When the government's tax discount increases, the manufacturer does not produce excess capacity because the manufacturer's recovery capacity has already met the market demand. Additionally, the manufacturer's capacity recovery effort level is only affected by the demand d and the capacity recovery coefficient α. An increase in market demand will increase the manufacturer's sales. At the same time, a decrease in the capacity recovery coefficient will decrease the manufacturer's recovery efficiency, which will encourage the manufacturer to increase the capacity recovery effort level to meet the demand. Therefore, when the government adopts the tax-preference policy, the upper limit of the tax discount can be set to achieve the optimal allocation of market resources, incentivize enterprises to recover capacity, and maximize utility.

If f is high, the manufacturer's optimal capacity recovery effort level decreases as the government's tax discount rate f increases. The government's tax preference is weak, and the manufacturer's recovery capacity cannot meet market demand. As the government tax discount rate decreases, the positive effect of the increased government tax preference offsets the negative effect of the manufacturer's increased capacity recovery effort level, the capacity recovery effort level that shows the manufacturer's willingness to increases. Therefore, when the scarcity products are disrupted, the government can subsidize enterprises through a tax-preference policy to incentivize production recovery.

From the perspective of the government's tax-discount policy, on the premise of considering the manufacturer's capacity recovery, the government sets the tax discount rate with the government utility function $\pi_G^T(f) = (s + gfp)\min\{\alpha e, d\}$, which leads to proposition 6. Proposition 6 summarizes the government's optimal decision under different situations.

Proposition 6. When the degree of product scarcity is low ($s \le \frac{2dk(1+\beta)}{\alpha^2} + c(1+\beta) - p$), the government's optimal tax discount rate is $f^{T2*} = \frac{p-c-s-c\beta}{2gp}$; when the degree of product scarcity is high ($s \ge \frac{2dk(1+\beta)}{\alpha^2} + c(1+\beta) - p$), the government's optimal tax discount rate is $f^{T1*} = \frac{((1-g)p-c)\alpha^2 - dk}{(dk+c\alpha^2)\beta}$.

We further analyze the relationship between the government's optimal tax discount rate and the relevant parameters.

Lemma 4. (a) When the degree of product scarcity is low ($s \le \frac{2dk(1+\beta)}{\alpha^2} + c(1+\beta) - p$), the government's optimal tax discount rate decreases with the increase of s. If $p < c + s + c\beta$, the rate increases with the increase of g; if $p > c + s + c\beta$, the rate decreases with the increase of g. (b) When the degree of product scarcity is high ($s \ge \frac{2dk(1+\beta)}{\alpha^2} + c(1+\beta) - p$), the government's optimal tax discount rate is independent of s. If $p < \frac{(dk+\alpha^2 c)(1+\beta)}{\alpha^2}$, the rate increases with the increase of g; if $p > \frac{(dk+\alpha^2 c)(1+\beta)}{\alpha^2}$, the rate decreases with the increase of g and increases with the increase of α. (c) The government's optimal tax discount rate continuously decreases as β increases and increases as p increases.

When the degree of product scarcity is low, the government's optimal tax discount rate decreases with the increase of product scarcity because, as the degree of product scarcity increases, the consequences of scarcity shortage are more severe, and the government needs to improve the tax preference to incentivize manufacturers to increase the capacity recovery effort level. Moreover, when the product price is low, the government's optimal tax discount rate increases as the tax rate increases because, as the tax rate increases, the government should improve the tax preference. However, if the price is low and the manufacturer does not pay much tax, the government will reduce the tax preference with the goal of utility maximization. On the contrary, if the price is high, the government's optimal tax discount rate decreases with the increase in the tax rate. Then, the tax paid by the manufacturer will increase. To subsidize the manufacturer, the government needs to improve the tax preference. Furthermore, when the degree of product scarcity is high, the government's optimal tax discount rate is independent of the degree of product scarcity. The manufacturer's recovery capacity has met the demand. Even if the government improves the tax preference, the manufacturer will not improve production. Moreover, the government does not need to consider the degree of product scarcity when formulating the tax-preference policy. The relationship between the government's tax discount rate and the tax rate is similar to the above. Finally, the government's optimal tax discount rate increases as the capacity recovery coefficient increases. An increase in the capacity recovery coefficient means that the efficiency of capacity recovery increases; thus, the government's tax preference can be reduced.

The government's optimal tax discount rate increases as the product price increases. As the product price increases, the manufacturer's sales increase, encouraging the manufacturer to increase the capacity recovery effort level even if the government's tax discount is lower. Moreover, the government's optimal tax discount rate decreases as the bank interest rate increases. As the interest rate increases, the tax paid by the manufacturer and the interest paid to the bank increase, and the cost increases. Therefore, to encourage the manufacturer to recover capacity and get loans from the bank, the government needs to improve the tax preference.

Lemma 5. Government's optimal utility increases with the increase of p and s. When the degree of product scarcity is low ($s \leq \frac{2dk(1+\beta)}{\alpha^2} + c(1+\beta) - p$), the government's optimal utility increases with the increase of β; when the degree of product scarcity is high ($s \geq \frac{2dk(1+\beta)}{\alpha^2} + c(1+\beta) - p$), the government's optimal utility decreases with the increase of β.

Government utility increases as price increases. When the price is higher, the government collects more taxes, and the government utility higher. The government utility is also influenced by interest rates and the degree of product scarcity. As the interest rate increases, the interest rate of manufacturers' loans increases, and the government needs to improve the tax preference to motivate manufacturers. At this time, when the degree of product scarcity is low, the manufacturer's capacity recovery effort level increases, and the government's tax revenue increases. The positive impact of increased tax revenue is greater than the negative impact of increased tax preference; thus, increasing government utility. When the degree of product scarcity is high, tax revenue remains the same, and the stronger tax preference will decrease government utility.

Lemma 6. When the degree of product scarcity is low ($s \leq \frac{2dk(1+\beta)}{\alpha^2} + c(1+\beta) - p$), the manufacturer's optimal profit increases with the increase of s. Furthermore, the manufacturer's optimal profit always increases with the increase of β.

When the degree of product scarcity is low, the manufacturer's optimal profit increases with the increase of product scarcity. At this time, as the degree of product scarcity increases, the government's tax preference improves, and the manufacturer's capacity recovery effort level increases, making the manufacturer's revenue increase and tax paid decrease; thus, the manufacturer's profit increases. Furthermore, the manufacturer's optimal profit increases with the increase in the interest rate. As the interest rate increases, on the one hand, the manufacturer's capacity recovery effort level and the recovery capacity decrease, hence the manufacturer's revenue decreases; on the other hand, the government's tax preference improves, and the manufacturer's tax paid decreases. The positive effects of increasing bank interest rates outweigh their negative effects, so the manufacturer's profit increases with increasing bank interest rates.

In summary, when the government applies a tax-preference policy, the main findings are as follows. First, when the manufacturer's expected recovery capacity is less than the demand, the manufacturer's capacity recovery effort level, the government's tax preference, the manufacturer's profit, and government utility all increase with the increase in product scarcity. Therefore, when the degree of product scarcity is higher, the government needs to adopt a stronger tax preference until the manufacturer's recovery capacity is qualified to meet the demand. Second, when the manufacturer's expected recovery can meet the demand, although the degree of product scarcity increases, the government's tax preference and the manufacturer's capacity recovery effort level are unrelated to the degree of product scarcity. Therefore, the government should apply tax preference to enterprises with certain limits to avoid wasting resources. Moreover, the government needs to consider the impact of market demand and capacity recovery efficiency. When the market demand is great, and the capacity recovery efficiency would be low, then the government needs to implement a stronger tax preference. Third, product price and bank interest rate also impact the decisions made by manufacturers and the government. When the product price is high, manufacturers will be motivated to increase the capacity recovery effort level, and the government can appropriately reduce the tax preference. When the bank interest rate is high, the government needs to improve the tax preference.

5. COMPARATIVE ANALYSIS OF INCENTIVE POLICY

We applied the game model under two incentive policies to find the optimal solution and complied with the optimal decision and profit in Table 2.

From the government's point of view, both policies of fiscal interest discount and tax preference cause reductions in fiscal revenue. Nevertheless, these policies are different for enterprises. (a) Fiscal interest discount is the government's expenditure, while tax preference helps reduce the expenditure of enterprises. (b) The application procedure for a fiscal interest discount is complicated, and the use of funds is generally regulated, while the tax relieved from the tax preference can be used freely. (c) Fiscal interest discount can be regarded as an "ex-ante incentive," while tax preference belongs to an "ex-post incentive."

Table 2. Optimal decision and profit of the government and manufacturers under two incentive policies.

Capacity Recovery Effort Level	Fiscal Interest-Discount Policy	$e^{I1*}=\frac{d}{\alpha}$. $e^{I2*}=\frac{c\alpha(-3^{2/3}H3+3H4(-9H2+\sqrt{3}H1)^{1/3}-3^{1/3}(-9H2+\sqrt{3}H1)^{2/3})}{3^{1/3}k(3^{1/3}H3+(-9H2+\sqrt{3}H1)^{2/3})}$.
	Tax-Preference Policy	$e^{T1*}=\frac{d}{\alpha}$. $e^{T2*}=\frac{\alpha(p+s-c(1+\beta))}{2k(1+\beta)}$.
The Manufacturer's Profit	Fiscal Interest-Discount Policy	$\pi_M^{I1*}(e^*;t^*)=\frac{d^2(1-g)kp}{2(dk+c\alpha^2)}$. $\pi_M^{I2*}(e^*;t^*)=-\frac{\alpha^2(3^{2/3}H3-3H4(-9H2+\sqrt{3}H1)^{1/3}+3^{1/3}(-9H2+\sqrt{3}H1)^{2/3})^2}{63^{1/3}k\beta^2(\sqrt{3}H1+cH4\beta^2(-9cH4(1+\beta)+(p+gp+2s)(-27H2+3\sqrt{3}H1)^{1/3}))}$.
	Tax-Preference Policy	$\pi_M^{T1*}(e^*;f^*)=\frac{d^2k(1+\beta)}{2\alpha^2}$. $\pi_M^{T2*}(e^*;f^*)=\frac{\alpha^2(p+s-c(1+\beta))^2}{8k(1+\beta)}$.
Government Utility	Fiscal Interest-Discount Policy	$\pi_G^{I1*}(t^*;e^*)=\frac{d(-d^2k^2(1+\beta)+dk\alpha^2(p+gp+2s-3c(1+\beta))+2c\alpha^4(p+s-c(1+\beta)))}{2\alpha^2(dk+c\alpha^2)}$. $\pi_G^{I2*}(t^*;e^*)=\frac{1}{23^{2/3}k\left(3^{1/3}H3+(-9H2+\sqrt{3}H1)^{2/3}\right)^2}\left(c\alpha^2\left(-3^{2/3}H3+3H4(-9H2+\sqrt{3}H1)^{1/3}-3^{1/3}(-9H2+\sqrt{3}H1)^{2/3}\right)\left(23^{1/3}(gp+s)\left(3^{1/3}H3+(-9H2+\sqrt{3}H1)^{2/3}\right)-c\beta\left(3^{2/3}H3+3H4(-9H2+\sqrt{3}H1)^{1/3}+3^{1/3}(-9H2+\sqrt{3}H1)^{2/3}\right)\left(1+\frac{1}{\beta}+\frac{(-9H2+\sqrt{3}H1)^{1/3}}{3^{2/3}c^2\beta^3}+\frac{(-1+g)p(p+gp+2s)\beta}{(-27H2+3\sqrt{3}H1)^{1/3}}\right)\right)\right)$.
	Tax-Preference Policy	$\pi_G^{T1*}(t^*;f^*)=\frac{\alpha^2(p+s-c-c\beta)-d(dk(1+\beta))}{\alpha^2}$. $\pi_G^{T2*}(f^*;e^*)=\frac{\alpha^2(p+s-c(1+\beta))^2}{4k(1+\beta)}$.

From the optimal solution, we can see when adopting the fiscal interest-discount policy, the increase in tax rate will reduce the manufacturer's profit and increase the government utility. When adopting the tax-preference policy, the manufacturer's profit and government utility is independent of the tax rate.

First, from the perspective of the government's policy, the conclusions can be drawn by comparing the manufacturer's optimal capacity recovery effort level under the two incentive policies.

Proposition 7. If $t^{I0} \le \frac{2p-4gp-2s+3p\beta-3gp\beta}{(p+gp+2s)\beta}$, $0 \le t^{I0} \le \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta}$, and $s \ge \frac{2dk(1+\beta)}{\alpha^2}+c(1+\beta)-p$, the manufacturer's capacity recovery effort level is constantly $e^{I1*}=e^{T1*}=\frac{d}{\alpha}$, unaffected by incentive policies.

When the degree of product scarcity is low, due to the computational difficulties, we made an overall comparison of the manufacturer's capacity recovery effort level via numerical simulation to demonstrate the difference between the two incentive policies. Although the actual data from enterprises is unavailable, we reasonably chose the parameter values according to relevant subsidy policies and literature. Assume $p=1, c=0.1, d=1, k=1, \beta=0.05, g=0.15$ and the domain satisfies $e>0$ and $0 \le t \le 1$. The effects of product scarcity and capacity recovery efficiency on the manufacturer's capacity recovery effort level shows in Figure 3.

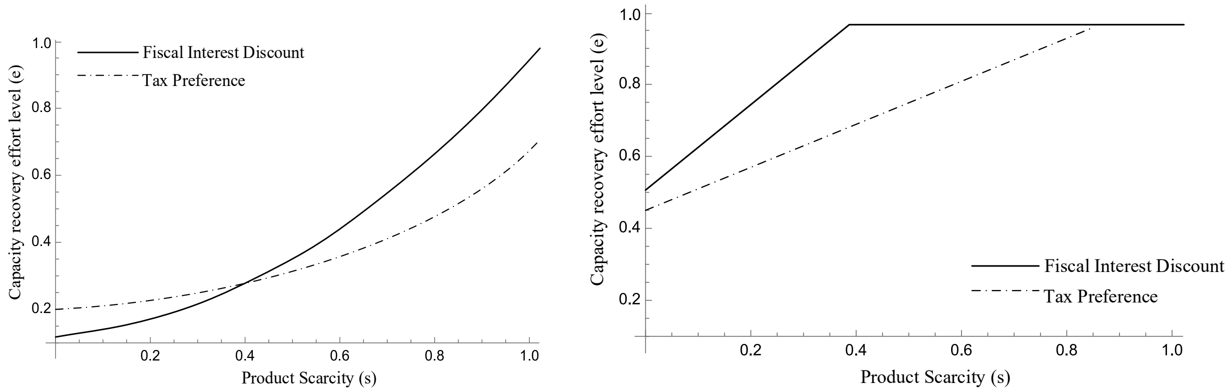

Figure 3 (a). $\alpha = 1$ Figure 3 (b). $\alpha = 1.3$

Figure 3. Effect of product scarcity on the capacity recover effort level ($\alpha = 1,\ \alpha = 1.3$).

From Figures 3(a) and 3 (b), it is noted that when the degree of product scarcity is low, the manufacturer's capacity recovery effort level is greater under the fiscal interest-discount policy. When the manufacturer's capacity recovery effort level continues to increase till the recovery capacity meets the demand, the manufacturer will stop recovering production. And then, the capacity recovery effort level is merely related to the demand and capacity recovery efficiency. In addition, when the degree of product scarcity and capacity recovery efficiency is high, the manufacturer's final capacity recovery effort level will be low. Therefore, when the degree of product scarcity is low, the fiscal interest-discount policy is better to recover more capacity. The conclusions can be drawn as follows by comparing the government's optimal utility under the two incentive policies.

Proposition 8. If $t^{I0} \le \frac{2p-4gp-2s+3p\beta-3gp\beta}{(p+gp+2s)\beta}$, $0 \le t^{I0} \le \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta}$, and $s \ge \frac{2dk(1+\beta)}{\alpha^2}+c(1+\beta)-p$, when $p < \frac{(dk+c\alpha^2)(1+\beta)}{(1-g)\alpha^2}$, the government utility of the fiscal-interest-discount-policy is higher than that of the tax-preference policy; when $p > \frac{(dk+c\alpha^2)(1+\beta)}{(1-g)\alpha^2}$, the government utility of the tax-preference policy is higher than that of the fiscal interest-discount policy.

The proposition shows that in the scenario of a high degree of product scarcity, the government utility will be higher under the fiscal interest-discount policy when the price is low. However, when the price is high, the government utility will be higher under the tax-preference policy.

When the degree of product scarcity is low, due to the computational difficulties, we made an overall comparison of the government's optimal utility via numerical simulation to show the difference in government utility under two incentive policies. The parameter values are as above. Figure 4 provides a numerical examination of the policy preferences outlined in Proposition 8, illustrating how product scarcity and capacity recovery efficiency jointly determine the superior policy.

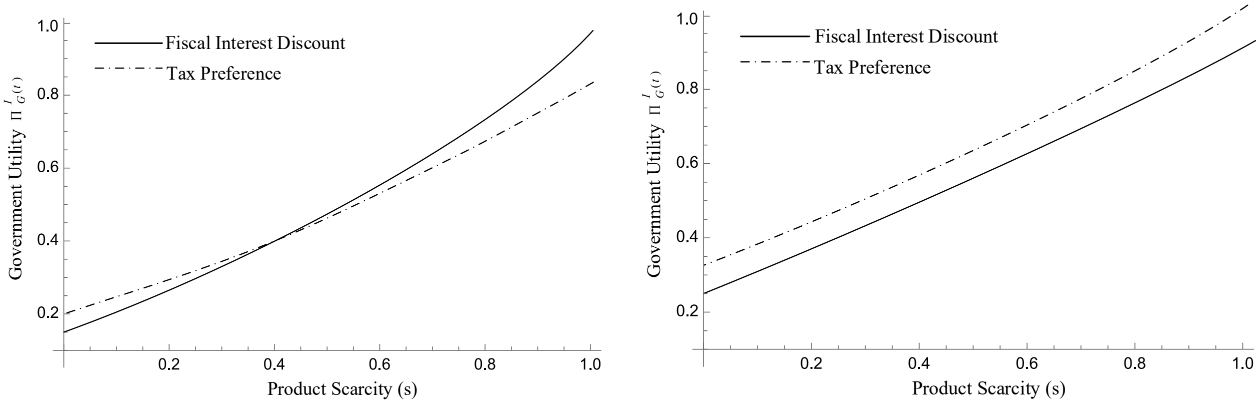

Figure 4 (a). $\alpha = 1$ Figure 4 (b). $\alpha = 1.3$

Figure 4. Effect of product scarcity on Government's incentive policies ($\alpha = 1,\ \alpha = 1.3$).

Consistent with the insights from Proposition 8, Figure 4 (a) and Figure 4 (b) illustrate that when the degree of product scarcity is low, the tax-preference policy leads to higher government utility. This aligns with the proposition's discussion of the price threshold; under low scarcity (which often correlates with competitive pricing dynamics), the tax-preference policy is preferred.

Furthermore, the figures extend the analytical findings by incorporating the role of capacity recovery efficiency. They show that when the degree of product scarcity is high and the capacity recovery efficiency is low, the government utility is higher under a fiscal interest-discount policy. This scenario corresponds to the condition in Proposition 7 where a high scarcity and a lower effective price (due to inefficient recovery) make the fiscal interest-discount policy more advantageous. Conversely, when the degree of product scarcity is high and the capacity recovery efficiency is also high, the government utility is higher under a tax-preference policy. This can be interpreted as a high-scarcity scenario combined with a high effective price (due to efficient recovery), crossing the critical threshold identified in Proposition 7 and shifting preference towards the tax instrument.

Thus, the numerical results not only validate the core tradeoff presented in Proposition 7 but also clarify that the manufacturer's capacity recovery efficiency is a critical factor modulating the price threshold effect, thereby having a greater impact on the incentive policy selection of the government.

Second, from the perspective of the manufacturer's decision, it can be concluded by comparing the manufacturer's optimal profit under the two incentive policies.

Proposition 9. If $t^{I0} \leq \frac{2p-4gp-2s+3p\beta-3gp\beta}{(p+gp+2s)\beta}$, $0 \leq t^{I0} \leq \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta}$, and $s \geq \frac{2dk(1+\beta)}{\alpha^2} + c(1+\beta) - p$, when $p < \frac{(dk+c\alpha^2)(1+\beta)}{(1-g)\alpha^2}$, the manufacturer's profit is higher under the tax-preference policy than that under the fiscal interest-discount policy; when $p > \frac{(dk+c\alpha^2)(1+\beta)}{(1-g)\alpha^2}$, the manufacturer's profit is higher under the fiscal-interest-discount policy than that under the tax-preference policy.

To complement the analytical findings of Proposition 9 and investigate the manufacturer's policy preferences across a broader spectrum of conditions, we numerically simulate its optimal profit. Figure 5 maps these preferences against product scarcity and the manufacturer's own capacity recovery efficiency, revealing how these key factors shape corporate financial incentives.

Strikingly, the manufacturer's perspective often diverges from the government's. As shown in Figures 5 (a) and 5 (b), the manufacturer's profit is maximized under the fiscal interest-discount policy in low-scarcity environments. This preference aligns with a fundamental corporate finance principle: the immediate reduction of financial costs (via interest discounts) provides more certain and direct value when market upside (demand) is limited.

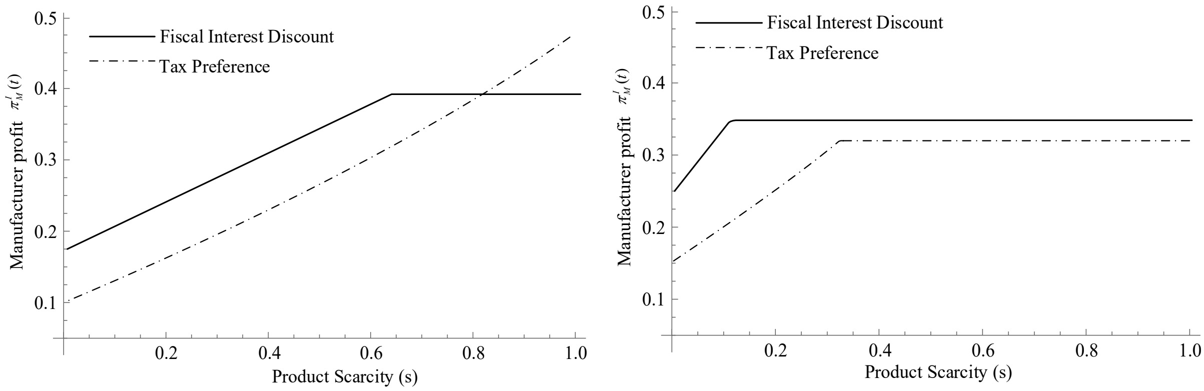

Figure 5 (a). $\alpha = 1$ Figure 5 (b). $\alpha = 1.3$

Figure 5. Effect of product scarcity on Manufacturer's incentive policies ($\alpha = 1,\ \alpha = 1.3$).

This divergence in objectives creates a principal-agent tension under high-scarcity scenarios. When scarcity is high, but the manufacturer's own recovery efficiency is low, it prefers the tax-preference policy. Here, the manufacturer appears to transfer more market risk to the government, as the tax benefit is contingent on future sales. However, when both scarcity and its recovery efficiency are high, the manufacturer's preference shifts back to the fiscal interest-discount policy. In this case, the firm is confident in its ability to capture the high-margin market and thus prioritizes minimizing the upfront capital cost required to scale production rapidly.

Therefore, Figure 5 does more than validate Proposition 8; it illuminates the underlying financial logic of the manufacturer. The choice between policies is a tradeoff between reducing upfront capital costs and sharing future market risk, a tradeoff that is critically modulated by the firm's internal capability (recovery efficiency) and external opportunity (product scarcity).

Synthesizing the preceding analyses, Figure 6 provides a comprehensive map of policy preferences, directly validating the contingent framework formalized in Proposition 10 and Table 3. This synthesis moves beyond individual perspectives to reveal the fundamental alignment and conflict between government and manufacturer objectives across the key contextual dimensions of recovery efficiency and product scarcity.

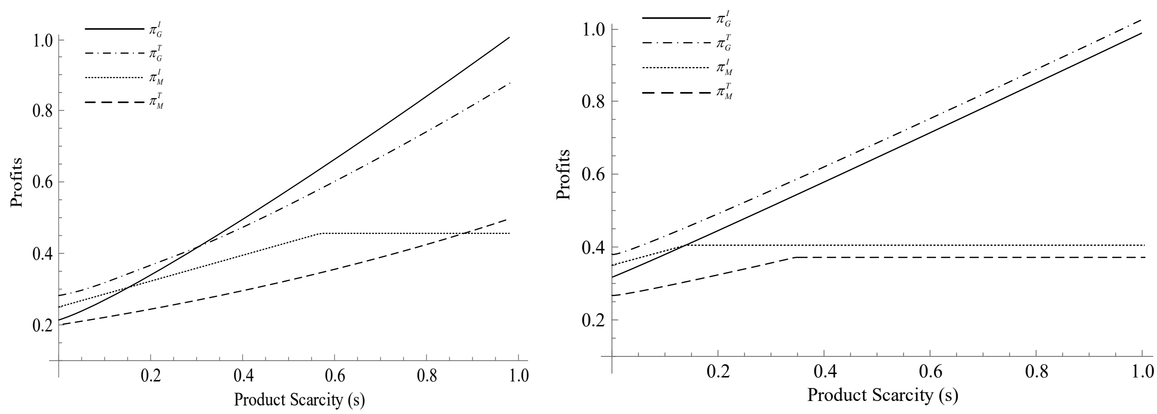

Figure 6 (a). $\alpha = 1$ Figure 6 (b). $\alpha = 1.3$

Figure 6. Effect of product scarcity on incentive policies for manufacturer and government ($\alpha = 1,\ \alpha = 1.3$).

The central insight from Figure 6 is the critical role of manufacturer recovery efficiency in determining goal congruence. When the manufacturer's capacity recovery efficiency is high (Figures 6 (a) and 6 (b), right-side regions), a stable, albeit conflicting, equilibrium emerges: the government consistently prefers the tax-preference policy while the manufacturer prefers the fiscal interest-discount policy. This universal divergence underscores a clear principal-agent problem under conditions of high competence; the government seeks to share in the success via taxes, while the firm seeks to minimize its cost of capital to fully exploit the opportunity.

The interplay between product scarcity, recovery efficiency, and policy incentives leads to complex and sometimes counterintuitive preferences for both the government and the manufacturer. The government's objective is to maximize social welfare (e.g., ensuring supply stability and efficient use of public funds), while the manufacturer seeks to maximize its own profit. The fiscal interest-discount policy directly alleviates the manufacturer's financial burden, which is particularly critical when recovery costs are high. In contrast, the tax-preference policy enhances profitability post-recovery, which is more valuable when market demand is assured. The following proposition synthesizes how these mechanisms manifest under different contingencies.

Proposition 10. (1) When a manufacturer has low capacity recovery efficiency, the optimal incentive policy for recovering from production interruptions is: if the product scarcity is low, the government would prefer the tax-preference policy and the manufacturer would prefer the fiscal interest-discount policy; If the product scarcity is high, the government would prefer the fiscal interest-discount policy and the manufacturer would prefer the tax-preference policy; if the product scarcity is moderate, both the government and the manufacturer would prefer the fiscal interest-discount policy. (2) When a manufacturer has high-capacity recovery efficiency, the government would prefer the tax-preference policy, and the manufacturer would prefer the fiscal interest-discount policy.

Table 3. The optimal incentive policy under different conditions.

Factors	**Variation**		**Government**	**Manufacturer**
Capacity Recovery Efficiency	Low	(s) Low	Tax-Preference Policy	Fiscal Interest-Discount Policy
		(s) High	Fiscal Interest-Discount Policy	Tax-Preference Policy
		(s) Moderate	Fiscal Interest-Discount Policy	Fiscal Interest-Discount Policy
	High		Tax-Preference Policy	Fiscal Interest-Discount Policy

Our findings yield a contingent decision-making framework for selecting post-disruption incentive policies, which is a central contribution of this study. The framework, as summarized in Proposition 10, reveals that the alignment of interests between the government and the manufacturer depends critically on the specific context:

For low-efficiency manufacturers, policy preference is highly sensitive to product scarcity. Notably, a goal conflict arises for both high- and low-scarcity products, where the government and the manufacturer prefer different policies. This highlights the need for negotiation or side payments in these scenarios. Only under moderate scarcity do their interests align in favor of the fiscal interest-discount policy.

For high-efficiency manufacturers, the preference structure is simpler but still marked by a fundamental goal conflict, with the government favoring the tax-preference policy and the manufacturer preferring the fiscal interest-discount policy, regardless of scarcity level.

Therefore, Figure 6 and Proposition 10 together deliver the pivotal managerial insight: achieving supply chain recovery is not merely about selecting policies, but about managing goal alignment. Policymakers must first diagnose the recovery efficiency of the industry. Inefficient sectors require a nuanced, scarcity-dependent strategy that anticipates and manages preference conflicts, whereas efficient sectors present a predictable, though persistent, need for balancing conflicting goals between public and private entities.

6. EXTENSION

Building upon the base model established in Section 3, which focused on the government's utility maximization, we now extend our analysis to evaluate the policies under a social welfare maximization objective. As noted in the abstract and introduced in Section 3, we express the government's expected utility through taxation, but policymakers should consider the benefits of the entire society. Therefore, we introduce social welfare and attempt to guide the government and manufacturers in terms of social welfare under the fiscal interest-discount policy and tax preference policy. As commonly done in the literature (e.g., Singh & Vives, 1984; Chen & Su, 2019), we assume the utility of a consumer purchasing a post-disruption recovery product under the two policies is,

$$CS = dq - \frac{1}{2}bq^2 - pq. \tag{5}$$

where d is the market demand, p is the unit sales price, q is the optimal quantity, $q = min\{\alpha e, d\}$, b denotes the rate of change of marginal utility, and is normalized to one for brevity. Then we have the total social welfare (see Singh & Vives, 1984; Chen & Su, 2019),

$$SW = \pi_M + \pi_G + CS. \tag{6}$$

Calculating social welfare under the two policies separately, and we use numerical analysis to represent changes in social welfare under two policies. Assume $p = 1, c = 0.1, d = 1, k = 1, \alpha = 1, \beta = 0.05, g = 0.15$ and the domain satisfies $e \geq 0$, $0 \leq f \leq 1$ and $0 \leq t \leq 1$. The effects of product scarcity on social welfare are shown in Figure 7.

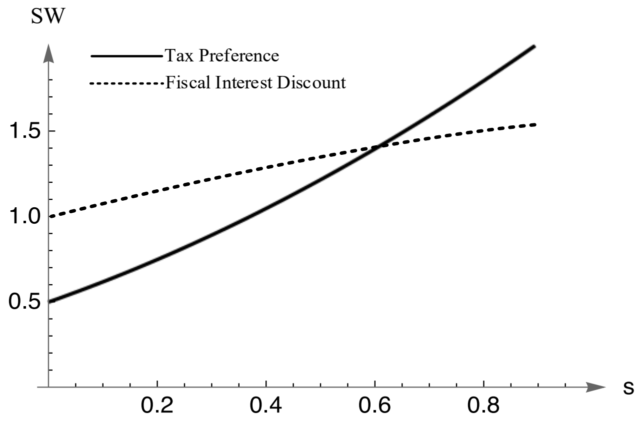

Figure 7. Effect of product scarcity on incentive policies for social welfare.

We are surprised to find that the optimal policy choice made from the perspective of social welfare is consistent with the government's optimal choice. Specifically, if the manufacturer's capacity recovery is inefficient, then for firms with a high level of scarcity, the optimal policy for the government is fiscal interest-discount policy, and the optimal policy for the manufacturer is a tax-preference policy. From the perspective of social welfare, when product scarcity is low, fiscal interest-discount policy is the optimal choice. Additionally, for products with a very low level of scarcity, the optimal policy for the government is a tax-preference policy, and the fiscal interest-discount policy is also the optimal choice for both manufacturer and social welfare under this premise.

7. CONCLUSIONS AND FUTURE RESEARCH

This study investigates the roles of fiscal interest-discount and tax-preference policies in enhancing supply chain resilience, considering product scarcity and manufacturer capacity recovery efficiency. Our game-theoretic analysis yields a contingent decision-making framework, which reveals complex alignments and conflicts between government and manufacturer preferences.

7.1 Main Findings and Managerial Implications

Our key finding is that the manufacturer's capacity recovery efficiency is a pivotal factor determining goal congruence. For manufacturers with low recovery efficiency, policy preferences are highly sensitive to product scarcity, leading to divergent interests in both high- and low-scarcity scenarios. Alignment occurs only under moderate scarcity, where both parties prefer the fiscal interest-discount policy. In contrast, for manufacturers with high recovery efficiency, a stable conflict emerges: the government consistently prefers the tax-preference policy, while the manufacturer favors the fiscal interest-discount policy, regardless of scarcity levels.

These findings offer clear managerial and policy guidance: policymakers should first diagnose the recovery efficiency of a critical industry. For inefficient sectors, policies must be tailored to product scarcity to manage inherent goal conflicts. For efficient sectors, the focus should be on balancing the persistent preference divergence through negotiation or side payments. Managers should recognize that investing in recovery capabilities not only improves operational performance but also expands their firm's policy preference set and strengthens its bargaining position.

7.2 Theoretical Contribution

Our findings necessitate a paradigm shift in traditional supply chain risk management (SCRM) frameworks. Conventional frameworks, often focused on firm-level mitigation of isolated risks, are inadequate for systemic disruptions. This study contributes by explicitly incorporating proactive government intervention as a critical layer of defense.

The contingent policy selection model developed herein offers a significant improvement. It moves beyond generic recommendations by providing a decision-aid tool for policymakers and managers to design interventions based on specific disruption contexts (e.g., product scarcity) and firm-specific capabilities (e.g., recovery efficiency). This addresses a key gap in existing frameworks, which often lack the granularity to guide public-private partnerships during crises. Therefore, the primary improvement to SCRM frameworks is the integration of a government incentive dimension. This enhanced framework acknowledges that building resilience is a shared objective, underscoring the need for dynamic, data-driven policies to safeguard global supply chains.

7.3 Future Research

This research contributes to theory by integrating proactive government intervention into supply chain risk management frameworks. Our contingent model provides a decision-aid tool that moves beyond generic recommendations, offering granularity for public-private partnerships during crises.

Future research could extend this work by: (1) introducing competition among manufacturers, (2) modeling asymmetric information to design screening mechanisms, and (3) incorporating stochastic demand to test policy robustness under dual uncertainty.

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APPENDIX

Proof of Lemma 1.

The problem of optimizing e is equivalent to $\max\{\mathrm{MI1}, \mathrm{MI2}\}$, where

$\mathrm{MI1}: \max\pi_M^{I1}(e;t) = (1-g)pd - (1+\beta t)\left(c\alpha e + \frac{k}{2}e^2\right)$ s.t. $e > \frac{d}{\alpha}$, $e \geq 0$;

$\mathrm{MI2}: \max\pi_M^{I2}(e;t) = (1-g)p\alpha e - (1+\beta t)\left(c\alpha e + \frac{k}{2}e^2\right)$ s.t. $0 \leq e \leq \frac{d}{\alpha}$.

Since $\frac{\partial \pi_M^{I1}(e;t)}{\partial_e} = -(1+t\beta)(ek+c\alpha)$ is less than zero,$\max\pi_M^{I1}(e;t)$ decreases as the capacity recovery effort level e varies, hence the optimal capacity recovery effort level is $e^{I*} = \frac{d}{\alpha}$ $(\frac{d}{\alpha} > 0)$. Since $\frac{\partial \pi_M^{I2}(e;t)}{\partial_e} = (1-g)p\alpha - (ek + c\alpha)(1+t\beta)$ and $\frac{\partial^2 \pi_M^{I2}(e;t)}{\partial_{e^2}} = -k(1+t\beta)$ is less than zero, hence $\pi_M^{I2}(e;t)$ is a convex function about e with a maximum value. Let the first derivative equal to zero, the maximum value be$\frac{\alpha((1-g)p-c-ct\beta)}{k+kt\beta}$, therefore, the optimal capacity recovery effort level is,

$$e^{I*} = \begin{cases} \frac{d}{\alpha} & if\ \frac{d}{\alpha} \leq \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta} \\ \frac{\alpha((1-g)p-c-ct\beta)}{k(1+t\beta)} & if\ \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta} \leq \frac{d}{\alpha} \end{cases},$$

i.e., $$e^{I*} = \begin{cases} \frac{d}{\alpha} & if\ 0 \leq t \leq \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta} \\ \frac{\alpha((1-g)p-c-ct\beta)}{k(1+t\beta)} & if\ \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta} \leq t \leq 1 \end{cases}.$$

Compare MI1 and MI2, the optimal capacity recovery effort level is,

$$e^{I*} = \begin{cases} \frac{d}{\alpha} & if\ 0 \leq t \leq \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta} \\ \frac{\alpha((1-g)p-c-ct\beta)}{k(1+t\beta)} & if\ \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta} \leq t \leq 1 \end{cases}.$$

Let $e^{I0} = \frac{\alpha((1-g)p-c-ct\beta)}{k(1+t\beta)}$, since $\frac{\partial e^{I0}}{\partial_t} = \frac{(g-1)p\alpha\beta}{k(1+t\beta)^2}$, $0 < g < 1$, and the first derivative is less than zero. Hence e^{I0} decreases as the interest discount rate t varies.

Proof of Proposition 1.

From the manufacturer's profit function $\pi_M(e) = (1-g)p\min\{\alpha e,\ d\} - (1+\beta t)\left(c\alpha e + \frac{k}{2}e^2\right)$, it can be seen that the manufacturer's profit function decreases as t increases, so that when $\frac{p\alpha^2 - dk - c\alpha^2 - gp\alpha^2}{(dk+c\alpha^2)\beta} \le t \le 1$ ($e^* = \frac{\alpha((1-g)p - c - ct\beta)}{k(1+t\beta)}$), the manufacturer's profit is positive and $\pi_M(e^*;t) = \frac{2\alpha^4(c+(-1+g)p+ct\beta)^2 - d^2(k+kt\beta)^2}{2\alpha^2(k+kt\beta)} > 0$. Since $2\alpha^2(k+kt\beta)$ is greater than zero, it is only necessary to judge that $2\alpha^4(c+(-1+g)p+ct\beta)^2 - d^2(k+kt\beta)^2$ is greater than zero, simplify the expression as $(\sqrt{2}\alpha^2(c+(-1+g)p+ct\beta))^2 - (dk(1+t\beta))^2$, and compare the value between $\left|\sqrt{2}\alpha^2(c+(-1+g)p+ct\beta)\right|$, and $|dk(1+t\beta)|$, then it can be obtained that $t < \bar{t} = \frac{\sqrt{2}(c+(-1+g)p)\alpha^2 - dk}{(dk-\sqrt{2}c\alpha^2)\beta}$. Since $\bar{t} \ge \frac{((1-g)p-c)\alpha^2 - dk}{(dk+c\alpha^2)\beta}$, $dk < \sqrt{2}c\alpha^2$.

Proof of Proposition 2.

To optimize $\max\{\mathrm{GI1}, \mathrm{GI2}\}$, proved by Theorem 1, substitute into $e^*(t)$, then

$$\mathrm{GI1}: \max\pi_G^{I1}(t;e^*) = (s+gp)d - (1-t)\beta\left(cd + \frac{d^2k}{2\alpha^2}\right) \quad \text{s.t. } 0 \le t \le \frac{((1-g)p-c)\alpha^2 - dk}{(dk+c\alpha^2)\beta}$$

$$\mathrm{GI2}: \max\pi_G^{I2}(t;e^*) = \frac{\alpha^2(p-c-gp-ct\beta)(2s+2st\beta+gp(2+\beta+t\beta)-(1-t)\beta(c+p+ct\beta))}{2k(1+t\beta)^2}.$$

$$\text{s.t. } \frac{((1-g)p-c)\alpha^2 - dk}{(dk+c\alpha^2)\beta} \le t \le 1$$

The equations are defined as follows for simplification.

$\mathrm{H1} = \sqrt{-c^6(-1+g)^3p^3\beta^{12}((p+gp+2s)^3 - 27c^2(-1+g)p(1+\beta)^2)}$, $\mathrm{H2} = c^4(-1+g)^2p^2\beta^6(1+\beta)$, $\mathrm{H3} = c^2(-1+g)p(p+gp+2s)\beta^4$, $\mathrm{H4} = c(-1+g)p\beta^2$。

Since $\frac{\partial \max\pi_G^{I1}(t;e^*)}{\partial_t} = \left(cd + \frac{d^2k}{2\alpha^2}\right)\beta$ is greater than zero, $\max\pi_G^{I1}(t;e^*)$ increases as the government's interest discount rate t varies. Hence, the optimal interest discount rate takes the boundary value, i.e., $t^{I1*} = \frac{((1-g)p-c)\alpha^2 - dk}{(dk+c\alpha^2)\beta}$.

Since the first derivative $\frac{\partial\pi_G^{I2}(t;e^*)}{\partial_t} = -\frac{\alpha^2\beta(c^2(1+t\beta)^3 - (-1+g)p(2(s+st\beta)+p(-1+3g+(-2+t+g(2+t))\beta)))}{2k(1+t\beta)^3}$ and the second derivative $\frac{\partial^2\pi_G^{I2}(t;e^*)}{\partial_{t^2}} = -\frac{(-1+g)p\alpha^2\beta^2(2(s+st\beta)+p(-2+(-3+t)\beta+g(4+(3+t)\beta)))}{k(1+t\beta)^4}$,

Let the first derivative $\frac{\partial\pi_G^{I2}(t;e^*)}{\partial_t} = 0$, then

$$t^{I0} = ((1-g)p(p+gp+2s)\beta)/(-27H2 + 3\sqrt{3}H1)^{1/3} - \frac{1}{\beta} - \frac{(-9H2+\sqrt{3}H1)^{1/3}}{3^{2/3}c^2\beta^3}.$$

When the second derivative is less than zero, i.e.,$\frac{\partial^2\pi_G^{I2}(t;e^*)}{\partial_{t^2}} < 0$, and $t^{I0} < \frac{2p-4gp-2s+3p\beta-3gp\beta}{(p+gp+2s)\beta}$, there is a maximum value of the government's interest discount rate. Therefore,

$$t^{I*} = \begin{cases} \frac{((1-g)p-c)\alpha^2 - dk}{(dk+c\alpha^2)\beta} & if\ 0 \le t^{I0} \le \frac{((1-g)p-c)\alpha^2 - dk}{(dk+c\alpha^2)\beta} \\ t^{I0} & if\ \frac{((1-g)p-c)\alpha^2 - dk}{(dk+c\alpha^2)\beta} \le t^{I0} \le 1 \end{cases}.$$

Compare GI1 and GI2, the government's optimal interest discount rate can be obtained as

$$t^{I*} = \begin{cases} \frac{((1-g)p-c)\alpha^2 - dk}{(dk+c\alpha^2)\beta} & if\ 0 \le t^{I0} \le \frac{((1-g)p-c)\alpha^2 - dk}{(dk+c\alpha^2)\beta} \\ t^{I0} & if\ \frac{((1-g)p-c)\alpha^2 - dk}{(dk+c\alpha^2)\beta} \le t^{I0} \le 1 \end{cases}.$$

Proof of Lemma 2.

$\frac{\partial t^{I*}}{\partial_d} = \frac{(-1+g)kp\alpha^2}{(dk+c\alpha^2)^2\beta}$, $\frac{\partial t^{I*}}{\partial_k} = \frac{(-1+g)kp\alpha^2}{(dk+c\alpha^2)^2\beta}$, since $(-1+g) < 0$, it is easy to prove $\frac{\partial t^{I*}}{\partial_d} < 0$, $\frac{\partial t^{I*}}{\partial_k} < 0$, $\frac{\partial t^{I*}}{\partial_\alpha} = \frac{2d(1-g)kp\alpha}{(dk+c\alpha^2)^2\beta}$; and since $(1-g) > 0$, it is easy to prove $\frac{\partial t^{I*}}{\partial_\alpha} > 0$.

Proof of Proposition 3.

By substituting proposition 2 into the manufacturer's profit function, the manufacturer's optimal profit can be obtained as follows.

$$\begin{cases} \pi_M^{I1*} = \frac{d^2(1-g)kp}{2(dk+c\alpha^2)} & if\ 0 \le t^{I0} \le \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta} \\ \pi_M^{I2*} & if\ \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta} \le t^{I0} \le 1 \end{cases},$$

where, $\pi_M^{I2*} = \frac{\alpha^2(3H4(-9H2+\sqrt{3}H1)^{1/3}-3^{2/3}H3-3^{1/3}(-9H2+\sqrt{3}H1)^{2/3})^2}{63^{1/3}k\beta^2(\sqrt{3}H1+cH4\beta^2(-9cH4(1+\beta)+(p+gp+2s)(-27H2+3\sqrt{3}H1)^{1/3}))}$.

Therefore, $\frac{\partial \pi_M^{I1*}(e^*;t^*)}{\partial_\alpha} = \frac{cd^2(-1+g)kp\alpha}{(dk+c\alpha^2)^2}$, $(-1+g) < 0$, it is easy to prove $\frac{\partial \pi_M^{I1*}(e^*;t^*)}{\partial_\alpha} < 0$.

Proof of Proposition 4.

By substituting proposition 2 into the government utility function, the optimal utility of the government is obtained as follows.

$$\begin{cases} \pi_G^{I1*}(t^{I*}; e^{I*}) & if\ 0 \le t^{I0} \le \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta} \\ \pi_G^{I2*}(t^{I*}; e^{I*}) & if\ \frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta} \le t^{I0} \le 1 \end{cases},$$

where $\pi_G^{I1*}(t^{I*}; e^{I*}) = \frac{d(-d^2k^2(1+\beta)+dk\alpha^2(p+gp+2s-3c(1+\beta))+2c\alpha^4(p+s-c(1+\beta)))}{2\alpha^2(dk+c\alpha^2)}$,

$$\pi_G^{I2*}(t^{I*}; e^{I*}) = \frac{1}{23^{2/3}k\left(3^{1/3}H3+(-9H2+\sqrt{3}H1)^{2/3}\right)^2}\left(c\alpha^2\left(-3^{2/3}H3 + 3H4(-9H2+\sqrt{3}H1)^{1/3} - 3^{1/3}(-9H2+\sqrt{3}H1)^{2/3}\right)\left(23^{1/3}(gp+s)\left(3^{1/3}H3+(-9H2+\sqrt{3}H1)^{2/3}\right) - c\beta\left(3^{2/3}H3+3H4(-9H2+\sqrt{3}H1)^{1/3} + 3^{1/3}(-9H2+\sqrt{3}H1)^{2/3}\right)\left(1+\frac{1}{\beta}+\frac{(-9H2+\sqrt{3}H1)^{1/3}}{3^{2/3}c^2\beta^3}+\frac{(-1+g)p(p+gp+2s)\beta}{(-27H2+3\sqrt{3}H1)^{1/3}}\right)\right)\right)$$

. Therefore, $\frac{\partial \pi_G^{I1*}(t^{I*};e^{I*})}{\partial_\beta} = -cd - \frac{d^2k}{2\alpha^2}$ and it is easy to prove $\frac{\partial \pi_G^{I1*}(t^{I*};e^{I*})}{\partial_\beta} < 0$; and since $\frac{\partial \pi_G^{I1*}(t^{I*};e^{I*})}{\partial_p} = \frac{d(d(1+g)k\alpha^2+2c\alpha^4)}{2\alpha^2(dk+c\alpha^2)}$, it is easy to prove $\frac{\partial \pi_G^{I1*}(t^{I*};e^{I*})}{\partial_\beta} > 0$.

Proof of Lemma 3.

$\frac{\partial e^{I1*}}{\partial_d} > 0$ and $\frac{\partial e^{I1*}}{\partial_\alpha} < 0$.

Proof of Proposition 5.

The problem of optimizing e is equivalent to $\max\{MI1, MI2\}$, where

MT1: $\max \pi_M^{T1}(e;t) = (1-fg)pd - (1+\beta)(c\alpha e + \frac{k}{2}e^2)$; s.t. $e > \frac{d}{\alpha}, e \ge 0$

MT2: $\max \pi_M^{T2}(e;t) = (1-fg)p\alpha e - (1+\beta)(c\alpha e + \frac{k}{2}e^2)$; s.t. $0 \le e \le \frac{d}{\alpha}$

Since $\frac{\partial \pi_M^{T1}(e;t)}{\partial_e} = -(ek + c\alpha)(1+\beta)$ is less than zero, $\max\pi_M^{T1}(e;t)$ decreases as the capacity recovery effort level e varies, and the optimal capacity recovery effort level is $e^{T*} = \frac{d}{\alpha}$ $(\frac{d}{\alpha} > 0)$. Since $\frac{\partial \pi_M^{T2}(e;t)}{\partial_e} = (1-fg)p\alpha - (ek + c\alpha)(1+\beta)$ and $\frac{\partial^2 \pi_M^{T2}(e;t)}{\partial_{e^2}} = -k(1+\beta)$ is less than zero, $\pi_M^{T2}(e;t)$ is a convex function about e with a maximum value. Let the first derivative equal to zero, the maximum value is $\frac{\alpha^2(p-c-c\beta)-dk(1+\beta)}{gp\alpha^2}$. Therefore, the optimal capacity recovery effort level is,

$$e^{T*} = \begin{cases} \frac{d}{\alpha} & if\ \frac{d}{\alpha} \le \frac{\alpha(p-c-fgp-c\beta)}{k(1+\beta)} \\ \frac{\alpha(p-c-fgp-c\beta)}{k(1+\beta)} & if\ \frac{\alpha(p-c-fgp-c\beta)}{k(1+\beta)} \le \frac{d}{\alpha} \end{cases},$$

i.e.,

$$e^{T*} = \begin{cases} \frac{d}{\alpha} & if\ 0 \le f \le \frac{\alpha^2(p-c-c\beta)-dk(1+\beta)}{gp\alpha^2} \\ \frac{\alpha(p-c-fgp-c\beta)}{k(1+\beta)} & if\ \frac{\alpha^2(p-c-c\beta)-dk(1+\beta)}{gp\alpha^2} \le f \le 1 \end{cases}.$$

Compare MT1 and MT2, the optimal capacity recovery effort level can be obtained as

$$e^{T*} = \begin{cases} \frac{d}{\alpha} & if\ 0 \le f \le \frac{\alpha^2(p-c-c\beta)-dk(1+\beta)}{gp\alpha^2} \\ \frac{\alpha(p-c-fgp-c\beta)}{k(1+\beta)} & if\ \frac{\alpha^2(p-c-c\beta)-dk(1+\beta)}{gp\alpha^2} \le f \le 1 \end{cases}.$$

Let $e^{T0} = \frac{\alpha(p-c-fgp-c\beta)}{k(1+\beta)}$, $\frac{\partial_{e^{T0}}}{\partial_f} = -\frac{gp}{k(1+\beta)}$, $0 < f < 1$, and the first derivative is less than zero. Thus e^{T0} decreases as the tax discount rate f varies.

Proof of Proposition 6.

Optimizing $\max\{GT1, GT2\}$, then substituting the above optimal value into $e^*(f)$, we can obtain

GT1: $max\pi_G^{T1}(f;e^*) = (s + gfp)d$

$$\text{s.t. } 0 \le f \le \frac{\alpha^2(p-c-c\beta) - dk(1+\beta)}{gp\alpha^2}$$

$$\text{GT2: } \max\pi_G^{T2}(f;e^*) = \frac{(fgp+s)\alpha^2(p-c-fgp-c\beta)}{k(1+\beta)}$$

$$\text{s.t. } \frac{\alpha(p-c-fgp-c\beta)}{k(1+\beta)} \le f \le 1$$

Since $\frac{\partial \pi_G^{T1}(f;e^*)}{\partial_f} = dgp$ is greater than zero, hence $\pi_G^{T1}(f;e^*)$ increases as the tax discount rate f varies. The optimal tax discount rate takes the boundary value, i.e., $f^{T1*} = \frac{\alpha^2(p-c-c\beta)-dk(1+\beta)}{gp\alpha^2}$. Since the first derivative $\frac{\partial \pi_G^{T2}(f;e^*)}{\partial_f} = \frac{\alpha^2(p-c-c\beta)-dk(1+\beta)}{gp\alpha^2}$ and the second derivative $\frac{\partial^2 \pi_G^{T2}(f;e^*)}{\partial_{f^2}} = -\frac{2g^2p^2\alpha^2}{k(1+\beta)}$ is constantly less than zero, $\pi_G^T(f)$ is a concave function about f, and there is a local maximum in the government utility. Let the first derivative $\frac{\partial \pi_G^{T2}(f;e^*)}{\partial_f} = 0$, $f^{T0} = \frac{p-c-s-c\beta}{2gp}$.

Since $f \ge \frac{\alpha^2(p-c-c\beta)-dk(1+\beta)}{gp\alpha^2}$,

$$f^{T*}=\begin{cases}\frac{\alpha^2(p-c-c\beta)-dk(1+\beta)}{gp\alpha^2} & if\ 0\le f^{T0}\le\frac{\alpha^2(p-c-c\beta)-dk(1+\beta)}{gp\alpha^2}\\ \frac{p-c-s-c\beta}{2gp} & if\ \frac{\alpha^2(p-c-c\beta)-dk(1+\beta)}{gp\alpha^2}\le f^{T0}\le 1\end{cases}.$$

Compare GT1 and GT2, then the government's optimal tax discount rate can be obtained as,

$$f^{T*}=\begin{cases}\frac{\alpha^2(p-c-c\beta)-dk(1+\beta)}{gp\alpha^2} & if 0\le f^{T0}\le\frac{\alpha^2(p-c-c\beta)-dk(1+\beta)}{gp\alpha^2}\\ \frac{p-c-s-c\beta}{2gp} & if\frac{\alpha^2(p-c-c\beta)-dk(1+\beta)}{gp\alpha^2}\le f^{T0}\le 1\end{cases},$$

i.e., $f^{T*}=\begin{cases}\frac{\alpha^2(p-c-c\beta)-dk(1+\beta)}{gp\alpha^2} & ifs\ge\frac{2dk(1+\beta)}{\alpha^2}+c(1+\beta)-p\\ \frac{p-c-s-c\beta}{2gp} & ifs\le\frac{2dk(1+\beta)}{\alpha^2}+c(1+\beta)-p\end{cases}.$

Proof of Lemma 4.

When $s\le\frac{2dk(1+\beta)}{\alpha^2}+c(1+\beta)-p$, it is easy to prove $\frac{\partial f^{T*}}{\partial_s}<0$, $\frac{\partial f^{T*}}{\partial_p}>0$, $\frac{\partial f^{T*}}{\partial_\beta}<0$, and $\frac{\partial f^{T*}}{\partial_g}=\frac{c+s+c\beta-p}{2g^2p}$. When $p<c+s+c\beta$, $\frac{\partial f^{T*}}{\partial_g}>0$; when $p>c+s+c\beta$, $\frac{\partial f^{T*}}{\partial_g}<0$. When $s\ge\frac{2dk(1+\beta)}{\alpha^2}+c(1+\beta)-p$, it is easy to prove $\frac{\partial f^{T*}}{\partial_k}<0$, $\frac{\partial f^{T*}}{\partial_\beta}<0$, $\frac{\partial f^{T*}}{\partial_\alpha}>0$, $\frac{\partial f^{T*}}{\partial_p}>0$, and $\frac{\partial f^{T*}}{\partial_g}=\frac{dk(1+\beta)-\alpha^2(p-c-c\beta)}{g^2p\alpha^2}$. When $p<\frac{(dk+\alpha^2c)(1+\beta)}{\alpha^2}$, $\frac{\partial f^{T*}}{\partial_g}>0$; when $p>\frac{(dk+\alpha^2c)(1+\beta)}{\alpha^2}$, $\frac{\partial f^{T*}}{\partial_g}<0$.

Proof of Lemma 5.

Substituting the above optimal value into the government utility function to obtain the government's optimal utility, we can get $\begin{cases}\pi_G^{T1*}(f^*;e^*)=\frac{d(\alpha^2(p+s-c-c\beta)-dk(1+\beta))}{\alpha^2} & ifs\ge\frac{2dk(1+\beta)}{\alpha^2}+c(1+\beta)-p\\ \pi_G^{T2*}(f^*;e^*)=\frac{\alpha^2(p+s-c(1+\beta))^2}{4k(1+\beta)} & ifs\le\frac{2dk(1+\beta)}{\alpha^2}+c(1+\beta)-p\end{cases}.$

When $s\le\frac{2dk(1+\beta)}{\alpha^2}+c(1+\beta)-p$, it is easy to prove $\frac{\partial\pi_G^{T2*}(f^*;e^*)}{\partial_p}>0$, $\frac{\partial\pi_G^{T2*}(f^*;e^*)}{\partial_\beta}>0$, and $\frac{\partial\pi_G^{T2*}(f^*;e^*)}{\partial_s}>0$; when $s\ge\frac{2dk(1+\beta)}{\alpha^2}+c(1+\beta)-p$, it is easy to prove $\frac{\partial\pi_G^{T1*}(f^*;e^*)}{\partial_\beta}<0$ and $\frac{\partial\pi_G^{T1*}(f^*;e^*)}{\partial_p}>0$.

Proof of Lemma 6.

Substitute the above optimal value into the manufacturer's profit function to obtain the manufacturer's optimal utility.

$$\begin{cases}\pi_M^{T1*}(e^*;f^*)=\frac{d^2k(1+\beta)}{2\alpha^2} & if\mathrm{s}\ge\frac{2dk(1+\beta)}{\alpha^2}+c(1+\beta)-p\\ \pi_M^{T2*}(e^*;f^*)=\frac{\alpha^2(p+s-c(1+\beta))^2}{8k(1+\beta)} & ifs\le\frac{2dk(1+\beta)}{\alpha^2}+c(1+\beta)-p\end{cases}.$$

When$s\le\frac{2dk(1+\beta)}{\alpha^2}+c(1+\beta)-p$, it is easy to prove $\frac{\partial_{\pi_M^{T2*}}}{\partial_s}>0$ and $\frac{\partial_{\pi_M^{T2*}}}{\partial_\beta}=\frac{\alpha^2(c-p-s+c\beta)(c+p+s+c\beta)}{8k(1+\beta)^2}$. When $\beta>\frac{p-c+s}{c}$, i.e., $s<\beta c-p+c$, then $\frac{\partial_{\pi_M^{T2*}}}{\partial_\beta}>0$. Moreover, since $s\le\frac{2dk(1+\beta)}{\alpha^2}+c(1+\beta)-p$, $\frac{\partial_{\pi_M^{T2*}}}{\partial_\beta}$ is constantly greater than zero. When $s\ge\frac{2dk(1+\beta)}{\alpha^2}+c(1+\beta)-p$, it is easy to prove $\frac{\partial_{\pi_M^{T1*}}}{\partial_\beta}>0$.

Proof of Proposition 7.

When $t^{I0}\le\frac{2p-4gp-2s+3p\beta-3gp\beta}{(p+gp+2s)\beta}$, $0\le t^{I0}\le\frac{((1-g)p-c)\alpha^2-dk}{(dk+c\alpha^2)\beta}$, and $\mathrm{s}\ge\frac{2dk(1+\beta)}{\alpha^2}+c(1+\beta)-p$, $\pi_G^{I1*}(t^*;e^*)=$

$\frac{d(-d^2k^2(1+\beta)+dk\alpha^2(p+gp+2s-3c(1+\beta))+2c\alpha^4(p+s-c(1+\beta)))}{2\alpha^2(dk+c\alpha^2)}$ and $\pi_G^{T1*}(t^*;f^*) = \frac{d\left(\alpha^2(p+s-c-c\beta)-dk(1+\beta)\right)}{\alpha^2}$. Let $\pi_G^{I1*}(t^*;e^*) > \pi_G^{T1*}(t^*;f^*)$, it is easy to obtain $p < \frac{(dk+c\alpha^2)(1+\beta)}{(1-g)\alpha^2}$; let $\pi_G^{I1*}(t^*;e^*) < \pi_G^{T1*}(t^*;f^*)$, it is easy to obtain $p > \frac{(dk+c\alpha^2)(1+\beta)}{(1-g)\alpha^2}$.

Proof of Proposition 8.

$\pi_M^{I1*}(e^*;t^*) = \frac{d^2(1-g)kp}{2(dk+c\alpha^2)}$ and $\pi_M^{T1*}(e^*;f^*) = \frac{d^2k(1+\beta)}{2\alpha^2}$. Let $\pi_M^{I1*}(e^*;t^*) > \pi_M^{T1*}(e^*;f^*)$, it is easy to obtain $p > \frac{(dk+c\alpha^2)(1+\beta)}{(1-g)\alpha^2}$; let $\pi_G^{I1*}(t^*;e^*) < \pi_G^{T1*}(t^*;f^*)$, it is easy to obtain $p < \frac{(dk+c\alpha^2)(1+\beta)}{(1-g)\alpha^2}$.